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STATISTICAL MECHANICS OF ONE-DIMENSIONAL S=1/2 ANISOTROPIC XY MODEL IN TRANSVERSE FIELD WITH DZYALOSHINSKII-MORIYA INTERACTION

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Abstract

The thermodynamical functions, static spin correlation functions, transverse dynamical spin correlation function and connected with it transverse dynamical susceptibility have been obtained for 1D s=1/2 anisotropic XY model in transverse field with Dzyaloshinskii-Moriya interaction using Jordan-Wigner transformation. It has been shown that Dzyaloshinskii-Moriya interaction essentially influences the calculated quantities.

1 Introduction

In 1961 E.Lieb, T.Schultz and D.Mattis in ref. [1] pointed out one type of the exactly solvable models of statistical mechanics so called 1D $s = \frac{1}{2}$ XY model. Rewriting the Hamiltonian of such chain

$$H = \sum_j \left[(1+\gamma)s_j^x s_{j+1}^x + (1-\gamma)s_j^y s_{j+1}^y\right], \ -1 \leq \gamma \leq 1, \tag{1.1}$$

$$\left[s_j^\alpha, s_m^\beta\right] = \imath\delta_{jm}s_m^\gamma, \ \ \alpha,\beta,\gamma = x,y,z + cyclic\, permutations \tag{1.2}$$

with the help of the raising and lowering operators $s_j^\pm \equiv s_j^x \pm \imath s_j^y$ in the form

$$H = \tfrac{1}{2}\sum_j \left[(\gamma s_j^+ s_{j+1}^+ + s_j^+ s_{j+1}^-) + h.c.\right], \tag{1.3}$$

they noted that the difficulty of diagonalization of the obtained quadratic in operators s^+, s^- form (1.3) is connected with the commutation rules that these operators do satisfy, namely, $\left[s_j^-, s_m^+\right] = \delta_{jm}(1-2s_m^+ s_m^-)$. Really, they are similar to Fermi-type commutation rules for operators at the same site and to Bose-type commutation rules for operators attached to different sites

$$\begin{gathered}\left\{s_j^-, s_j^+\right\} = 1, \ (s_j^+)^2 = (s_j^-)^2 = 0; \\ \left[s_j^-, s_m^+\right] = \left[s_j^+, s_m^+\right] = \left[s_j^-, s_m^-\right] = 0, \ j \neq m.\end{gathered} \tag{1.4}$$

That is why one should perform at first Jordan-Wigner transformation (see, besides ref. [1], also refs.[2-4])

$$\begin{aligned} c_1 &= s_1^-, \quad c_j = s_j^- P_{j-1} = P_{j-1} s_j^-, \quad j = 2, \ldots, N, \\ c_1^+ &= s_1^+, \quad c_j^+ = s_j^+ P_{j-1} = P_{j-1} s_j^+, \quad j = 2, \ldots, N, \end{aligned} \tag{1.5}$$

where Jordan-Wigner factor is denoted by $P_j \equiv \prod_{n=1}^{j}(-2s_n^z)$. The introduced operators really obey Fermi commutation rules. From (1.5) it follows that

$$c_j^+ c_j = s_j^+ P_{j-1}^2 s_j^- = s_j^+ s_j^-, \quad c_j c_j^+ = s_j^- s_j^+, \quad c_j^+ c_j^+ = s_j^+ s_j^+, \quad c_j c_j = s_j^- s_j^-, \tag{1.6}$$

since $P_j^2 = \prod_{n=1}^{j}(-2s_n^z)^2 = \prod_{n=1}^{j} 4(s_n^z)^2 = 1$, and the commutation rules at the same site remain of Fermi-type. Consider then a product of c-operators at different sites

$$c_n^+ c_m = s_n^+ \prod_{p=1}^{n-1}(-2s_p^z) \prod_{j=1}^{m-1}(-2s_j^z) s_m^- = s_n^+ \prod_{j=n}^{m-1}(-2s_j^z) s_m^-, \tag{1.7}$$

putting here for definiteness $n < m$. Since $s_j^{\pm}(-2s_j^z) = \pm s_j^{\pm}$ and $(-2s_j^z)s_j^{\pm} = \mp s_j^{\pm}$, and consequently

$$c_m c_n^+ = s_m^- \prod_{j=n}^{m-1}(-2s_j^z) s_n^+ = -s_n^+ \prod_{j=n}^{m-1}(-2s_j^z) s_m^-, \tag{1.8}$$

one gets $c_n^+ c_m = -c_m c_n^+$. Similarly one finds that $c_n^+ c_m^+ = -c_m^+ c_n^+$, $c_n c_m = -c_m c_n$. Thus the introduced in (1.5) operators are Fermi-type operators

$$\left\{c_j, c_l^+\right\} = \delta_{jl}, \ \left\{c_j^+, c_l^+\right\} = \{c_j, c_l\} = 0. \tag{1.9}$$

Since $P_j^2 = 1$, $P_j = \exp(\pm i\pi \sum_{n=1}^{j} s_n^+ s_n^-)$ (because $\exp\left[\pm i\pi \sum_{n=1}^{j}(\frac{1}{2} + s_n^z)\right] = \prod_{n=1}^{j}(-2s_n^z)$), $s_j^+ s_j^- = c_j^+ c_j$, it is easy to write the inverse to (1.5) transformation

$$\begin{aligned} s_1^- &= c_1, \quad s_j^- = c_j \exp(\pm i\pi \sum_{n=1}^{j-1} c_n^+ c_n) = \exp(\pm i\pi \sum_{n=1}^{j-1} c_n^+ c_n) c_j, \quad j = 2, \ldots, N, \\ s_1^+ &= c_1^+, \quad s_j^+ = c_j^+ \exp(\pm i\pi \sum_{n=1}^{j-1} c_n^+ c_n) = \exp(\pm i\pi \sum_{n=1}^{j-1} c_n^+ c_n) c_j^+, \quad j = 2, \ldots, N. \end{aligned} \tag{1.10}$$

Returning to the Hamiltonian (1.3) one notes that the products of two Pauli operators at neighbouring sites transform into products of Fermi operators:

$$\begin{aligned} c_j^+ c_{j+1}^+ &= s_j^+(-2s_j^z)s_{j+1}^+ &= s_j^+ s_{j+1}^+, \\ c_j^+ c_{j+1} &= s_j^+(-2s_j^z)s_{j+1}^- &= s_j^+ s_{j+1}^-, \\ c_j c_{j+1}^+ &= s_j^-(-2s_j^z)s_{j+1}^+ &= -s_j^- s_{j+1}^+, \\ c_j c_{j+1} &= s_j^-(-2s_j^z)s_{j+1}^- &= -s_j^- s_{j+1}^-. \end{aligned} \tag{1.11}$$

Usually bearing in mind the study of thermodynamical properties of the system that requires the performance of thermodynamical limit $N \to \infty$, the periodic boundary conditions are implied

$$s^{\alpha}_{N+1} \equiv s^{\alpha}_{1}, \quad \alpha = x, y, z. \tag{1.12}$$

In accordance with this in (1.3) appear products of the following form

$$\begin{gathered} s^{+}_{N} s^{+}_{N+1} = s^{+}_{N} s^{+}_{1} = s^{+}_{1} s^{+}_{N} = c^{+}_{1} c^{+}_{N} \prod_{p=1}^{N-1} (-2 s^{z}_{p}) = c^{+}_{1} c^{+}_{N} P, \quad P \equiv P_{N}, \\ s^{+}_{N} s^{-}_{N+1} = c_{1} c^{+}_{N} P, \quad s^{-}_{N} s^{+}_{N+1} = -c^{+}_{1} c_{N} P, \quad s^{-}_{N} s^{-}_{N+1} = -c_{1} c_{N} P. \end{gathered} \tag{1.13}$$

Gathering the similar terms one finds the following representation for the ring:

$$H = H^{-} + BP^{+} = H^{+} P^{+} + H^{-} P^{-}. \tag{1.14}$$

Here

$$H^{\pm} \equiv \tfrac{1}{2} \sum_{j=1}^{N} \left[(\gamma c^{+}_{j} c^{+}_{j+1} + c^{+}_{j} c_{j+1}) + h.c. \right], \tag{1.15}$$

the difference between H^{+} and H^{-} is only in the implied boundary conditions: for H^{+} they are antiperiodic

$$c^{+}_{j} = -c^{+}_{j+N}, \; c_{j} = -c_{j+N}, \tag{1.16}$$

and for H^{-} they are periodic

$$c^{+}_{j} = c^{+}_{j+N}, \; c_{j} = c_{j+N}; \tag{1.17}$$

$B \equiv H^{+} - H^{-} = - \left[\left(\gamma c^{+}_{N} c^{+}_{1} + c^{+}_{N} c_{1} \right) + h.c. \right]$ is the boundary term; $P^{\pm} \equiv (1 \pm P)/2$ are the orthogonal projectors ($P^{+} + P^{-} = 1$, $(P^{\pm})^{2} = P^{\pm}$, $P^{\pm} P^{\mp} = 0$), besides this $[H^{\pm}, P] = [H^{\pm}, P^{\pm}] = 0$. For the open chain with free ends (then in the sum in (1.1) the summation index spans values $j = 1, \ldots, N-1$) the Hamiltonian after fermionization has the form

$$H = \tfrac{1}{2} \sum_{j=1}^{N-1} \left[(\gamma c^{+}_{j} c^{+}_{j+1} + c^{+}_{j} c_{j+1}) + h.c. \right]. \tag{1.18}$$

Formulae (1.14), (1.15) or (1.18) realize the reformulation of the initial Hamiltonian (1.1) in terms of fermions. They are the starting point for further study of statistical mechanics of models like (1.1). Besides it appears [5,6] that for calculation of free energy

$$f \equiv -\frac{1}{\beta} \lim_{N \to \infty} \left[\frac{1}{N} \, Sp \exp(-\beta H) \right] \tag{1.19}$$

or static spin correlation functions

$$< s^{\alpha_1}_{j_1} \ldots s^{\alpha_n}_{j_n} > \equiv \lim_{N \to \infty} \left\{ Sp \left[\exp(-\beta H) s^{\alpha_1}_{j_1} \ldots s^{\alpha_n}_{j_n} \right] / Sp \exp(-\beta H) \right\} \tag{1.20}$$

the boundary term may be omitted and hence one has to consider a system of free fermions. It is more difficult to calculate the dynamical correlation functions. Really,

$$\begin{aligned} s_j^z(t) &\equiv \exp(\imath Ht)s_j^z\exp(-\imath Ht) = \\ &= \exp(\imath H^+t)s_j^z\exp(-\imath H^+t)P^+ + \exp(\imath H^-t)s_j^z\exp(-\imath H^-t)P^- = \\ &= P^+\exp(\imath H^+t)s_j^z\exp(-\imath H^+t) + P^-\exp(\imath H^-t)s_j^z\exp(-\imath H^-t) \end{aligned} \tag{1.21}$$

(owing to the following relation that is valid for arbitrary function of $H = H^+P^+ + H^-P^-$: $f(H) \equiv \sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}(H^+P^+ + H^-P^-)^n = \sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!} \times \times[(H^+)^nP^+ + (H^-)^nP^-] = f(H^+)P^+ + f(H^-)P^- = P^+f(H^+) + P^-f(H^-)$) in contrast to

$$\begin{aligned} &s_j^{x,y}(t) = \\ &= \exp(\imath H^+t)\, s_j^{x,y}\exp(-\imath H^-t)P^- + \exp(\imath H^-t)\, s_j^{x,y}\exp(-\imath H^+t)P^+ = \\ &= P^+\exp(\imath H^+t)\, s_j^{x,y}\exp(-\imath H^-t) + P^-\exp(\imath H^-t)\, s_j^{x,y}\exp(-\imath H^+t). \end{aligned} \tag{1.22}$$

In accordance with (1.21) the pair transverse correlation function in the thermodynamical limit can be written as

$$< s_j^z(t)s_{j+n}^z >= \frac{Sp\left[\exp(-\beta H^-)\exp(\imath H^-t)s_j^z\exp(-\imath H^-t)s_{j+n}^z\right]}{Sp\exp\left(-\beta H^-\right)} \tag{1.23}$$

and hence may be calculated with c-cyclic Hamiltonian. Whereas the pair longitudinal correlation function in accordance with (1.22) in the thermodynamical limit can be written as

$$< s_j^x(t)s_{j+n}^x >= \frac{Sp\left[\exp(-\beta H^-)\exp(\imath H^-t)s_j^x\exp(-\imath H^-t)O^-(t)s_{j+n}^x\right]}{Sp\exp\left(-\beta H^-\right)} \tag{1.24}$$

where $O^-(t) \equiv \exp(\imath H^-t)\exp\left(-\imath(H^- + B)t\right)$. The calculation with c-cyclic Hamiltonian that neglects the boundary term B yields the approximate result that, in particular, is incorrect in the limit of Ising model ($\gamma = 1$) (see [7], for instance). It is interesting to note that the calculation of the four-spin correlation function in the thermodynamical limit involves only c-cyclic Hamiltonian

$$< s_{j_1}^x(t)s_{j_2}^x(t)s_{j_3}^x s_{j_4}^x >= \frac{Sp\left[\exp(-\beta H^-)\exp(\imath H^-t)s_{j_1}^x s_{j_2}^x\exp(-\imath H^-t)s_{j_3}^x s_{j_4}^x\right]}{Sp\exp(-\beta H^-)}. \tag{1.25}$$

Thus here as in the case (1.23) one comes to calculation of the dynamical correlation functions of the system of non-interacting fermions (see [8]). The calculation of the pair longitudinal correlation function, in spite of a great number of papers dealing with this problem, remains an open problem of statistical mechanics of 1D $s = \frac{1}{2}$ XY models. Among other interesting and principal questions of the theory of 1D $s = \frac{1}{2}$ XY models one may mention the investigation of nonequilibrium properties of such models (see [9], for example) and the examination of the properties of disordered versions of such models (see [10], for example).

