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# FROM THE HAMILTONIAN MECHANICS TO A CONTINUOUS MEDIA. DISSIPATIVE STRUCTURES. CRITERIA OF SELF-ORGANIZATION

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The paper is aimed at presenting some main ideas and results of the modern statistical theory of macroscopic open systems.

We begin from the demonstration of the necessity and the possibility of the unified description of kinetic, hydrodynamic and diffusion processes in nonlinear macroscopic open systems on the base of a generalized kinetic equations.

A derivation of the generalized kinetic equations is based on the concrete physical definition of continuous media. A"point" of a continuous medium is determined by definition of physically infinitesimal scales. On the same base the definition of the Gibbs ensemble is given. The Boltzman gas and a fully ionized plasma as the test systems are used.

For the transition from the reversible Hamilton equations to the generalized kinetic equations the dynamic instability of the motion of particles plays the constructive role.

The kinetic equation for the Boltzman gas consists of the two dissipative terms: 1. The "collision integral" is defined by processes in a velocity space; 2.The additional dissipative term of the diffusion type in the coordinate space. Owing to the later the unified description of the kinetic, hydrodynamic and diffusion processes for all values of the Knudsen number becomes possible.

The $H$-theorem for he generalized kinetic equation is proved. An entropy production is defined by the sum of two independent positive terms corresponding to redistribution of the particles in velocity and coordinate space respectively.

An entropy flux also consists of two parts. The one is proportional to the entropy, and other one is proportional to the gradient of entropy. The existence of second term allows to give the general definition of the heat flux for any values of the Knudsen number which is proportional to the gradient of entropy. This general definition for small Knudsen number and a constant pressure leads to the Fourier law.

The equations of gas dynamic for special class of distribution functions follow from the generalized kinetic equation without the perturbation theory for the Knudsen number. These equations differ from the traditional ones by taking the self- diffusion processes into account.

The generalized kinetic equation for description the Brownian motion and of autowave processes in active media are considered. The connection with reaction diffusion equations - the Fisher-Kolmogorov-Petrovski-Piscunov and Ginzburg-Landau equations, are established. We discuss the connection between the diffusion of particles in a restricted system with the natural flicker ($1/f$) noise in passive and active systems.

The Criteria of the relative degree of order of the states of open system — the criteria of self-organization, are presented.

## Contents

# 1. The Transition from Reversible Equations of Mechanics to Irreversible Equations of the Statistical Theory

## 1.1. Physical Definition of Continuous Medium

In order to describe the transition to irreversible equations of the statistical theory it is necessary to take into account the structure of the "continuous medium". In other words, it is necessary to give the concrete definition of the infinitesimal scales — to give the physical definition of the notion "point" [1–5].

Obviously, it is impossible to give a unified definition of the physically infinitesimal scales for all systems. We shall introduce these scales for two simplest cases: the Boltzman gas, and a fully ionized plasma. Such definitions are different for the kinetic and hydrodynamic regions of scales.

The kinetic region. A rarefied (Boltzman) gas and a rarefied fully ionized plasma are characterized respectively by the dimensionless small density and plasma parameters

$$\epsilon = nr_0^3, \qquad \mu = 1/nr_D^3. \tag{1.1}$$

$r_0$ is the diameter of atoms, $r_D$ is the Debye radius, $n$ is the mean density of the number of particles. These parameters determine the connection between the infinitesimal scales $\tau_{ph}$, $l_{ph}$ and the corresponding "collision" parameters $\tau$, $l$ of the Boltzman gas and Debye plasma.

We denote by $N_{ph} = nV_{ph}$ the number of particles in the physically infinitesimal volume and define the physically infinitesimal time as

$$\tau/N_{ph} = \tau_{ph}, \qquad N_{ph} = nV_{ph} \sim nl_{ph}^3, \qquad \tau_{ph} = l_{ph}/v_T. \tag{1.2}$$

Thus defined $\tau_{ph}$ is the time within which any particle out of the number $N_{ph}$ in the volume $V_{ph}$ undergoes a collision.

The physically infinitesimal scales for the Boltzman gas follow from the expressions (1.2) (Klimontovich [1-5]).

$$\tau_{ph} \sim \sqrt{\epsilon}\tau \ll \tau, \qquad l_{ph} \sim \sqrt{\epsilon}l \ll l, \qquad N_{ph} \sim \frac{1}{\sqrt{\epsilon}} \gg 1. \tag{1.3}$$

The corresponding kinetic characteristics for a rarefied plasma are ($a = e, i$)

$$\tau_{ph}^{(a)} \sim \mu\tau_{(a)}, \qquad l_{ph}^{(a)} \sim r_D \sim \mu l_{(a)} \gg l_{(a)}, \qquad N_{ph} \sim 1/\mu \gg 1. \tag{1.4}$$

We see that for the Boltzman gas and a rarefied plasmas the general conditions imposed on the physically infinitesimal scales are satisfied.