It is necessary to stress the essential features of the present consideration:

- the dimension of space D=1;
- the value of spin $s = \frac{1}{2}$;
- interactions occur only between neighbouring spins (otherwise the Hamiltonian will contain the terms that are the products of more than two Fermi operators);
- only x and y components of spins interact and the field that may be included should be transverse (the interaction of z components, for instance, leads to the appearance of the terms that are the products of four Fermi operators in the Hamiltonian).

In connection with this it is easy to point out the model that has more general than in (1.1) form of interspin interaction, and that still allows the described consideration. Really, considering the additional terms in the Hamiltonian that have form $\sum_j \left(J^{xy}s_j^x s_{j+1}^y + J^{yx}s_j^y s_{j+1}^x\right)$ one notes that after fermionization they do not change the form of the Hamiltonian (1.14), (1.15) or (1.18), and lead only to changes in the values of constants. The Hamiltonian of the generalized 1D $s = \frac{1}{2}$ anizotropic XY model in transverse field that as a matter of fact will be studied in the present paper is given by

$$H = \Omega \sum_j s_j^z + \sum_j \left(J^{xx}s_j^x s_{j+1}^x + J^{xy}s_j^x s_{j+1}^y + J^{yx}s_j^y s_{j+1}^x + J^{yy}s_j^y s_{j+1}^y\right). \tag{1.26}$$

Before starting the examination of this model it is worthwhile to mention its possible physical application [11]. For this purpose let's perform the transformation of rotation around axis z over an angle α

$$\begin{array}{lll} \tilde{s}_j^x = s_j^x \cos\alpha + s_j^y \sin\alpha, & \tilde{s}_j^y = -s_j^x \sin\alpha + s_j^y \cos\alpha, & \tilde{s}_j^z = s_j^z; \\ s_j^x = \tilde{s}_j^x \cos\alpha - \tilde{s}_j^y \sin\alpha, & s_j^y = \tilde{s}_j^x \sin\alpha + \tilde{s}_j^y \cos\alpha, & s_j^z = \tilde{s}_j^z. \end{array} \tag{1.27}$$

Then rewritting at first new terms in sum in the Hamiltonian (26) in the form

$$\frac{J^{xy}+J^{yx}}{2}\left(s_j^x s_{j+1}^y + s_j^y s_{j+1}^x\right) + \frac{J^{xy}-J^{yx}}{2}\left(s_j^x s_{j+1}^y - s_j^y s_{j+1}^x\right), \tag{1.28}$$

taking into account that the terms $\left(s_j^x s_{j+1}^y - s_j^y s_{j+1}^x\right)$ are invariant under transformation (1.27) and that

$$\begin{aligned} &J^{xx}s_j^x s_{j+1}^x + \tfrac{J^{xy}+J^{yx}}{2}\left(s_j^x s_{j+1}^y + s_j^y s_{j+1}^x\right) + J^{yy}s_j^y s_{j+1}^y = \\ &= \left(J^{xx}\cos^2\alpha + \tfrac{J^{xy}+J^{yx}}{2}\sin 2\alpha + J^{yy}\sin^2\alpha\right)\tilde{s}_j^x\tilde{s}_{j+1}^x + \\ &+ \left(\tfrac{J^{yy}-J^{xx}}{2}\sin 2\alpha + \tfrac{J^{xy}+J^{yx}}{2}\cos 2\alpha\right)\left(\tilde{s}_j^x\tilde{s}_{j+1}^y + \tilde{s}_j^y\tilde{s}_{j+1}^x\right) + \\ &+ \left(J^{xx}\sin^2\alpha - \tfrac{J^{xy}+J^{yx}}{2}\sin 2\alpha + J^{yy}\cos^2\alpha\right)\tilde{s}_j^y\tilde{s}_{j+1}^y, \end{aligned} \tag{1.29}$$

and choosing the parameter of transformation α from the condition $(J^{xy}+$ $+J^{yx})\cos 2\alpha - (J^{xx} - J^{yy})\sin 2\alpha = 0$, one will have

$$H = \Omega \sum_j \tilde{s}_j^z + \sum_j \left[J^x\tilde{s}_j^x\tilde{s}_{j+1}^x + J^y\tilde{s}_j^y\tilde{s}_{j+1}^y + D(\tilde{s}_j^x\tilde{s}_{j+1}^y - \tilde{s}_j^y\tilde{s}_{j+1}^x)\right], \tag{1.30}$$

where

$$\begin{aligned} J^x &\equiv J^{xx}\cos^2\alpha + \frac{J^{xy}+J^{yx}}{2}\sin 2\alpha + J^{yy}\sin^2\alpha,\\ J^y &\equiv J^{xx}\sin^2\alpha - \frac{J^{xy}+J^{yx}}{2}\sin 2\alpha + J^{yy}\cos^2\alpha,\\ D &\equiv \frac{J^{xy}-J^{yx}}{2},\quad \tan 2\alpha = \frac{J^{xy}+J^{yx}}{J^{xx}-J^{yy}}. \end{aligned} \tag{1.31}$$

One easily recognizes z component of vector $[\vec{s}_j \times \vec{s}_{j+1}]$ in the term that is proportional to D that is the so called Dzyaloshinskii-Moriya interaction. It was at first introduced phenomenologically by I.E. Dzyaloshinskii [12] and then derived by T.Moriya [13] by extending Anderson's theory of superexchange interactions [14] to include spin-orbital coupling (see, for example, ref.[15]). The model with relativistic Dzyaloshinskii-Moriya interaction together with ANNNI model are widely used in microscopic theory of crystals with incommensurate phase [16,17]. In the classical case Dzyaloshinskii-Moriya interaction may lead to the appearance of the spiral spin structure. The possibility of the appearance of spiral structure in quantum case has been studied in ref.[11] where for this purpose pair static spin correlation functions have been estimated.

Except the above mentioned paper [11] the problem of statistical mechanics of 1D $s=\frac{1}{2}$ XY type model with the Hamiltonian (1.26) or (1.30) as to our knowledge has not been considered yet [1]. In the present paper an attempt to fill up this gap by the generalization for this case of the well-known scheme has been made. In section 2 the transformation of the Hamiltonian to the initial form for further examination of statistical properties is presented. In section 3 the thermodynamical properties of the model are considered, and in section 4 it is shown how to calculate the static spin correlation functions. The dynamics of transverse spin correlations and the transverse dynamical susceptibility are studied in section 5. The conclusions form section 6.

2 Transformation of the Hamiltonian

In the spirit of above described approach the Hamiltionian of the model (26) at first should be rewritten with the help of the raising and lowering operators in the form that is similar to (1.3)

$$\begin{aligned} H &= \Omega\sum_{j=1}^{N}\left(s_j^+s_j^- - \tfrac{1}{2}\right) +\\ &+\sum_{j=1}^{N}\left(J^{++}s_j^+s_{j+1}^+ + J^{+-}s_j^+s_{j+1}^- + J^{-+}s_j^-s_{j+1}^+ + J^{--}s_j^-s_{j+1}^-\right),\\ J^{++} &\equiv \left[J^{xx}-J^{yy}-\imath(J^{xy}+J^{yx})\right]/4 = (J^{--})^*,\\ J^{+-} &\equiv \left[J^{xx}+J^{yy}+\imath(J^{xy}-J^{yx})\right]/4 = (J^{-+})^*; \end{aligned} \tag{2.1}$$

here the periodic boundary conditions (1.12) are imposed. The Hamiltonian of the model (2.1) after Jordan-Wigner transformation (1.5), (1.10) will have the form that is similar to (1.14), (1.15)

$$H = H^+P^+ + H^-P^-,$$

$$H^\pm \equiv \Omega\sum_{j=1}^{N}\left(c_j^+c_j^- - \frac{1}{2}\right) + \sum_{j=1}^{N}\left(J^{++}c_j^+c_{j+1}^+ + J^{+-}c_j^+c_{j+1} - \right. \tag{2.2}$$

$$\left. -J^{-+}c_jc_{j+1}^+ - J^{--}c_jc_{j+1}\right); \tag{2.3}$$

[1] In ref.[18] on the base of the model with Hamiltonian (1.26) the problem of the validity of the Bose commutation rules approximation for spin operators has been examined.

besides H^- is c-cyclic and H^+ is c-anticyclic quadratic forms in Fermi operators. After Fourier transformation

$$\begin{array}{ll} c_\kappa^+ = \frac{1}{\sqrt{N}}\sum_{j=1}^N e^{-\imath\kappa j} c_j^+, & c_\kappa = \frac{1}{\sqrt{N}}\sum_{j=1}^N e^{\imath\kappa j} c_j, \\ c_j^+ = \frac{1}{\sqrt{N}}\sum_\kappa e^{\imath\kappa j} c_\kappa^+, & c_j = \frac{1}{\sqrt{N}}\sum_\kappa e^{-\imath\kappa j} c_\kappa, \end{array} \tag{2.4}$$

with $\kappa = \kappa^- \equiv 2\pi n/N$ for H^- and $\kappa = \kappa^+ \equiv 2\pi(n+1/2)/N$ for H^+, $n = -N/2, -N/2+1, \ldots, N/2-1$ (for N even), $n = -(N-1)/2, -(N-1)/2+1, \ldots, (N-1)/2$ (for N odd) $H^\pm$ can be rewritten in the form

$$\begin{array}{l} H^\pm = \sum_\kappa \left[-\frac{\Omega}{2} + \epsilon_\kappa c_\kappa^+ c_\kappa - \imath \sin\kappa \left(J^{++} c_\kappa^+ c_{-\kappa}^+ + J^{--} c_\kappa c_{-\kappa} \right) \right], \\ \epsilon_\kappa \equiv \epsilon_\kappa^{(+)} + \epsilon_\kappa^{(-)},\ \epsilon_\kappa^{(+)} \equiv \Omega + \frac{J^{xx}+J^{yy}}{2}\cos\kappa,\ \epsilon_\kappa^{(-)} \equiv \frac{J^{xy}-J^{yx}}{2}\sin\kappa. \end{array} \tag{2.5}$$

Here we made use of the following relations:

$$\sum_\kappa e^{-\imath\kappa} c_\kappa^+ c_{-\kappa}^+ = -\imath \sum_\kappa \sin\kappa\, c_\kappa^+ c_{-\kappa}^+, \quad \sum_\kappa e^{\imath\kappa} c_\kappa c_{-\kappa} = \imath \sum_\kappa \sin\kappa\, c_\kappa c_{-\kappa}. \tag{2.6}$$

Bogolyubov transformation completes the diagonalization of the quadratic forms:

$$\begin{array}{ll} \beta_\kappa = x_\kappa c_\kappa + y_\kappa c_{-\kappa}^+, & \beta_{-\kappa}^+ = y_{-\kappa}^* c_\kappa + x_{-\kappa}^* c_{-\kappa}^+, \\ c_\kappa = \left(-x_{-\kappa}^* \beta_\kappa + y_\kappa \beta_{-\kappa}^+ \right)/\Delta_\kappa, & c_{-\kappa}^+ = \left(y_{-\kappa}^* \beta_\kappa - x_\kappa \beta_{-\kappa}^+ \right)/\Delta_\kappa, \\ \Delta_\kappa \equiv y_\kappa y_{-\kappa}^* - x_\kappa x_{-\kappa}^* \neq 0. & \end{array} \tag{2.7}$$

β-operators remain of Fermi type if

$$|x_\kappa|^2 + |y_\kappa|^2 = 1, \quad \frac{x_\kappa}{y_\kappa} + \frac{x_{-\kappa}}{y_{-\kappa}} = 0. \tag{2.8}$$

The transformed Hamiltonian contains the operator terms being proportional only to $\beta_\kappa^+ \beta_\kappa$ if

$$\epsilon_\kappa^{(+)} + \imath \sin\kappa \left(J^{++} \frac{x_\kappa}{y_\kappa} - J^{--} \frac{y_{-\kappa}}{x_{-\kappa}} \right) = 0. \tag{2.9}$$

The condition (2.9) and the second condition in (2.8) yield

$$\begin{array}{l} \frac{x_\kappa}{y_\kappa} = \imath \frac{\epsilon_\kappa^{(+)} \mp \mathcal{E}_\kappa}{2|J^{++}|\sin\kappa} \exp\left(-\imath \arg J^{++}\right), \\ \mathcal{E}_\kappa \equiv \sqrt{(\epsilon_\kappa^{(+)})^2 + 4|J^{++}|^2 \sin^2\kappa}. \end{array} \tag{2.10}$$

Taking into account the first condition in (2.8) one finds that for lower sign in (2.10)

$$\begin{array}{l} x_\kappa = 2\imath|J^{++}|\sin\kappa \exp(-\imath \arg J^{++}) / \sqrt{2\mathcal{E}_\kappa(\mathcal{E}_\kappa - \epsilon_\kappa^{(+)})}, \\ y_\kappa = \sqrt{(\mathcal{E}_\kappa - \epsilon_\kappa^{(+)})/2\mathcal{E}_\kappa}; \end{array} \tag{2.11}$$

besides $\Delta_\kappa = 1$, $E_\kappa = \epsilon_\kappa^{(-)} + \mathcal{E}_\kappa$. For upper sign in (2.10)

$$x_\kappa = -2\imath|J^{++}|\sin\kappa\exp(-\imath\arg J^{++})/\sqrt{2\mathcal{E}_\kappa(\mathcal{E}_\kappa+\epsilon_\kappa^{(+)})},\qquad y_\kappa = \sqrt{(\mathcal{E}_\kappa+\epsilon_\kappa^{(+)})/2\mathcal{E}_\kappa}; \tag{2.12}$$

besides $\Delta_\kappa = 1$, $E_\kappa = \epsilon_\kappa^{(-)} - \mathcal{E}_\kappa$. Thus in a result of Bogolyubov transformation (2.7) one gets

$$H^\pm = \sum_\kappa E_\kappa\left(\beta_\kappa^+\beta_\kappa - 1/2\right), \ \{\beta_\kappa, \beta_\pi^+\} = \delta_{\kappa\pi}, \ \{\beta_\kappa, \beta_\pi\} = \{\beta_\kappa^+, \beta_\pi^+\} = 0. \tag{2.13}$$

It is important to note that in contrast to anisotropic XY model because of inequality $J^{xy} \neq J^{yx}$ one has $E_\kappa \neq E_{-\kappa}$. This is connected with the absence of symmetry with respect to spatial inversion. Really, the Hamiltonian of the model (1.26) $H(\Omega, J^{xx}, J^{xy}, J^{yx}, J^{yy})$ under the action of spatial inversion, that leads to change of indexes j to $-j$ or $N-j$, $j+1$ to $N-j-1$, transforms into $H(\Omega, J^{xx}, J^{yx}, J^{xy}, J^{yy})$.

It is shown in fig. 1 how the presence of Dzyaloshinskii-Moriya interaction influences the dependence of $E_\kappa = \epsilon_\kappa^{(-)} + \mathcal{E}_\kappa$ on κ in de Gennes model ($1: D = 0, \ \Omega = 0, \ \ 1': D = J^{xx}, \ \Omega = 0; \ \ 2: D = 0, \ \Omega = J^{xx}, \ 2': D = J^{xx}, \ \Omega = J^{xx}$). In fig. 2 the same is depicted for the case of isotropic XY model.

It is worthwhile to note that the spectrum of elementary excitations in the model under consideration as it follows from the expression for ground state energy (3.3) is given by $|E_\kappa|$.

3 Thermodynamics

Let's calculate the free energy per site in the limit $N \to \infty$ in order to study the thermodynamical properties of the model under consideration. One can use c-cyclic Hamiltonian for such calculation [5,6] and thus

$$f = -\frac{1}{\beta}\lim_{N\to\infty}\left[\frac{1}{N}\ln Sp\exp\left(-\beta H^-\right)\right]. \tag{3.1}$$

The diagonalized quadratic in Fermi operators form H^- involved in (3.1) has the form (2.13), and owing to this one easily obtains the desired result

$$\begin{aligned} f &= -\frac{1}{\beta}\lim_{N\to\infty}\left\{\frac{1}{N}\ln Sp\exp\left[-\beta\sum_\kappa E_\kappa\left(\beta_\kappa^+\beta_\kappa - \frac{1}{2}\right)\right]\right\} = \\ &= -\frac{1}{\beta}\lim_{N\to\infty}\left\{\frac{1}{N}\ln Sp\prod_\kappa\exp\left[-\beta E_\kappa\left(\beta_\kappa^+\beta_\kappa - \frac{1}{2}\right)\right]\right\} = \\ &= -\frac{1}{\beta}\lim_{N\to\infty}\left[\frac{1}{N}\ln\left(\prod_\kappa 2\cosh\frac{\beta E_\kappa}{2}\right)\right] = -\frac{1}{\beta}\lim_{N\to\infty}\left[\frac{1}{N}\sum_\kappa\ln(2\cosh\frac{\beta E_\kappa}{2})\right] = \\ &= -\frac{1}{\beta}\frac{1}{2\pi}\int_{-\pi}^{\pi}d\kappa\ln\left(2\cosh\frac{\beta E_\kappa}{2}\right). \end{aligned} \tag{3.2}$$

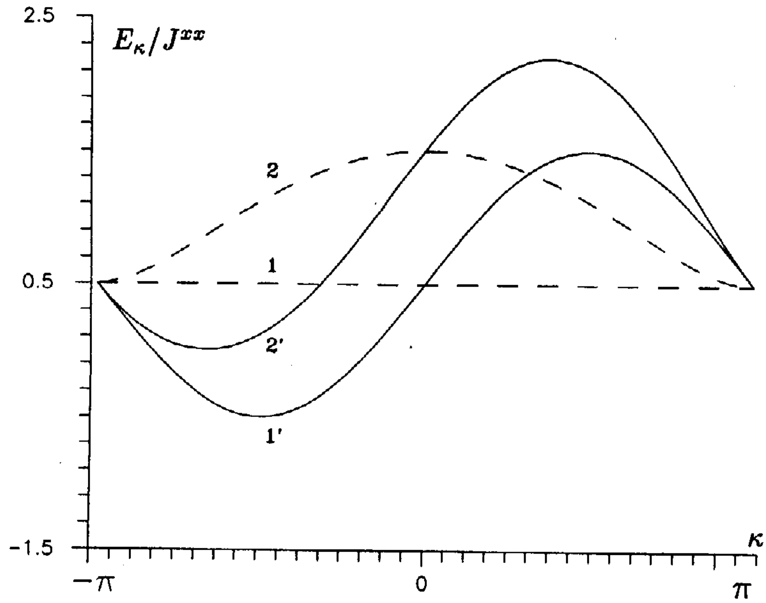

Figure 1: $E_\kappa/J^{xx} = (\epsilon_\kappa^{(-)} + \mathcal{E}_\kappa)/J^{xx}$ vs. κ; $J^{yy} = 0$.

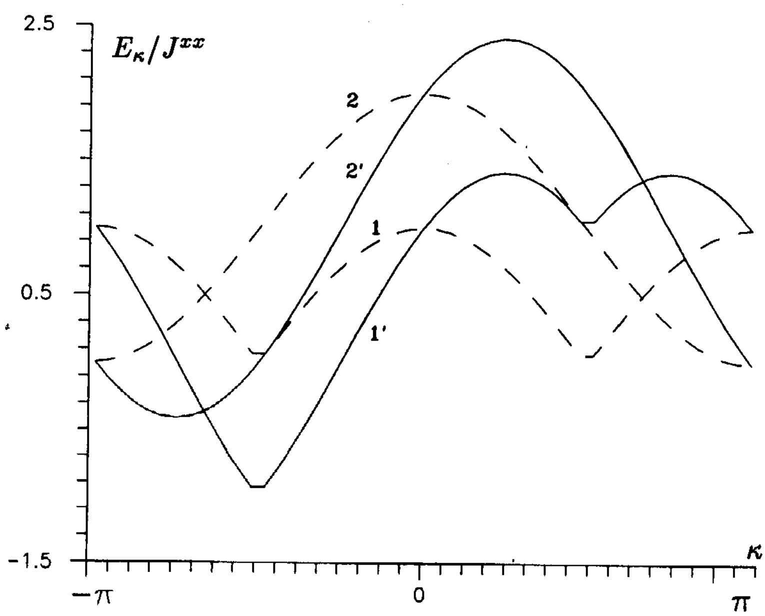

Figure 2: $E_\kappa/J^{xx} = (\epsilon_\kappa^{(-)} + \mathcal{E}_\kappa)/J^{xx}$ vs. κ; $J^{yy} = J^{xx}$.

Knowing the free energy (3.2) one finds the energy of the ground state

$$e = \lim_{\beta\to\infty} f = -\frac{1}{4\pi}\int_{-\pi}^{\pi} d\kappa\,|E_\kappa|, \tag{3.3}$$

the entropy

$$s = \beta^2\frac{\partial f}{\partial\beta} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\kappa \ln\left(2\cosh\frac{\beta E_\kappa}{2}\right) - \frac{1}{2\pi}\int_{-\pi}^{\pi} d\kappa \frac{\beta E_\kappa}{2}\tanh\frac{\beta E_\kappa}{2}, \tag{3.4}$$

the specific heat

$$c = -\beta\frac{\partial s}{\partial\beta} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\kappa \left(\frac{\beta E_\kappa}{2}\right)^2\left(\cosh\frac{\beta E_\kappa}{2}\right)^{-2}, \tag{3.5}$$

the transverse magnetization

$$< \frac{1}{N}\sum_{j=1}^{N} s_j^z >= \frac{\partial f}{\partial\Omega} = -\frac{1}{4\pi}\int_{-\pi}^{\pi} d\kappa \frac{\partial E_\kappa}{\partial\Omega}\tanh\frac{\beta E_\kappa}{2}, \tag{3.6}$$

the static transverse susceptibility

$$\begin{aligned}\chi_{zz} &= \frac{\partial < \frac{1}{N}\sum_{j=1}^{N} s_j^z >}{\partial\Omega} = \\ &= -\frac{1}{4\pi}\int_{-\pi}^{\pi} d\kappa \frac{\partial^2 E_\kappa}{\partial\Omega^2}\tanh\frac{\beta E_\kappa}{2} - \frac{\beta}{8\pi}\int_{-\pi}^{\pi} d\kappa\left(\frac{\partial E_\kappa}{\partial\Omega}\right)^2\left(\cosh\frac{\beta E_\kappa}{2}\right)^{-2}.\end{aligned} \tag{3.7}$$

In order to illustrate the influence of the additional interactions on thermodynamical properties let's present the results of numerical calculations of the specific heat (3.5) as a function of temperature (figs. 3,4 (1: $D=0,\ \Omega=0,\ 1': D=J^{xx},\ \Omega=0;\ \ 2: D=0,\ \Omega=J^{xx},\ 2': D=J^{xx},\ \Omega=J^{xx}$)) and of transverse magnetization (49) as a function of transverse field (figs. 5,6 (1: $D=0,\ \beta=1/J^{xx},\ 1': D=J^{xx},\ \beta=1/J^{xx};\ \ 2: D=0,\ \beta=1000/J^{xx},\ 2': D=J^{xx},\ \beta=1000/J^{xx}$)) and of temperature (figs. 7,8 (1: $D=0,\ \Omega=0,\ 1': D=J^{xx},\ \Omega=0;\ \ 2: D=0,\ \Omega=J^{xx},\ 2': D=J^{xx},\ \Omega=J^{xx}$)) for de Gennes model and for isotropic XY model in the presence of Dzyaloshinskii-Moriya interaction.

4 Static spin correlation functions

Let's introduce the static spin correlation functions for the investigation of the spin stucture in the model under consideration. Due to the possibility of their calculation with the help of c-cyclic Hamiltonian the initial formula for their evaluation can be rewritten in the form

$$< s_{j_1}^{\alpha_1}\dots s_{j_n}^{\alpha_n} >= \lim_{N\to\infty}\left\{Sp\left[\exp(-\beta H^-)s_{j_1}^{\alpha_1}\dots s_{j_n}^{\alpha_n}\right]/Sp\exp(-\beta H^-)\right\}. \tag{4.1}$$

Let's introduce then φ-operators that owing to (2.7), (2.11) are the linear combinations of β-operators:

$$\begin{aligned}\varphi_j^\pm &\equiv c_j^+ \pm c_j = \textstyle\sum_\kappa\left(\lambda_{j\kappa}^\pm\beta_\kappa^+ \pm \mu_{j\kappa}^\pm\beta_{-\kappa}\right),\\ \lambda_{j\kappa}^\pm &\equiv \frac{1}{\sqrt{N}}e^{\imath\kappa j}(x_\kappa \pm y_\kappa),\quad \mu_{j\kappa}^\pm \equiv \frac{1}{\sqrt{N}}e^{\imath\kappa j}[x_\kappa\exp(2\imath\arg J^{++}) \pm y_\kappa].\end{aligned} \tag{4.2}$$

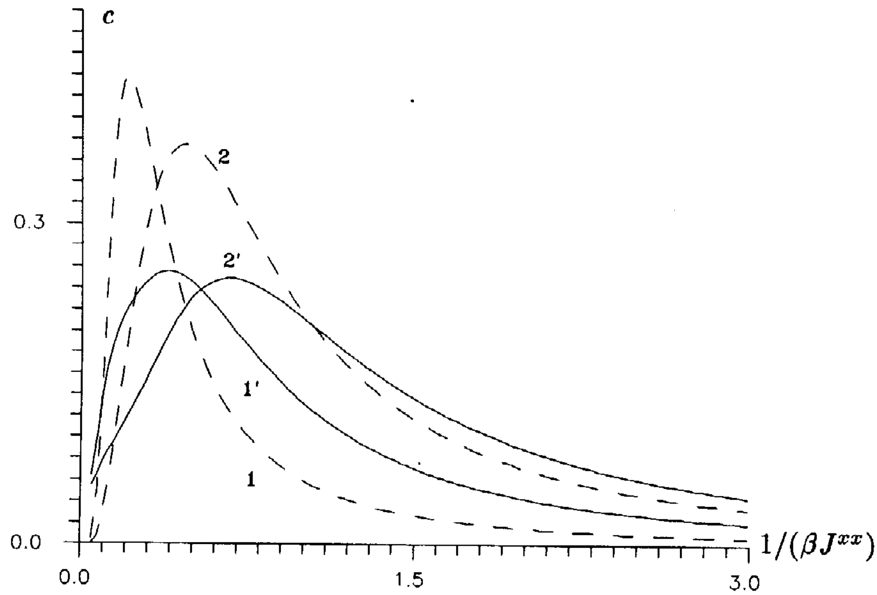

Figure 3: c vs. $1/(\beta J^{xx})$; $J^{yy} = 0$.

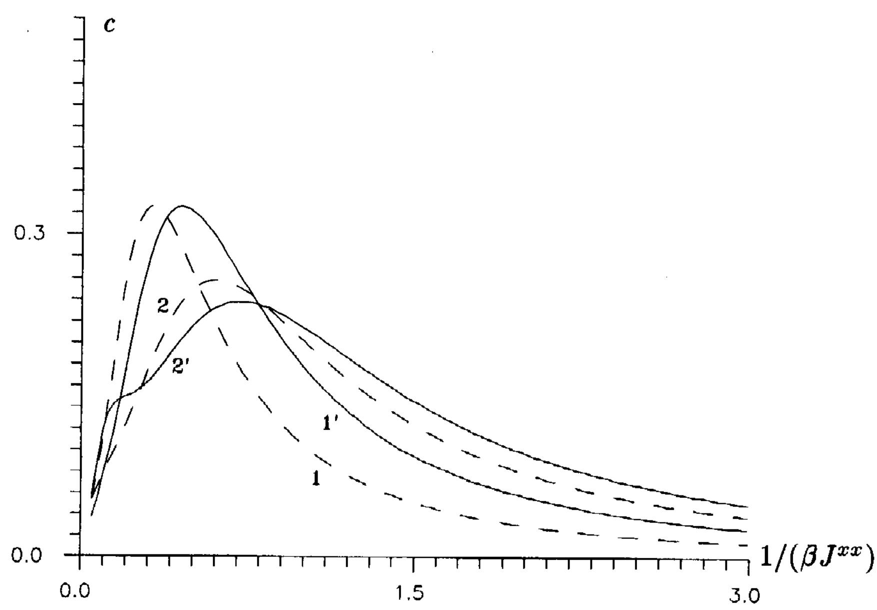

Figure 4: c vs. $1/(\beta J^{xx})$; $J^{yy} = J^{xx}$.