**The gas dynamic region.** In this case the relaxation time is defined by the characteristic scale of the system $L : \tau_D = L^2/D$. The role $D$ is played by viscosity $\nu$, thermal conductivity $\chi$, and the self-diffusion coefficient $D$. Here $D = \nu = \chi$. The physically infinitesimal characteristics are defined now by relations

$$\tau_{ph}^G \sim \frac{\tau_D}{N_{ph}^G}, \qquad l_{ph}^G \sim \frac{L}{N^{1/5}}, \qquad N_{ph}^G \sim N^{2/5}. \tag{1.5}$$

Here $N = nL^3$.

The definition of physically infinitesimal scales allows to use the notion of the "continuous medium", respectively, for the kinetic and hydrodynamic description. In order to illustrate the importance of this notion, we consider the following example.

**The maximum of the Reynolds number in the Kolmogorov theory.**

In the Kolmogorov theory of a developed turbulence the number of collective (turbulent) degree of freedom $N_{turb} = (L/L_0)^3$ is connected with the Reynolds number by the relation [7,8,4]

$$N_{turb} \sim Re^{9/4}. \tag{1.6}$$

Here $L$, and $L_0$ are, respectively, the largest and the smallest scales of the developed turbulent motion.

From the definition $N_{turb}$ follows that the maximum value of the Reynolds number in the Kolmogorov theory is restricted by the condition $L_0 > l^G_{ph}$ and, as a result

$$(Re)_{max} \sim N^{4/15}, \quad N = nL^3. \tag{1.7}$$

We see that, as consequence, the limit $Re \to \infty$, which is used frequently in the theory of turbulent motion, cannot be realized for the "physical continuous medium".

## 1.2. The Gibbs Ensemble for Nonequilibrium Processes

The statistical ensemble was introduced for describing the equilibrium state. In this case the number of controllable degrees of freedom is very small, and hence the indeterminacy in the microscopic states of the system is extremely high. This way of defining of statistical ensemble of arbitrary nonequilibrium states is not suitable. What can be done?

Assume that we have chosen the concrete definition of the physically infinitesimal scales. We can then assume that the indeterminacy of defining the microscopic states of systems in the Gibbs ensemble is governed by the indeterminacy in the states of particles confined within the volume $V_{ph}$. This allows us to carry out the operation of averaging, or smoothing of dynamic distributions over a physically infinitesimal volume.

## 1.3. The Unified Definition of "Continuous Medium". Averaging over Physically Infinitesimal Volume [27,28]

In order to develop the unified description of the kinetic and hydrodynamic processes, it is necessary to use the equation $(V^G_{ph})_{min} = V_{ph}$ to express the minimal physically infinitesimal volume $(V^G_{ph})_{min} \equiv L^3_{min}$ via small density parameter $\epsilon$.

$$L_{min} \sim (N_{ph})^{1/2} l_{ph} \sim 1/(N_{ph})^{1/2}, \qquad N_{min} \sim nL^3_{min} \sim \epsilon^{-5/4}. \tag{1.8}$$

Thus, for example, at atmospheric pressure the density parameter $\epsilon \approx 10^{-4}$ and therefore the number of particles in the "point" $n_{min}$ is of order $10^5$.

Let us consider a local random function $N(r,p,t)$ — the microscopic phase density in the six-dimensional space of coordinate and momenta

$$N(r,p,t) = \sum_{1 \le i \le N} \delta\Big(r - r_i(t)\Big)\delta\Big(p - p_i(t)\Big) \tag{1.9}$$

and introduce the new function smoothed over the distribution

$$\begin{aligned} F(\rho) &= (2\pi L_{min}^2)^{-3/2} \exp\left(-\frac{\rho}{(L)_{min}^2}\right), \\ \tilde{N}(r,p,t) &= \int N(r-\rho,p,t)F(\rho)d\rho. \end{aligned} \tag{1.10}$$

As initial we shall use the following dynamic reversible equations for the microscopical function (1.9) [1-6,9]

$$\begin{gathered} \frac{\partial N}{\partial t} + v\frac{\partial N}{\partial r} + F^m(r,t)\frac{\partial N}{\partial p} = 0, \\ F^m(r,t) = F_0 - \frac{\partial}{\partial r}\int \Phi(|r-r'|)N(r',p',t)dr'dp'. \end{gathered} \tag{1.11}$$

Here $F^m$ is the microscopic force.