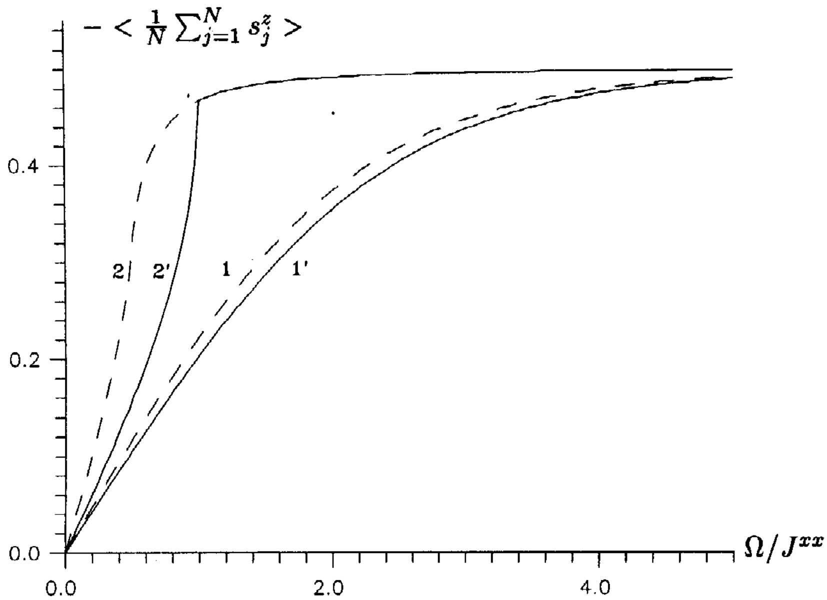

Figure 5: $-<\frac{1}{N}\sum_{j=1}^{N}s_j^z>$ vs. Ω/J^{xx}; $J^{yy}=0$.

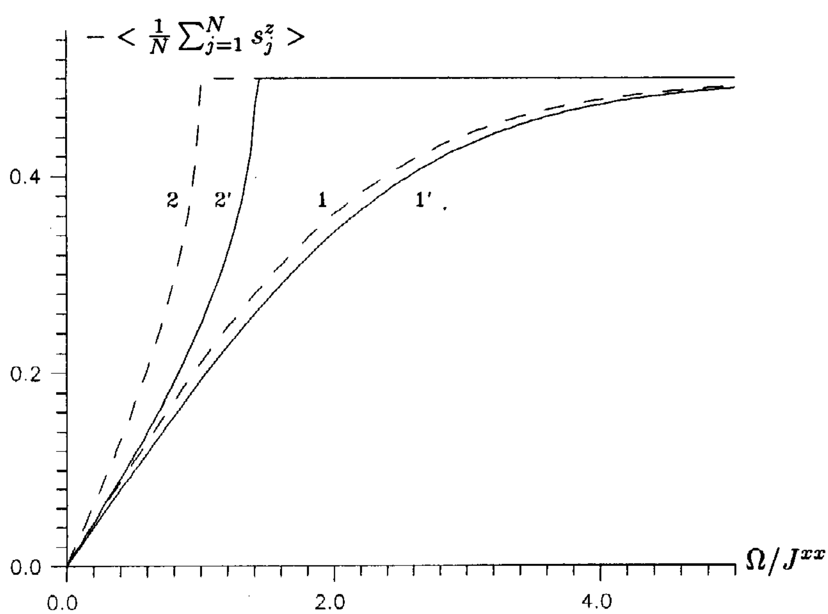

Figure 6: $-<\frac{1}{N}\sum_{j=1}^{N}s_j^z>$ vs. Ω/J^{xx}; $J^{yy}=J^{xx}$.

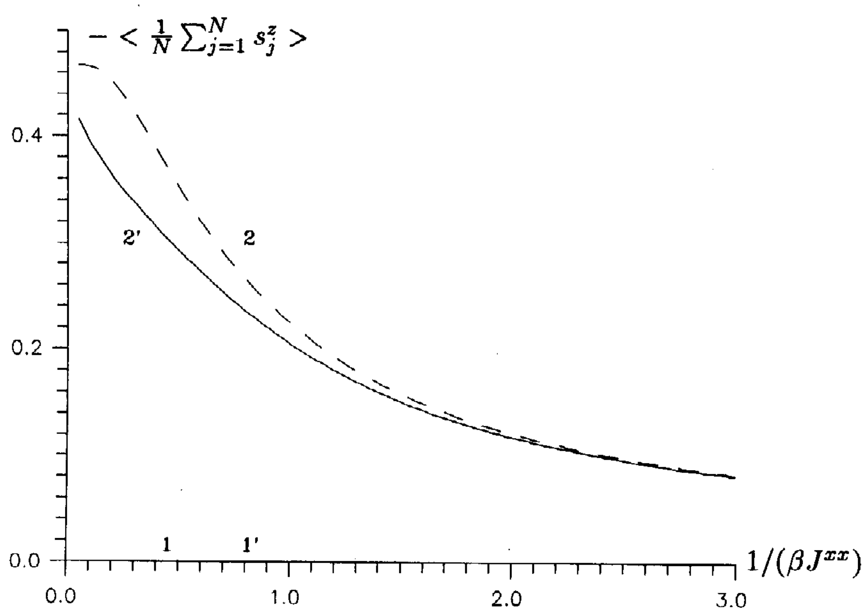

Figure 7: $- < \frac{1}{N} \sum_{j=1}^{N} s_j^z >$ vs. $1/(\beta J^{xx})$; $J^{yy} = 0$.

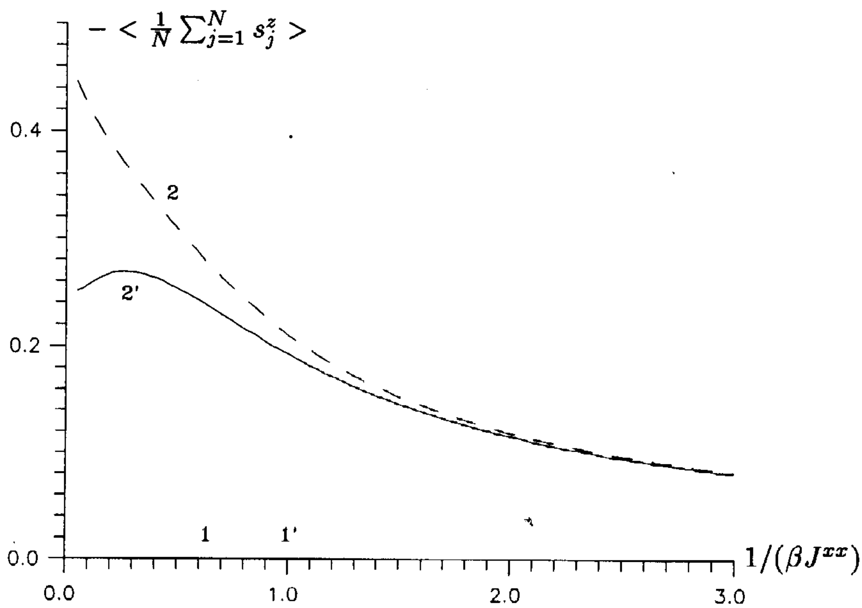

Figure 8: $- < \frac{1}{N} \sum_{j=1}^{N} s_j^z >$ vs. $1/(\beta J^{xx})$; $J^{yy} = J^{xx}$.

The spin operators can be presented in terms of φ-operators:

$$s_n^x=\frac{1}{2}\prod_{j=1}^{n-1}\left(\varphi_j^+\varphi_j^-\right)\varphi_n^+,\quad s_n^y=\frac{1}{2\imath}\prod_{j=1}^{n-1}\left(\varphi_j^+\varphi_j^-\right)\varphi_n^-,\quad s_n^z=\frac{1}{2}\varphi_n^-\varphi_n^+. \tag{4.3}$$

φ-operators obey the following commutation relations

$$\left\{\varphi_i^+,\varphi_j^-\right\}=0,\quad \left\{\varphi_i^+,\varphi_j^+\right\}=2\delta_{ij},\quad \left\{\varphi_i^-,\varphi_j^-\right\}=-2\delta_{ij}, \tag{4.4}$$

besides $(\varphi_j^+\varphi_j^-)^2=1$ and

$$\left[\varphi_i^\pm,\varphi_j^+\varphi_j^-\right]=2\delta_{ij}\varphi_i^\mp,\quad \left[\varphi_i^+\varphi_i^-,\varphi_j^+\varphi_j^-\right]=0. \tag{4.5}$$

That is why the calculation of static spin correlation functions after substitution of (4.3) into (4.1) and exploiting of (4.4), (4.5) reduces to application of Wick-Bloch-de Dominicis theorem. The theorem states that the mean value of the product of even number of φ operators with the Hamiltonian H^- (2.13) is equal to the sum of all possible full systems of contractions of this product; if the number of φ operators in the product is odd the mean value of the product is equal to zero. The full system of contractions of the product of even number of Fermi-type operators forms so called Pfaffian the square of which is equal to the determinant of antisymmetric matrix costructed in a certain way from elementary contractions [19,20]. Thus let's consider the calculation of elementary contractions. One has

$$<\varphi_j^+\varphi_{j+n}^+>=\sum_{\kappa_1,\kappa_2}\left(\lambda_{j\kappa_1}^+\mu_{j+n,\kappa_2}^+<\beta_{\kappa_1}^+\beta_{-\kappa_2}>+\mu_{j\kappa_1}^+\lambda_{j+n,\kappa_2}^+<\beta_{-\kappa_1}\beta_{\kappa_2}^+>\right) \tag{4.6}$$

(here evident relations $<\beta_{\kappa_1}^+\beta_{\kappa_2}^+>=<\beta_{-\kappa_1}\beta_{-\kappa_2}>=0$ were used). Since

$$\begin{aligned}<\beta_{\kappa_1}^+\beta_{-\kappa_2}>&=\delta_{\kappa_1,-\kappa_2}/(1+e^{\beta E_{\kappa_1}})=\delta_{\kappa_1,-\kappa_2}f_{\kappa_1},\\ <\beta_{-\kappa_1}\beta_{\kappa_2}^+>&=\delta_{-\kappa_1,\kappa_2}e^{\beta E_{\kappa_2}}/(1+e^{\beta E_{\kappa_2}})=\delta_{-\kappa_1,\kappa_2}e^{\beta E_{\kappa_2}}f_{\kappa_2},\end{aligned} \tag{4.7}$$

where $f_\kappa\equiv 1/(1+e^{\beta E_\kappa})$ and in accordance with (4.2)

$$\begin{gathered}\lambda_{j\kappa}^+\mu_{j+n,-\kappa}^+=\frac{1}{N}e^{-\imath\kappa n}\left(1+S_\kappa\right),\ \mu_{j,-\kappa}^+\lambda_{j+n,\kappa}^+=\frac{1}{N}e^{\imath\kappa n}\left(1+S_\kappa\right),\\ S_\kappa\equiv\frac{2|J^{++}|\sin\left(\arg J^{++}\right)\sin\kappa}{\mathcal{E}_\kappa}\end{gathered} \tag{4.8}$$

one has

$$<\varphi_j^+\varphi_{j+n}>=\frac{1}{N}\sum_\kappa e^{\imath\kappa n}\left(1+S_\kappa\right)-\frac{2\imath}{N}\sum_\kappa\sin\left(\kappa n\right)(1+S_\kappa)f_\kappa. \tag{4.9}$$

Similarly one finds that

$$<\varphi_j^-\varphi_{j+n}^->=-\frac{1}{N}\sum_\kappa e^{\imath\kappa n}\left(1-S_\kappa\right)+\frac{2\imath}{N}\sum_\kappa\sin\left(\kappa n\right)(1-S_\kappa)f_\kappa \tag{4.10}$$

and

$$< \varphi_j^+ \varphi_{j+n}^- >= = \frac{1}{N}\sum_\kappa e^{\imath\kappa n}\left(\frac{\epsilon_\kappa^{(+)}}{\mathcal{E}_\kappa} + \imath C_\kappa\right) - \frac{2}{N}\sum_\kappa\left[\cos(\kappa n)\frac{\epsilon_\kappa^{(+)}}{\mathcal{E}_\kappa} - \sin(\kappa n)C_\kappa\right] f_\kappa, \tag{4.11}$$

$$< \varphi_j^- \varphi_{j+n}^+ >= = \frac{1}{N}\sum_\kappa e^{\imath\kappa n}\left(\frac{-\epsilon_\kappa^{(+)}}{\mathcal{E}_\kappa} + \imath C_\kappa\right) + \frac{2}{N}\sum_\kappa\left[\cos(\kappa n)\frac{\epsilon_\kappa^{(+)}}{\mathcal{E}_\kappa} + \sin(\kappa n)C_\kappa\right] f_\kappa, \tag{4.12}$$
$$C_\kappa \equiv \frac{2|J^{++}|\cos(\arg J^{++})\sin\kappa}{\mathcal{E}_\kappa}.$$

The essential simplifications in expressions for contractions (4.9)-(4.12) take place in the case of model (1.30), that is when $J^{xy} = -J^{yx} = D$. Then

$$\begin{aligned} &< \varphi_j^+ \varphi_{j+n}^+ >= \delta_{n,0} + \frac{\imath}{N}\sum_\kappa \sin(\kappa n)\tanh\frac{\beta E_\kappa}{2} \equiv E(n), \\ &< \varphi_j^- \varphi_{j+n}^- >= -E(n), \\ &< \varphi_j^+ \varphi_{j+n}^- >= \frac{1}{N}\sum_\kappa \cos(\kappa n + \psi_\kappa)\tanh\frac{\beta E_\kappa}{2} \equiv G(n), \\ &< \varphi_j^- \varphi_{j+n}^+ >= -G(-n), \end{aligned} \tag{4.13}$$

where $\cos\psi_\kappa \equiv \epsilon_\kappa^{(+)}/\mathcal{E}_\kappa$, $\sin\psi_\kappa \equiv 2J^{++}\sin\kappa/\mathcal{E}_\kappa$.

Let's return to the evaluation of equal-time spin correlation functions and consider, for instance, $< s_j^x s_{j+n}^x >$. For this correlation function with the utilization of (4.3)-(4.5) one derives

$$< s_j^x s_{j+n}^x >= \frac{1}{4} < \varphi_j^- \varphi_{j+1}^+ \varphi_{j+1}^- \varphi_{j+2}^+ \cdots \varphi_{j+n-1}^+ \varphi_{j+n-1}^- \varphi_{j+n}^+ >, \tag{4.14}$$

and after exploiting Wick-Bloch-de Dominicis theorem in r.h.s. of (4.14) for its square one gets the following expression

$$\left[4 < s_j^x s_{j+n}^x >\right]^2 = \tag{4.15}$$

$$\begin{vmatrix} 0 & -E(1) & \ldots -E(n-1) & -G(-1) & -G(-2) & \ldots & -G(-n) \\ E(1) & 0 & \ldots -E(n-2) & -G(0) & -G(-1) & \ldots & -G(-n+1) \\ E(2) & E(1) & \ldots -E(n-3) & -G(1) & -G(0) & \ldots & -G(-n+2) \\ \vdots & \vdots & \vdots\quad\vdots & \vdots & \vdots & \vdots & \vdots \\ E(n-1) & E(n-2) & \ldots \quad 0 & -G(n-2) & -G(n-3) & \ldots & -G(-1) \\ G(-1) & G(0) & \ldots\ G(n-2) & 0 & E(1) & \ldots & E(n-1) \\ G(-2) & G(-1) & \ldots\ G(n-3) & -E(1) & 0 & \ldots & E(n-2) \\ G(-3) & G(-2) & \ldots\ G(n-4) & -E(2) & -E(1) & \ldots & E(n-3) \\ \vdots & \vdots & \vdots\quad\vdots & \vdots & \vdots & \vdots & \vdots \\ G(-n) & G(-n+1) & \ldots\ G(-1) & -E(n-1) & -E(n-2) & \ldots & 0 \end{vmatrix}.$$

In a similar way for other pair spin correlators one obtains

$$\left[4\imath < s_j^x s_{j+n}^y >\right]^2 = \tag{4.16}$$

$$
\begin{vmatrix}
0 & -E(1) & \ldots & -E(n-1) & -E(-n) & -G(-1) & \ldots & -G(-n+1) \\
E(1) & 0 & \ldots & -E(n-2) & -E(n-1) & -G(0) & \ldots & -G(-n+2) \\
E(2) & E(1) & \ldots & -E(n-3) & -E(n-2) & -G(1) & \ldots & -G(-n+3) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
E(n-1) & E(n-2) & \ldots & 0 & -E(1) & -G(n-2) & \ldots & -G(0) \\
E(n) & E(n-1) & \ldots & E(1) & 0 & -G(n-1) & \ldots & -G(1) \\
G(-1) & G(0) & \ldots & G(n-2) & G(n-1) & 0 & \ldots & E(n-2) \\
G(-2) & G(-1) & \ldots & G(n-3) & G(n-2) & -E(1) & \ldots & E(n-3) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
G(-n+1) & G(-n+2) & \ldots & G(0) & G(1) & -E(n-2) & \ldots & 0
\end{vmatrix},
$$

$$
< s_j^x s_{j+n}^z >= 0; \tag{4.17}
$$

$$
\left[4\imath < s_j^y s_{j+n}^x >\right]^2 = \tag{4.18}
$$

$$
\begin{vmatrix}
0 & -E(1) & \ldots & -G(1) & -G(0) & -G(-1) & \ldots & -G(-n+1) \\
E(1) & 0 & \ldots & -G(2) & -G(1) & -G(0) & \ldots & -G(-n+2) \\
E(2) & E(1) & \ldots & -G(3) & -G(2) & -G(1) & \ldots & -G(-n+3) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
G(1) & G(2) & \ldots & 0 & E(1) & E(2) & \ldots & E(n) \\
G(0) & G(1) & \ldots & -E(1) & 0 & E(1) & \ldots & E(n-1) \\
G(-1) & G(0) & \ldots & -E(2) & -E(1) & 0 & \ldots & E(n-2) \\
G(-2) & G(-1) & \ldots & -E(3) & -E(2) & -E(1) & \ldots & E(n-3) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
G(-n+1) & G(-n+2) & \ldots & -E(n) & -E(n-1) & -E(n-2) & \ldots & 0
\end{vmatrix},
$$

$$
\left[4 < s_j^y s_{j+n}^y >\right]^2 = \tag{4.19}
$$

$$
\begin{vmatrix}
0 & -E(1) & \ldots & -E(n-1) & -G(1) & -G(0) & \ldots & -G(-n+2) \\
E(1) & 0 & \ldots & -E(n-2) & -G(2) & -G(1) & \ldots & -G(-n+3) \\
E(2) & E(1) & \ldots & -E(n-3) & -G(3) & -G(2) & \ldots & -G(-n+4) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
E(n-1) & E(n-2) & \ldots & 0 & -G(n) & -G(n-1) & \ldots & -G(1) \\
G(1) & G(2) & \ldots & G(n) & 0 & E(1) & \ldots & E(n-1) \\
G(0) & G(1) & \ldots & G(n-1) & -E(1) & 0 & \ldots & E(n-2) \\
G(-1) & G(0) & \ldots & G(n-2) & -E(2) & -E(1) & \ldots & E(n-3) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
G(-n+2) & G(-n+3) & \ldots & G(1) & -E(n-1) & -E(n-2) & \ldots & 0
\end{vmatrix},
$$

$$
< s_j^y s_{j+n}^z >= 0; \tag{4.20}
$$

$$
< s_j^z s_{j+n}^x >= 0, \tag{4.21}
$$

$$
< s_j^z s_{j+n}^y >= 0, \tag{4.22}
$$

$$
\left[4 < s_j^z s_{j+n}^z >\right]^2 = \begin{vmatrix}
0 & E(n) & G(0) & G(n) \\
-E(n) & 0 & G(-n) & G(0) \\
-G(0) & -G(-n) & 0 & -E(n) \\
-G(n) & -G(0) & E(n) & 0
\end{vmatrix}. \tag{4.23}
$$

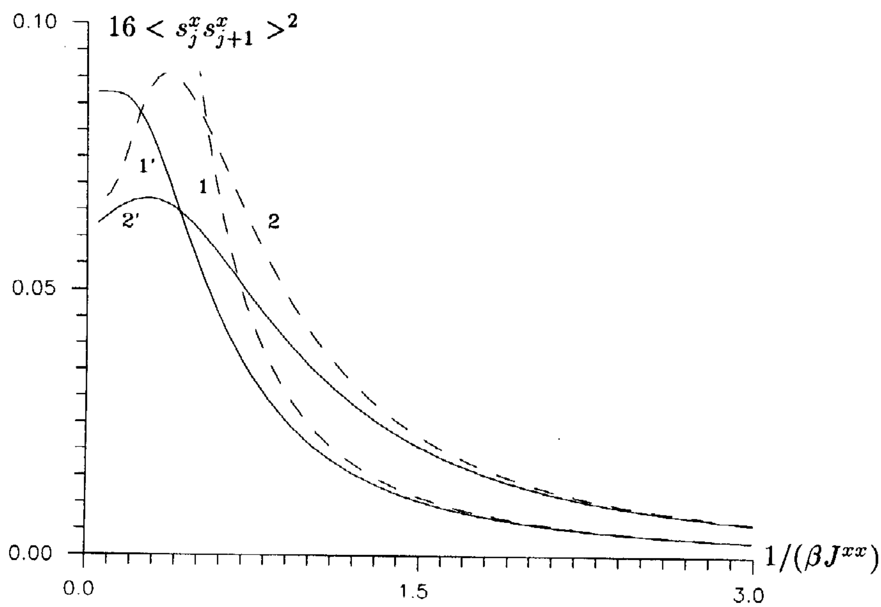

Figure 9: $16 < s_j^x s_{j+1}^x >^2$ vs. $1/(\beta J^{xx})$; $J^{yy} = 0$.

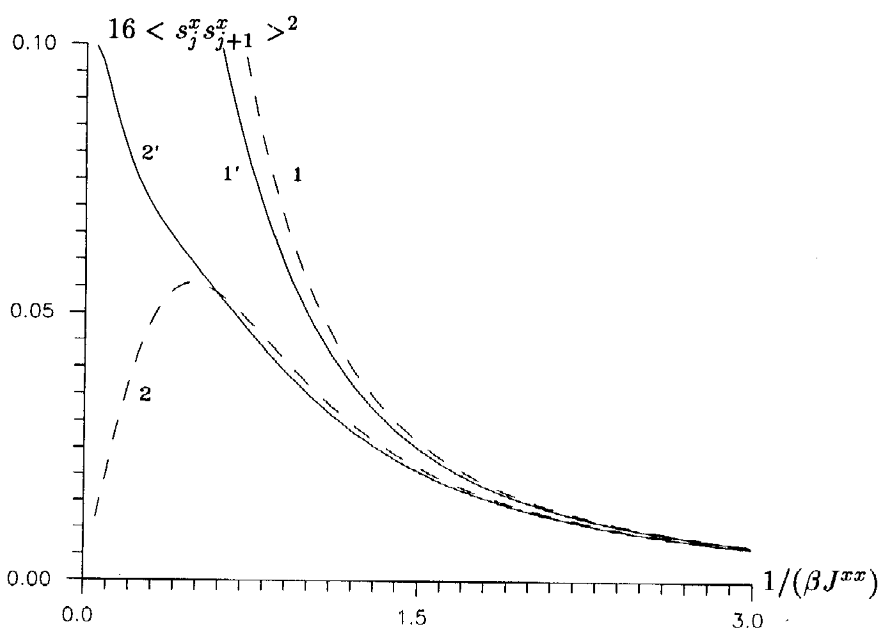

Figure 10: $16 < s_j^x s_{j+1}^x >^2$ vs. $1/(\beta J^{xx})$; $J^{yy} = J^{xx}$.

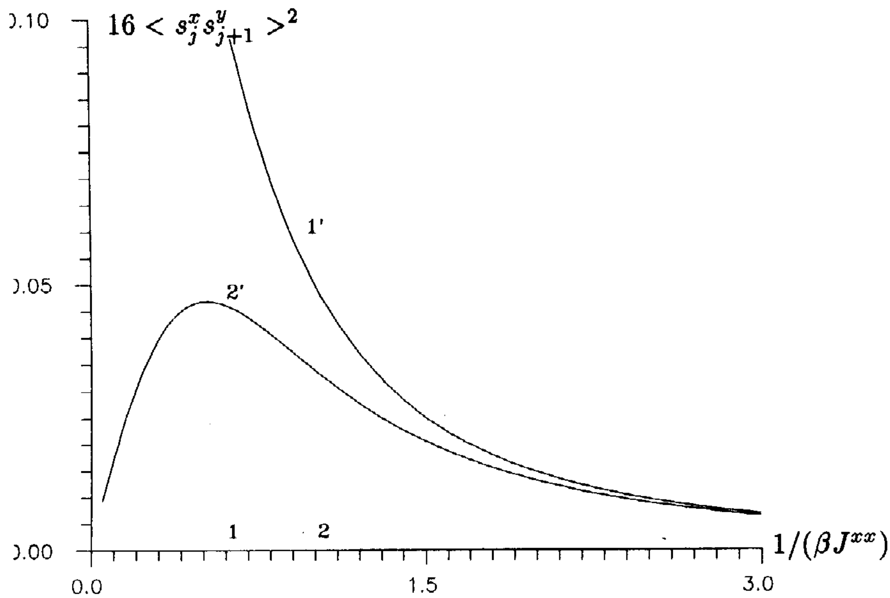

Figure 11: $16 < s_j^x s_{j+1}^y >^2$ vs. $1/(\beta J^{xx})$; $J^{yy} = 0$.

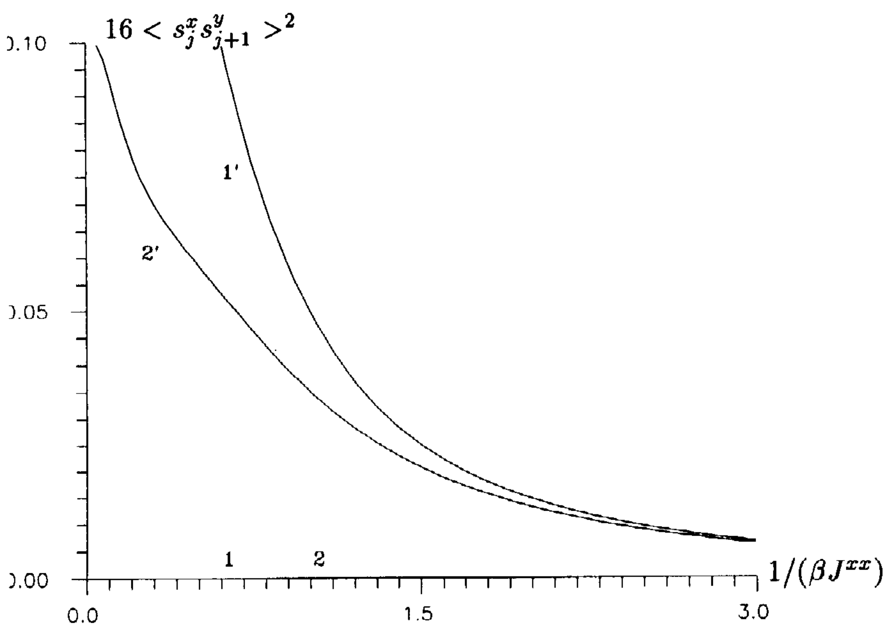

Figure 12: $16 < s_j^x s_{j+1}^y >^2$ vs. $1/(\beta J^{xx})$; $J^{yy} = J^{xx}$.