In order to obtain the generalized kinetic equation on the basis of this dynamic reversible equation, it is necessary, on the first step to the irreversible equation, to take into account the dynamic instability of the motion of atoms.

## 1.4. The Constructive Role of the Dynamic Instability of the Motion of Atoms

The kinetic theory of gases is traditionally constructed (the Bogolubov-Born-Green-Kirkwood-Yvon theory, for example) without using the principal ideas of the dynamic theory — the concepts of dynamic instability, dynamic chaos, $K$-entropy, mixing. Let us show that these enable us to elucidate more fully the causes of irreversibility and to obtain the generalized kinetic equation.

The first steps in this direction were made by N.S.Krylov in 1950 [10] dealing with the substantiation of statistical physics. The importance of the dynamic instability of motion in justifying the irreversible equations of the statistical physics is discussed elsewhere Prigogine [1], and in [13,14].

Let us make some elementary quantitative estimates that will connect the time interval $t$, introduced here by way of the physically infinitesimal time scale, with the minimum time of development of dynamic instability .

The motion of atom spheres in a Boltzman gas, like the motion of balls in the Sinai billiards, is dynamically unstable. For a single atom the time of development of instability — or, in other words, the characteristic time of exponential divergence of initially close trajectories of two atoms — is of the order of the free path time [13,14]:

$$\tau_{inst} \sim \tau. \tag{1.12}$$

This estimate relates to the path of a single chosen atom. We take into account that in the process of smoothing over the physically infinitesimal volume $V_{ph}$ all $N_{ph}$ particles contained therein are indistinguishable. We can therefore introduce, along with $\tau_{inst}$,the characteristic time of development of instability for any one particle within $V_{ph}$ . Let us denote the new quantity by $(\tau_{inst})_{min}$. This time it is $N_{ph}$ times shorter than $\tau_{inst}$, and hence, given the definition (1.12), we come to the following estimate for the minimum characteristic time of development of dynamic instability in the motion of atoms in a Boltzman gas:

$$(\tau_{inst})_{min} \sim \tau_{ph}. \tag{1.13}$$

Thus, the characteristic time for the development of instability of motion per particle within $N_{ph}$ is of the order of the physically infinitesimal time scale $\tau_{ph}$. This is another argument in favour of our method of defining $\tau_{ph}$.

The dynamic instability of motion of atoms in the Boltzman gas leads to mixing and thus facilitates the transition from the reversible Hamiltonian equations to the much simpler Boltzman kinetic equation. This we see as the constructive role of the dynamic instability of motion in the formulation of the statistical theory of nonequilibrium processes.

Macroscopic characteristics, for instance, the distribution function can be termed the functions of order, insofar as they single out and describe a more ordered motion against the background of molecular chaotic motion. In other words, they reveal the statistical laws.

This illustration, drawn for a Boltzman gas, in no way exhausts the constructive role of dynamic instability.

In fact, in statistical theory the dynamic instability can be associated not only with the atoms, but also with the macroscopic characteristics of the system. The latter kind of instability was first discovered with the model equations of thermal convection in a fluid, with the Lorenz equations [13].

An example of a physically feasible system, in which a large amount of feedback gives rise to dynamic instability of the macroscopic characteristics, is the lasers and electrical devices [15–18,5,6].

In connection with the above view concerning the constructive role of dynamic instability of motion of atoms in a Boltzman gas, some questions inevitably arise. Can the dynamic instability of motion of the macroscopic characteristics also play a constructive role? Will this kind of instability lead to more sophisticated dissipative structures, or to "dynamic chaos"? In what relation does physical chaos stand to dynamic chaos?

To answer these questions, we need criteria for the relative degree of order or organization (or, alternatively, chaoticity) the nonequilibrium states of open systems. We shall use the Boltzman-Gibbs-Shannon entropy, renormalized to a given mean effective energy — the effective Hamiltonian function of the open system.

The quantitative assessment of the relative order will be based on the $S$-Theorem.

The $S$-theorem will be used to check on the proper choice of the controlling parameters, and then to assess the relative degree of order. It is possible also to organize the optimization of the search for the most ordered states in the space of the controlling parameters of the open systems.

Basing our reasoning on the criteria of the relative degree of order, we shall use some examples to demonstrate that the processes of self-organization are also possible in the presence of the dynamic instability of motion of the macroscopic characteristics of open systems. However, we shall first continue to discuss the question about the generalized kinetic equation.