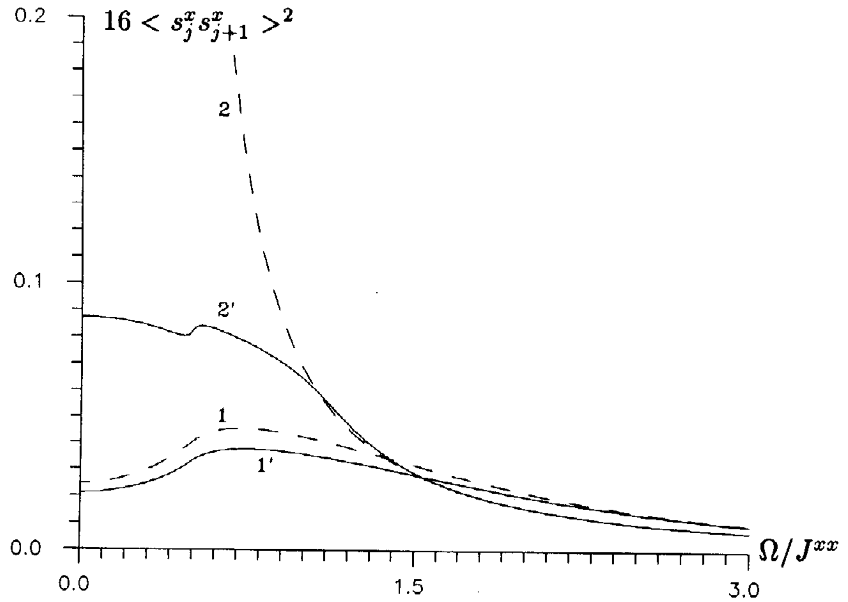

Figure 13: $16 < s_j^x s_{j+1}^x >^2$ vs. Ω/J^{xx}; $J^{yy} = 0$.

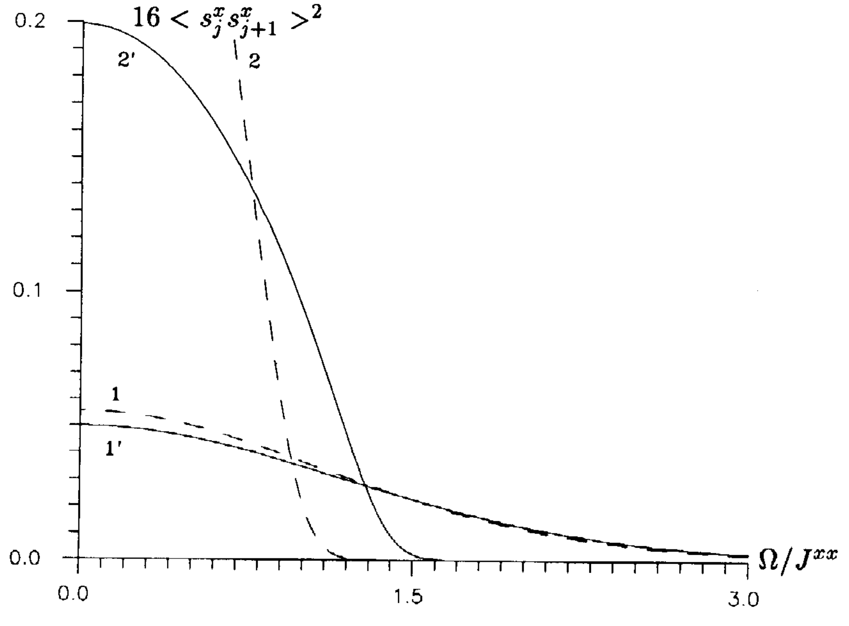

Figure 14: $16 < s_j^x s_{j+1}^x >^2$ vs. Ω/J^{xx}; $J^{yy} = J^{xx}$.

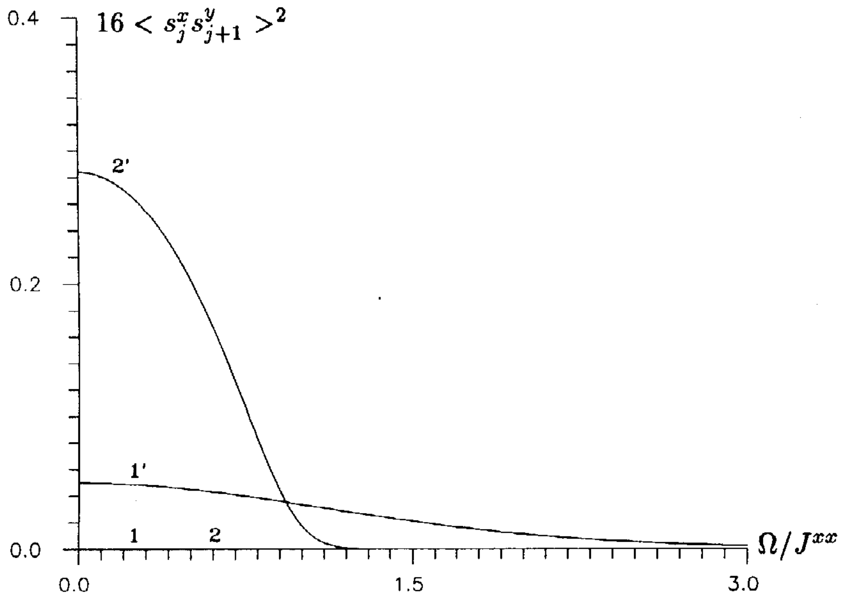

Figure 15: $16 < s_j^x s_{j+1}^y >^2$ vs. Ω/J^{xx}; $J^{yy} = 0$.

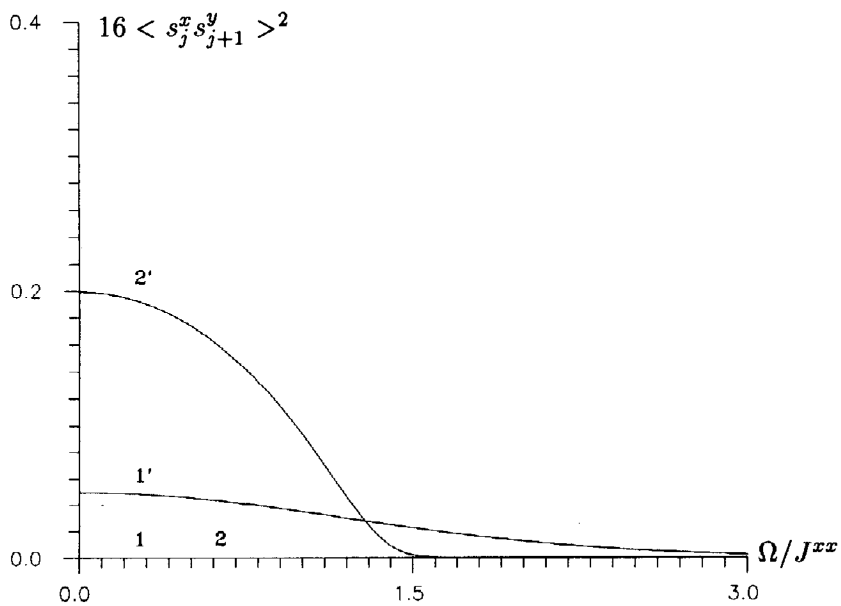

Figure 16: $16 < s_j^x s_{j+1}^y >^2$ vs. Ω/J^{xx}; $J^{yy} = J^{xx}$.

In figs. 9-12 the temperature dependences (1: $D=0,\ \Omega=0,\ 1':D=J^{xx},\ \Omega=0$; 2: $D=0,\ \Omega=J^{xx},\ 2':D=J^{xx},\ \Omega=J^{xx}$) and in figs. 13-16 the dependences on transverse field (1: $D=0,\ \beta=1/J^{xx},\ 1':D=J^{xx},\ \beta=1/J^{xx}$ 2 : $D=0,\ \beta=10/J^{xx},\ 2':D=J^{xx},\ \beta=10/J^{xx}$) for some pair spin static correlation functions are shown. It is necessary to underline the peculiarities caused by the presence of Dzyaloshinskii-Moriya interaction. First, only $<s_j^x s_{j+n}^z>$, $<s_j^y s_{j+n}^z>$, $<s_j^z s_{j+n}^x>$, $<s_j^z s_{j+n}^y>$ are equal to zero, but not $<s_j^x s_{j+n}^y>$ and $<s_j^y s_{j+n}^x>$. The last two correlators tend to zero when $J^{xy}=J^{yx}=0$. In this case $E(n)=0$ for $n\neq 0$ and hence (4.16) and (4.18) may be rewritten as determinants of matrices with only non-zero rectangle (but not square) submatrices on their diagonals; such determinants are equal to zero. Second, the dependence of pair static correlation functions on n is nonmonotonic (in accordance with ref.[11] this fact indicates the appearance of the incommensurate spiral spin structure).

5 Dynamics of transverse spin correlations and dynamical transverse susceptibility

Let's consider the dynamics of transverse spin correlations calculating for this purpose the transverse time-dependent (dynamical) pair spin correlation function $<s_j^z(t)s_{j+n}^z>$. Due to the possibility of exploiting for its calculation c-cyclic Hamiltonian H^- (2.13) the evaluation of this correlation function in accordance with (4.3) and (4.2) reduces to estimation of dynamical correlation functions density-density for the system of non-interacting fermions

$$
\begin{aligned}
&4<s_j^z(t)s_{j+n}^z>=\\
&=\sum_{\kappa_1,\kappa_2,\kappa_3,\kappa_4}<\left[\lambda^+_{j\kappa_1}\beta^+_{\kappa_1}(t)+\mu^+_{j\kappa_1}\beta_{-\kappa_1}(t)\right]\left[\lambda^-_{j\kappa_2}\beta^+_{\kappa_2}(t)-\mu^-_{j\kappa_2}\beta_{-\kappa_2}(t)\right]\times\\
&\times\left[\lambda^+_{j+n,\kappa_3}\beta^+_{\kappa_3}+\mu^+_{j+n,\kappa_3}\beta_{-\kappa_3}\right]\left[\lambda^-_{j+n,\kappa_4}\beta^+_{\kappa_4}-\mu^-_{j+n,\kappa_4}\beta_{-\kappa_4}\right]>=\\
&=\sum_{\kappa_1,\kappa_2,\kappa_3,\kappa_4}\Big[-\lambda^+_{j\kappa_1}\lambda^-_{j\kappa_2}\mu^+_{j+n,\kappa_3}\mu^-_{j+n,\kappa_4}<\beta^+_{\kappa_1}\beta^+_{\kappa_2}\beta_{-\kappa_3}\beta_{-\kappa_4}>e^{\imath(E_{\kappa_1}+E_{\kappa_2})t}+\\
&+\lambda^+_{j\kappa_1}\mu^-_{j\kappa_2}\lambda^+_{j+n,\kappa_3}\mu^-_{j+n,\kappa_4}<\beta^+_{\kappa_1}\beta_{-\kappa_2}\beta^+_{\kappa_3}\beta_{-\kappa_4}>e^{\imath(E_{\kappa_1}-E_{-\kappa_2})t}-\\
&-\lambda^+_{j\kappa_1}\mu^-_{j\kappa_2}\mu^+_{j+n,\kappa_3}\lambda^-_{j+n,\kappa_4}<\beta^+_{\kappa_1}\beta_{-\kappa_2}\beta_{-\kappa_3}\beta^+_{\kappa_4}>e^{\imath(E_{\kappa_1}-E_{-\kappa_2})t}-\\
&-\mu^+_{j\kappa_1}\lambda^-_{j\kappa_2}\lambda^+_{j+n,\kappa_3}\mu^-_{j+n,\kappa_4}<\beta_{-\kappa_1}\beta^+_{\kappa_2}\beta^+_{\kappa_3}\beta_{-\kappa_4}>e^{\imath(E_{\kappa_2}-E_{-\kappa_1})t}+\\
&+\mu^+_{j\kappa_1}\lambda^-_{j\kappa_2}\mu^+_{j+n,\kappa_3}\lambda^-_{j+n,\kappa_4}<\beta_{-\kappa_1}\beta^+_{\kappa_2}\beta_{-\kappa_3}\beta^+_{\kappa_4}>e^{\imath(E_{\kappa_2}-E_{-\kappa_1})t}-\\
&-\mu^+_{j\kappa_1}\mu^-_{j\kappa_2}\lambda^+_{j+n,\kappa_3}\lambda^-_{j+n,\kappa_4}<\beta_{-\kappa_1}\beta_{-\kappa_2}\beta^+_{\kappa_3}\beta^+_{\kappa_4}>e^{-\imath(E_{-\kappa_1}+E_{-\kappa_2})t}\Big].
\end{aligned}
\tag{5.1}
$$

In r.h.s. of (5.1) only non-zero averages of β-operators are written down and the following relations

$$\beta^+_\kappa(t)=\beta^+_\kappa\exp(\imath E_\kappa t),\quad \beta_\kappa(t)=\beta_\kappa\exp(-\imath E_\kappa t) \tag{5.2}$$

were used. The averages of β-operators can be calculated using Wick-Bloch-de Dominicis theorem, e.g.

$$
\begin{aligned}
&<\beta^+_{\kappa_1}\beta^+_{\kappa_2}\beta_{-\kappa_3}\beta_{-\kappa_4}>=\\
&=-\frac{\delta_{\kappa_1,-\kappa_3}}{1+e^{\beta E_{\kappa_1}}}\frac{\delta_{\kappa_2,-\kappa_4}}{1+e^{\beta E_{\kappa_2}}}+\frac{\delta_{\kappa_1,-\kappa_4}}{1+e^{\beta E_{\kappa_1}}}\frac{\delta_{\kappa_2,-\kappa_3}}{1+e^{\beta E_{\kappa_2}}}=\\
&=-f_{\kappa_1}f_{\kappa_2}\delta_{\kappa_1,-\kappa_3}\delta_{\kappa_2,-\kappa_4}+f_{\kappa_1}f_{\kappa_2}\delta_{\kappa_1,-\kappa_4}\delta_{\kappa_2,-\kappa_3}
\end{aligned}
\tag{5.3}
$$

etc. After computation of these averages one finds that the coefficients near the averages contain the following products $\lambda^{+}_{j\kappa}\mu^{+}_{j+n,-\kappa}$, $\lambda^{+}_{j+n,\kappa}\mu^{+}_{j,-\kappa}$, $\lambda^{-}_{j\kappa}\mu^{-}_{j+n,-\kappa}$, $\lambda^{-}_{j+n,\kappa}\mu^{-}_{j,-\kappa}$, $\lambda^{+}_{j\kappa}\mu^{-}_{j+n,-\kappa}$, $\lambda^{+}_{j+n,\kappa}\mu^{-}_{j,-\kappa}$, $\lambda^{-}_{j\kappa}\mu^{+}_{j+n,-\kappa}$, $\lambda^{-}_{j+n,\kappa}\mu^{+}_{j,-\kappa}$. They can be found with the help of (4.2). For simplicity in what follows their values will be used in the case when $J^{xy} = -J^{yx} = D$. Then

$$\begin{aligned}
\lambda^{+}_{j\kappa}\mu^{+}_{j+n,-\kappa} &= \tfrac{1}{N}e^{-\imath\kappa n} = \lambda^{-}_{j\kappa}\mu^{-}_{j+n,-\kappa}, \\
\lambda^{+}_{j+n,\kappa}\mu^{+}_{j,-\kappa} &= \tfrac{1}{N}e^{\imath\kappa n} = \lambda^{-}_{j+n,\kappa}\mu^{-}_{j,-\kappa}, \\
\lambda^{+}_{j\kappa}\mu^{-}_{j+n,-\kappa} &= \tfrac{1}{N}e^{-\imath(\kappa n+\psi_\kappa)}, \qquad \lambda^{+}_{j+n,\kappa}\mu^{-}_{j,-\kappa} = \tfrac{1}{N}e^{\imath(\kappa n-\psi_\kappa)}, \\
\lambda^{-}_{j\kappa}\mu^{+}_{j+n,-\kappa} &= \tfrac{1}{N}e^{-\imath(\kappa n-\psi_\kappa)}, \qquad \lambda^{-}_{j+n,\kappa}\mu^{+}_{j,-\kappa} = \tfrac{1}{N}e^{\imath(\kappa n+\psi_\kappa)}.
\end{aligned} \tag{5.4}$$

Gathering (5.1)-(5.4) together one derives the desired expression for transverse time-dependent correlation function for the model (1.30)

$$\begin{aligned}
&4 < s^z_j(t)s^z_{j+n} >= \\
&= \left[\frac{1}{N}\sum_\kappa \frac{\cosh\left(-\imath E_\kappa t + \imath\kappa n + \frac{\beta E_\kappa}{2}\right)}{\cosh\left(\frac{\beta E_\kappa}{2}\right)}\right]^2 + \left[\frac{1}{N}\sum_\kappa \frac{\sinh\left(\imath\psi_\kappa + \frac{\beta E_\kappa}{2}\right)}{\cosh\left(\frac{\beta E_\kappa}{2}\right)}\right]^2 - \\
&- \left[\frac{1}{N}\sum_\kappa \frac{\sinh\left(-\imath E_\kappa t + \imath\kappa n + \imath\psi_\kappa + \frac{\beta E_\kappa}{2}\right)}{\cosh\left(\frac{\beta E_\kappa}{2}\right)}\right] \times \\
&\times \left[\frac{1}{N}\sum_\kappa \frac{\sinh\left(-\imath E_\kappa t + \imath\kappa n - \imath\psi_\kappa + \frac{\beta E_\kappa}{2}\right)}{\cosh\left(\frac{\beta E_\kappa}{2}\right)}\right].
\end{aligned} \tag{5.5}$$

Although E_κ and $\cos\psi_\kappa$, $\sin\psi_\kappa$ in (5.5) are determined by formulae (2.13), (4.13) for the case $J^{xy} = -J^{yx} = D$ the obtained result covers the case (1.26) as well. Keeping in mind formulae (1.27) and (1.31) one should simply use $J^{xx}\cos^2\alpha + \frac{J^{xy}+J^{yx}}{2}\sin 2\alpha + J^{yy}\sin^2\alpha$ instead of J^x, $J^{xx}\sin^2\alpha - \frac{J^{xy}+J^{yx}}{2}\sin 2\alpha + J^{yy}\cos^2\alpha$ instead of J^y, and $\frac{J^{xy}-J^{yx}}{2}$ instead of D with $\tan 2\alpha = (J^{xy}+J^{yx})/(J^{xx}-J^{yy})$. If one puts $D=0$ in (5.5) it transforms into the well-known result obtained by Th.Niemeijer [21]. The depicted in figs. 17-20 dependence of the transverse dynamical autocorrelation function (5.5) on time ($\beta = 10/J^{xx}$; $1: D=0,\ \Omega=0,\ 1': D=J^{xx},\ \Omega=0$; $2: D=0,\ \Omega=J^{xx},\ 2': D=J^{xx},\ \Omega=J^{xx}$) shows substantial changes caused by Dzyaloshinskii-Moriya interaction.

The dynamical susceptibility

$$\chi_{\alpha\beta}(\kappa,\omega) \equiv \sum_{n=1}^{N} e^{\imath\kappa n}\int_0^\infty dt e^{\imath(\omega+\imath\epsilon)t}\frac{1}{\imath} < [s^\alpha_j(t), s^\beta_{j+n}] > \tag{5.6}$$

is of great interest from the point of view of observable properties of the system. The obtained result (5.5) permits one to calculate the transverse dynamical susceptibility. Really, taking into account the translation invariance one gets

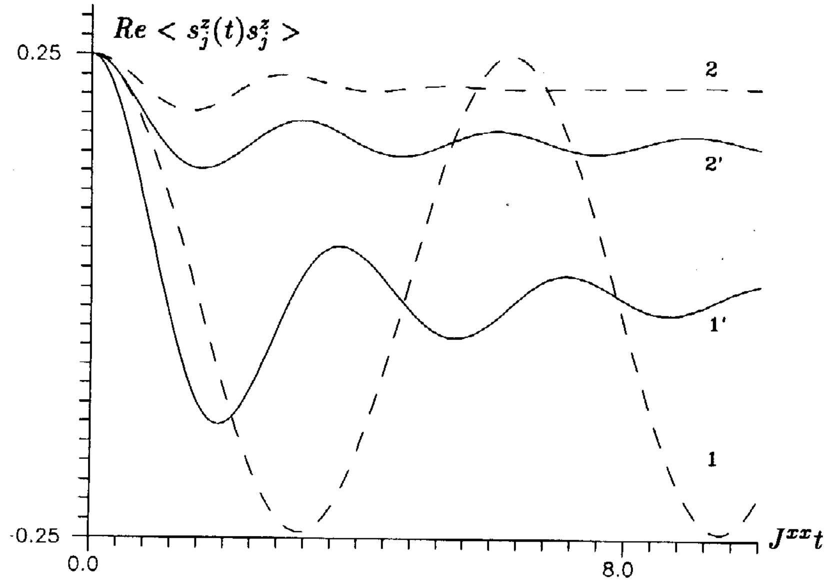

Figure 17: $Re < s_j^z(t) s_j^z >$ vs. $J^{xx}t$; $J^{yy} = 0$.

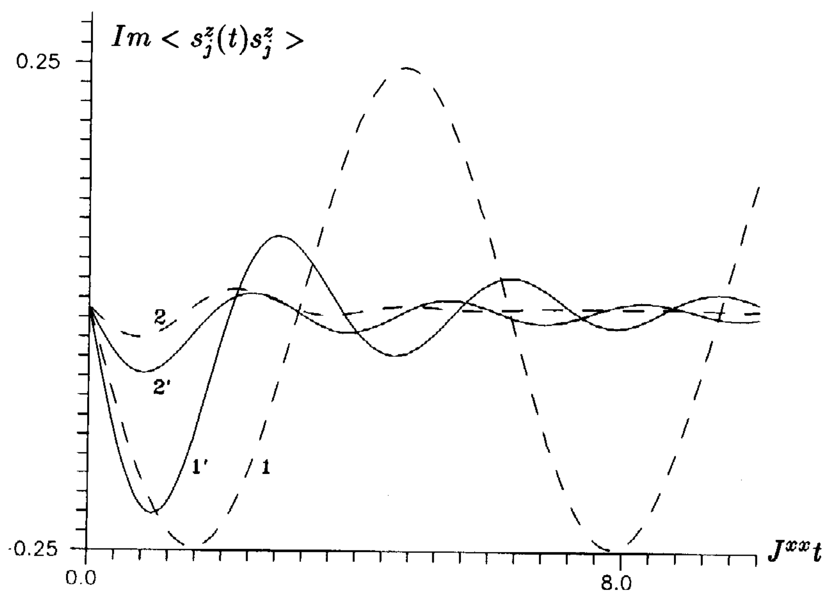

Figure 18: $Im < s_j^z(t) s_j^z >$ vs. $J^{xx}t$; $J^{yy} = 0$.

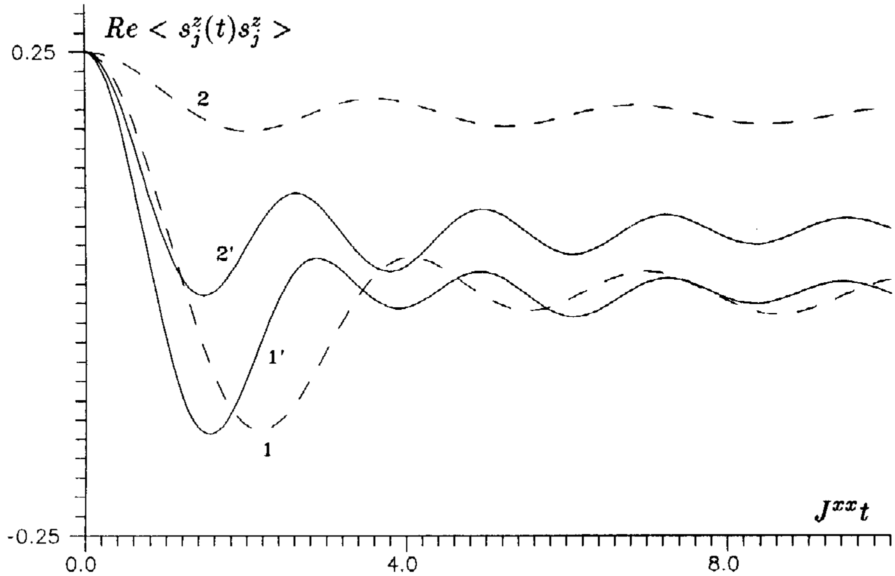

Figure 19: $Re < s_j^z(t)s_j^z >$ vs. $J^{xx}t$; $J^{yy} = J^{xx}$.

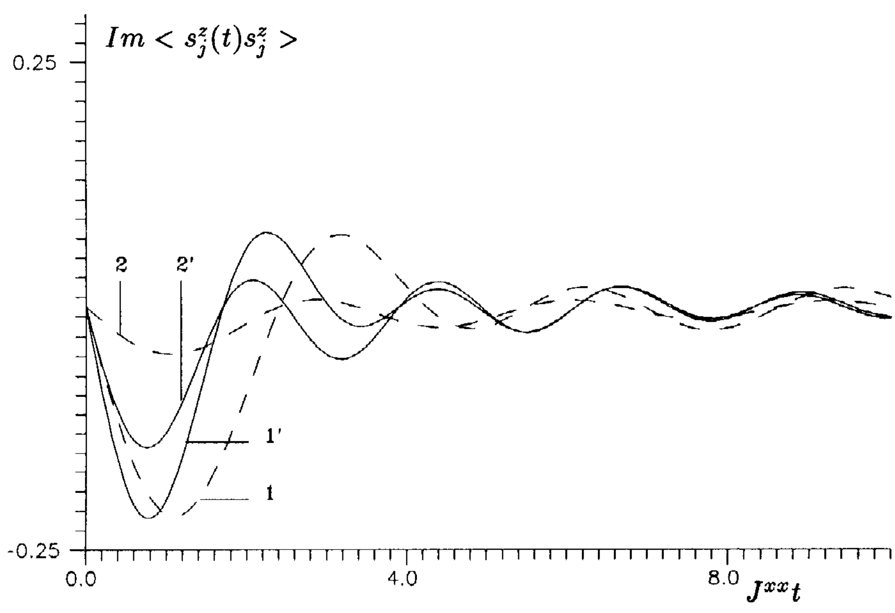

Figure 20: $Im < s_j^z(t)s_j^z >$ vs. $J^{xx}t$; $J^{yy} = J^{xx}$.

$$
\begin{aligned}
&< [s_j^z(t),\, s_{j+n}^z] >=< s_j^z(t)s_{j+n}^z > - < s_j^z(-t)s_{j-n}^z >= \\
&= \imath \left[\tfrac{1}{N}\textstyle\sum_\kappa \cos(\kappa n - E_\kappa t)\right]\left[\tfrac{1}{N}\textstyle\sum_\kappa \sin(\kappa n - E_\kappa t)\tanh\tfrac{\beta E_\kappa}{2}\right] - \\
&-\tfrac{\imath}{2}\left\{\left[\tfrac{1}{N}\textstyle\sum_\kappa \cos(\kappa n - E_\kappa t - \psi_\kappa)\tanh\tfrac{\beta E_\kappa}{2}\right]\left[\tfrac{1}{N}\textstyle\sum_\kappa \sin(\kappa n - E_\kappa t + \psi_\kappa)\right] + \right.\\
&\left. + \left[\tfrac{1}{N}\textstyle\sum_\kappa \cos(\kappa n - E_\kappa t + \psi_\kappa)\tanh\tfrac{\beta E_\kappa}{2}\right]\left[\tfrac{1}{N}\textstyle\sum_\kappa \sin(\kappa n - E_\kappa t - \psi_\kappa)\right]\right\}.
\end{aligned} \tag{5.7}
$$