## 2. The Unified Description of Kinetic and Hydrodynamic Motion. The Generalized Kinetic Equation [5,27,28]

In order to take into account the existence of the dynamic instability of the motion of atoms, we can introduce in the reversible dynamic equation (1.11) the following term

$$-\frac{1}{\tau_{ph}}\Big(N(r,p,t)-\tilde{N}(r,p,t)\Big). \qquad (2.1)$$

which describe the relaxation from the dynamic microscopic phase density $N(r,p,t)$ to the smoothed distribution $\tilde{N}(r,p,t)$. After the averaging over the Gibbs ensemble we can obtain the equation for the distribution function $f(r,p,t) = \langle N(r,p,t)\rangle / n$ with the additional relaxation term

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial r} + F(r,t)\frac{\partial f}{\partial p} = -\frac{1}{n}\frac{\partial\langle\delta F\delta N\rangle}{\partial p} - \frac{1}{\tau_{ph}}\Big(f(r,p,t) - \tilde{f}(r,p,t)\Big),$$
$$F(r,t) = F_0 - n\int \frac{\partial\Phi(|r - r'|)}{\partial r} f(r',p',t)dr'dp'. \tag{2.2}$$

In the right side of this equation now there are two dissipative terms. The first one describes the dissipation by processes in the velocity space — by "collisions" [1–6,8]. This term can be presented as the Boltzman collision integral for a rarefied gas, as the Landau or the Balescu-Lenard "collision integral" for a fully ionized plasmas.

In order to obtain the generalized kinetic equation for the unified description of kinetic and hydrodynamic processes it is naturally to use the most simple form for this "collision integral. Indeed, we saw that number of particle $N_{\min} \sim \epsilon^{-5/4}$ and for a rarefied gas much more unity ($N_{\min} \approx 10^5$ at atmospheric pressure), therefore it is possible to use not the Boltzman form but the nonlinear Fokker-Planck form for the "collision integral"

$$-\frac{1}{n}\frac{\partial\langle\delta F\delta N\rangle}{\partial p} \equiv I_{(v)}(r,p,t) = \frac{\partial}{\partial v}\Big[D_{(v)}(r,t)\frac{\partial f}{\partial v}\Big] + \frac{\partial}{\partial v}\Big[\frac{1}{\tau}\Big(v - u(r,t)\Big)f\Big]. \tag{2.3}$$

The diffusion coefficient in the velocity space is defined by the local temperature

$$D_{(v)}(r,t) = \frac{1}{\tau}\kappa T(r,t) = \frac{m}{3}\int \Big(v - u(r,t)\Big)^2 f(r,p,t)dv, \tag{2.4}$$

$u(r,t)$ is a local hydrodynamic velocity, $\tau$ is a "collision" time.

In order to obtain the final form for the second "collision integral", we can use the expansion on the "physical Knudsen number"

$$(Kn)_{ph} = \frac{l_{ph}}{L}, \quad \text{Here} \quad l_{ph} = L_{\min}. \tag{2.5}$$

If the mean force $F(r,t)$ is not zero, then it is necessary to change the definition (1.10) for the smoothing function: the mean value $\langle\rho\rangle = (b/m)F\tau_{ph}$. Here $b$ is the mobility of atoms.

By the expansion of the additional dissipative term in the equation (2.3) on the "physical Knudsen parameter" (2.5), we have the following expression for the new "collision integral"

$$-\frac{1}{\tau_{ph}}(f - \tilde{f}) \equiv I_r(r,p,t) = \frac{\partial}{\partial r}\Big(\frac{L_{\min}^2}{\tau_{ph}}\frac{\partial f}{\partial r}\Big) - \frac{\partial}{\partial r}\Big(\frac{b}{m}Ff\Big). \tag{2.6}$$

In order to obtain the final form for the new "collision integral", we must take into account the definitions (1.3), (1.8) for the scales $\tau_{ph}, L_{\min}$. In result we have:

$$\frac{L_{\text{min}}^2}{\tau_{ph}} \approx \frac{l^2}{\tau} = D = b\frac{\kappa T}{m}. \tag{2.7}$$

Here $D$ is the space self-diffusion coefficient which is connected with the mobility coefficient by the Einstein relation. For nonequilibrium states the Einstein relation connects local characteristics, therefore the final form for the new "collision" integral is

$$I_r(r,p,t) = \frac{\partial}{\partial r}\Big[D(r,t)\frac{\partial f}{\partial r} - \frac{b}{m}Ff\Big], \quad D(r,t) = b\frac{\kappa T}{m}. \tag{2.8}$$

It is useful to remind that $D$ is one of the three kinetic coefficients: kinematic viscosity $\nu$, temperature conductivity $\chi$, and self-diffusion D. For our model these three coefficients are equal, $D = \nu = \chi$. The distinction between $D$, $\nu$, $\chi$ it is possible to take into account in the equation of hydrodynamics.