Using for summation over sites in (5.6) the lattice sum $\frac{1}{N}\sum_{n=1}^{N} e^{\imath\kappa n} = \delta_{\kappa,0}$, evaluating the integrals over t of the form

$$
\int_0^\infty dt e^{\imath(\omega+F_\kappa+\imath\varepsilon)t} = \frac{\imath}{\omega + F_\kappa + \imath\varepsilon}, \tag{5.8}
$$

bearing in mind the definition of functions $\cos\psi_\kappa$, $\sin\psi_\kappa$, and performing thermodynamical limit one obtains:

$$
\begin{aligned}
\chi_{zz}(\kappa,\omega) = \tfrac{1}{8\pi}\int_{-\pi}^{\pi} d\rho &\left[\tfrac{1+\cos(\psi_\rho+\psi_{\rho-\kappa})}{E_{\rho-\kappa}-E_\rho-\omega-\imath\varepsilon} + \tfrac{1-\cos(\psi_\rho+\psi_{\rho-\kappa})}{-E_{\kappa-\rho}-E_\rho-\omega-\imath\varepsilon} - \right.\\
&\left. - \tfrac{1+\cos(\psi_\rho+\psi_{\rho+\kappa})}{-E_{\rho+\kappa}+E_\rho-\omega-\imath\varepsilon} - \tfrac{1-\cos(\psi_\rho+\psi_{\rho+\kappa})}{E_{-\rho-\kappa}+E_\rho-\omega-\imath\varepsilon}\right]\tanh\tfrac{\beta E_\rho}{2}.
\end{aligned} \tag{5.9}
$$

Using the relation

$$
\frac{1}{F_\rho - \omega - \imath\varepsilon} = \mathcal{P}\frac{1}{F_\rho - \omega} + \imath\pi\delta(F_\rho - \omega), \tag{5.10}
$$

for real and imaginary parts of transverse susceptibility one gets final expressions

$$
\begin{aligned}
Re\chi_{zz}(\kappa,\omega) = \tfrac{1}{8\pi}\mathcal{P}\int_{-\pi}^{\pi} d\rho &\left[\tfrac{1+\cos(\psi_\rho+\psi_{\rho-\kappa})}{E_{\rho-\kappa}-E_\rho-\omega} + \tfrac{1-\cos(\psi_\rho+\psi_{\rho-\kappa})}{-E_{\kappa-\rho}-E_\rho-\omega} - \right.\\
&\left. - \tfrac{1+\cos(\psi_\rho+\psi_{\rho+\kappa})}{-E_{\rho+\kappa}+E_\rho-\omega} - \tfrac{1-\cos(\psi_\rho+\psi_{\rho+\kappa})}{E_{-\rho-\kappa}+E_\rho-\omega}\right]\tanh\tfrac{\beta E_\rho}{2},
\end{aligned} \tag{5.11}
$$

$$
\begin{aligned}
Im\chi_{zz}(\kappa,\omega) = \tfrac{1}{8}\int_{-\pi}^{\pi} d\rho\,\{ &[1+\cos(\psi_\rho+\psi_{\rho-\kappa})]\,\delta(E_{\rho-\kappa}-E_\rho-\omega) + \\
&+ [1-\cos(\psi_\rho+\psi_{\rho-\kappa})]\,\delta(-E_{\kappa-\rho}-E_\rho-\omega) - \\
&- [1+\cos(\psi_\rho+\psi_{\rho+\kappa})]\,\delta(-E_{\rho+\kappa}+E_\rho-\omega) - \\
&- [1-\cos(\psi_\rho+\psi_{\rho+\kappa})]\,\delta(E_{-\rho-\kappa}+E_\rho-\omega)\}\tanh\tfrac{\beta E_\rho}{2}.
\end{aligned} \tag{5.12}
$$

These are the main results of the present paper.

It is useful to look at the particular case $\kappa = 0$. In this case one has

$$
\begin{aligned}
\chi_{zz}(0,\omega) = \tfrac{1}{4\pi}\int_{-\pi}^{\pi} d\rho \sin^2\psi_\rho &\left[\tfrac{1}{-E_\rho-E_{-\rho}-\omega-\imath\varepsilon} - \right.\\
&\left. - \tfrac{1}{E_\rho+E_{-\rho}-\omega-\imath\varepsilon}\right]\tanh\tfrac{\beta E_\rho}{2},
\end{aligned} \tag{5.13}
$$

and for the imaginary part:

$$
Im\chi_{zz}(0,\omega) = -\frac{1}{4}\int_{-\pi}^{\pi} d\rho \sin^2\psi_\rho \delta(2\mathcal{E}_\rho - \omega)\tanh\frac{\beta E_\rho}{2}. \tag{5.14}
$$

In the case of isotropic XY model with Dzyaloshinskii-Moriya interaction $\sin\psi_\rho = 0$ and $Im\chi_{zz}(0,\omega) = 0$ as one should expect because in this case

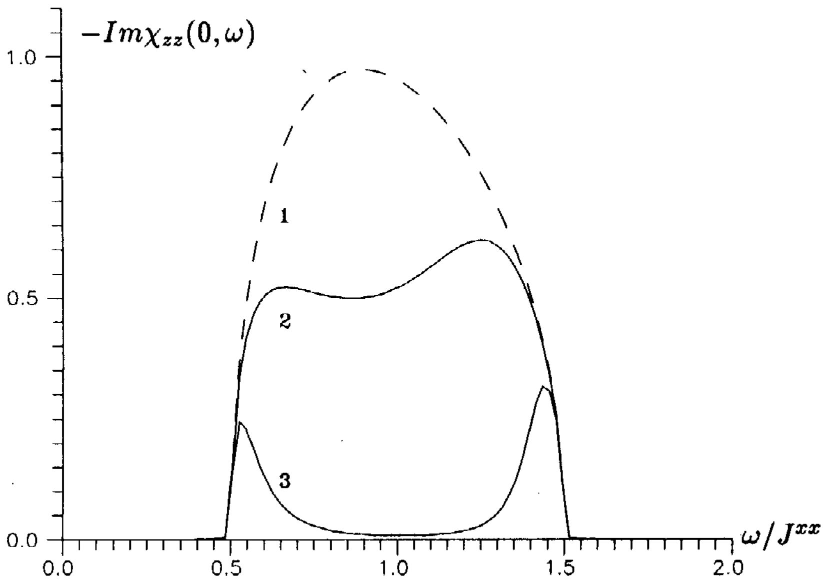

Figure 21: $-Im\chi_{zz}(0,\omega)$ vs. ω/J^{xx}; $\Omega/J^{xx} = 0.25$.

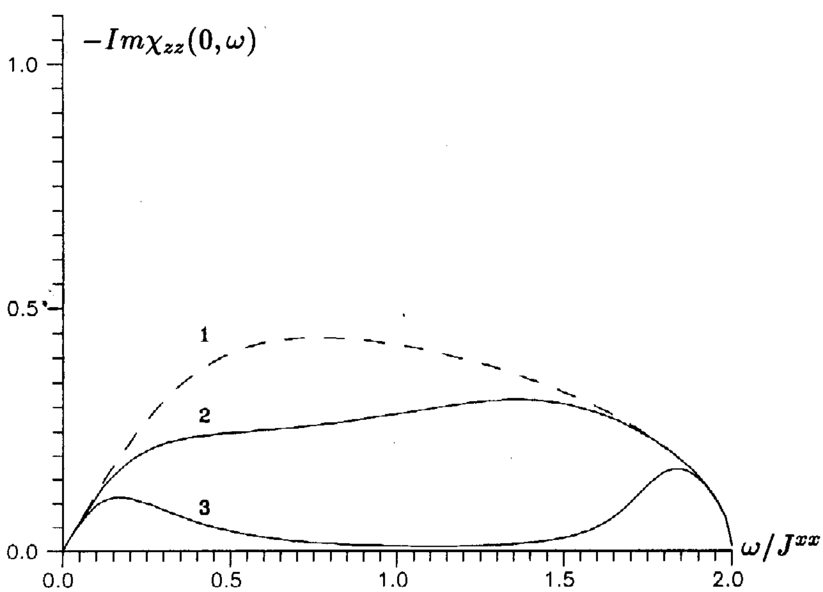

Figure 22: $-Im\chi_{zz}(0,\omega)$ vs. ω/J^{xx}; $\Omega/J^{xx} = 0.5$.

$\left[\sum_{j=1}^{N} s_j^z, H\right] = 0$. In the case of de Gennes model with Dzyaloshinskii-Moriya interaction when $\mathcal{E}_\rho = \sqrt{\Omega^2 + \Omega J \cos\rho + J^2/4}$ one can integrate in (5.14) over ρ using the relation

$$\delta(2\mathcal{E}_\rho - \omega) = \sum_{\rho_0} \frac{\delta(\rho - \rho_0)}{2\left|\frac{\partial \mathcal{E}_\rho}{\partial \rho}\right|}, \tag{5.15}$$

where by $\rho_0 = \rho_0(\omega)$ the solutions of the equation $2\mathcal{E}_{\rho_0} - \omega = 0$ are denoted. This equation can be written in the form

$$\cos\rho_0 = \frac{\omega^2 - J^2 - 4\Omega^2}{4\Omega J}, \tag{5.16}$$

and when ω satisfies inequalities

$$-1 \leq \frac{\omega^2 - J^2 - 4\Omega^2}{4\Omega J} \leq 1 \tag{5.17}$$

or for $\Omega, J > 0$

$$|J - 2\Omega| \leq \omega \leq J + 2\Omega, \tag{5.18}$$

equation (5.16) has two solutions in the region of integration $\rho_0 \geq 0$ and $-\rho_0$. Besides $\partial\mathcal{E}_\rho/\partial\rho = -\Omega J \sin\rho/2\mathcal{E}_\rho$, $\sin\psi_\rho = J\sin\rho/2\mathcal{E}_\rho$, so that in the case of de Gennes model with Dzyaloshinskii-Moriya interaction one gets the following final result

$$Im\chi_{zz}(0,\omega) = \\ = \begin{cases} \frac{-J|\sin\rho_0|}{16\mathcal{E}_{\rho_0}\Omega}\left(\tanh\frac{\beta E_{\rho_0}}{2} + \tanh\frac{\beta E_{-\rho_0}}{2}\right), \text{if} |J - 2\Omega| \leq \omega \leq J + 2\Omega, \\ 0, \text{ otherwise.} \end{cases} \tag{5.19}$$

The presented in figs. 21,22 results of the numerical calculations of frequency dependence of $Im\chi_{zz}(0,\omega)$ (5.13) for de Gennes model with Dzyaloshinskii-Moriya interaction ($\beta = 10/J^{xx}$; $1: D = 0$, $2: D = 0.5J^{xx}$, $3: D = J^{xx}$) show that the presence of this interaction dramatically changes the frequency dependence. This fact seems to be of great importance in connection with the possible experimental prove of evidence of Dzyaloshinskii-Moriya interaction on the base of experimental measurements of $Im\chi_{zz}(0,\omega)$.

Conclusions

Let's sum up the results of present study of statistical mechanics of 1D $s = \frac{1}{2}$ XY anisotropic ring in transverse field with Dzyaloshinskii-Moriya interaction. This interaction keeps the model in the class of 1D $s = \frac{1}{2}$ XY models because after fermionization of the Hamiltonian one is faced with the quadratic in Fermi operators forms. However, after their diagonalization one finds that the spectrum E_κ no longer is even function of κ. This leads only to some technical complications in computations. The obtained thermodynamical functions and static spin correlation functions essentially depend on the value of Dzyaloshinskii-Moriya interaction. For instance, these interaction decreases the transverse magnetization at certain transverse field in de Gennes model and in isotropic XY model (figs. 5,6). They lead to appearance of non-zero spin correlators $< s_j^x s_{j+n}^y >$ and $< s_j^y s_{j+n}^x >$ and to nonmonotonic dependence of pair spin correlation functions on n. The evaluation of transverse dynamical correlation function and the corresponding susceptibility shows that Dzyaloshinskii-Moriya interaction essentially influences on the dynamics of transverse spin correlations (figs. 17-20) and drastically changes the dynamical susceptibility (figs. 21,22).

It is necessary to note in addition that if $J^{xy} = J^{yx} = 0$ all obtained results transform into the corresponding results for anisotropic XY model. Really, in this limit $E_\kappa \to \mathcal{E}_\kappa \to \sqrt{\epsilon_\kappa^{(+)2} + 4(J^{++})^2 \sin^2 \kappa}$ and $F_\kappa = E_{-\kappa}$ The thermodynamical functions due to this simplification because of parity of integrands contain $2\int_0^\pi d\kappa(\ldots)$ instead of $\int_{-\pi}^{\pi} d\kappa(\ldots)$. In contractions (4.9)-(4.13) $< \varphi_j^+ \varphi_{j+n}^+ > \to \delta_{n,0}$, $< \varphi_j^- \varphi_{j+n}^- > \to -\delta_{n,0}$, $< \varphi_j^+ \varphi_{j+n}^- > \to \frac{1}{\pi}\int_0^\pi d\kappa \cos(\kappa n + \psi_\kappa) \tanh \frac{\beta E_\kappa}{2} \equiv G(n)$, $< \varphi_j^- \varphi_{j+n}^+ > \to -G(-n)$ so that $4 < s_j^x s_{j+n}^x >$, $4 < s_j^y s_{j+n}^y >$, $4 < s_j^z s_{j+n}^z >$ (but not their squares) can be rewritten as $N \times N$ determinants and $< s_j^x s_{j+n}^y > = < s_j^y s_{j+n}^x > = 0$. The transverse dynamical correlation function transforms into the corresponding expressions obtained in ref.[21].

Our investigations follow earlier works [18,22-29] considering the derivation of exact results in statistical mechanics of 1D $s = \frac{1}{2}$ systems with Dzyaloshinskii-Moriya interaction.

At last it should be mentioned that for a quite a lot of magnetic and ferroelectric materials, showing nearly 1D behavior above their ordering temperatures, a variety of experimental data are now available [30-40] and thus theoretical investigations of statistical mechanics of 1D spin models may be of great interest for clarifying whether the properties of such simple spin models are capable to caricature the measurements.

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