The inclusion of self-diffusion into the equations of hydrodynamics has been suggested more than once [see in 5,6]. It was argued that, in the first, self-diffusion distorts the conventional structure of the equations of hydrodynamics, and, in the second, from the kinetic Boltzman equation it follows that self-diffusion is absent from the equations of hydrodynamics, since the transfer of matter is completely determined by the convective flow $\rho u$. (Landau and Lifshits [8], p. 274 of the Russian edition) observed that the inclusion of self-diffusion into the hydrodynamic equations may result in the violation of the condition that the entropy production in a closed system should be positive, thus violating the second law of thermodynamics.

Of course, the last two arguments cannot be dismissed, and in the next sections we shall discuss them.

At this point we just note the following.

In the equilibrium state the kinetic equation (2.2) is satisfied by the Maxwell-Boltzman distribution. The left-hand side, determined by the nondissipative terms, goes to zero independently of the dissipative terms on the right-hand side, each of which also goes to zero.

The first of the latter, which is either the Boltzman collision integral or more simple "collision integral" (2.3) through cancelling out the collisions of two types, forms the Maxwell distribution.

The second dissipative term on the right-hand side also describes the balance of two dissipative flows. One of these is caused by the external force and is proportional to the mobility, while the second is the flow of matter due to self-diffusion. In this way the existence of the equilibrium Boltzman distribution can be regarded as a manifestation of self-diffusion.

## 3. The Equation of Entropy Balance. The Heat Flow

The local Boltzman entropy is defined by the expression

$$S(r,t) \equiv \rho(r,t)s(r,t) = -\kappa n\int \ln\Big(nf(r,p,t)\Big)f(r,p,t)dp. \tag{3.1}$$

At the condition $F = const$ the equation of entropy balance for the generalized kinetic equation (2.2) it is possible to present in the following elegant form

$$\frac{\partial\rho s}{\partial t} + \frac{\partial}{\partial r}\Big[(\rho u - D\frac{\partial\rho}{\partial r} + \frac{b}{m}F)s\Big] = \frac{\partial}{\partial r}\Big(D\rho\frac{\partial s}{\partial r}\Big) + \sigma(r,t). \tag{3.2}$$

The full entropy production is defined by the expression

$$\sigma(t) = \kappa n \int D_v f\Big(\frac{\partial}{\partial v} ln \frac{f}{f_{loc}}\Big)^2 drdp + \kappa n \int Df\Big(\frac{\partial}{\partial r} ln \frac{f}{f_0}\Big)^2 drdp \geq 0. \quad (3.3)$$

Here $f_0$ is the Maxwell distribution, and the $f_{loc}$ is the local one.

We see that the entropy production is the sum of two positive terms are defined, respectively, by the changing of the distribution function in the velocity and coordinate spaces.

The entropy flow $j_S$ consists of two parts:

$$j_S(r,t) = j_{con} + j_{dif}. \quad (3.4)$$

The convective part $j_{con}$ is proportional to the full flow of matter and the entropy $s(r,t)$ but the diffusion part $j_{dif}$ is proportional to the gradient of entropy $s(r,t)$. It is naturally to define the heat flow by the expression [27,28]

$$q(r,t) = T(r,t) j_{dif} = -T(r,t) D\rho \frac{\partial s}{\partial r}. \quad (3.5)$$

For the region of small values of the Knudsen number, when exists the local equilibrium, the heat flow is defined by the gradients of density and temperature:

$$q(r,t) = \frac{\kappa}{m} D \frac{\partial \rho}{\partial r} - c_v \rho \chi \frac{1}{T} \frac{\partial T}{\partial r}, \quad c_v = \frac{3}{2} \frac{\kappa}{m}. \quad (3.6)$$

Only for slow processes, when the pressure $p = const$, and the gradient of density is proportional to the temperature gradient, the heat flow is defined by the Fourier law

$$q(r,t) = -\lambda \frac{\partial T}{\partial r}, \quad \lambda = c_p \rho \chi, \quad c_p = \frac{5}{2} \frac{\kappa}{m}. \quad (3.7)$$

Here $c_p$ is a heat capacity at constant pressure.

## 4. Equations of Hydrodynamics with Self-Diffusion

The representation of the "collision integral" $I_v(r,p,t)$ in the form (2.3) based on the fact that the number of the particles in a "point" $N_{\min} \sim \epsilon^{-5/4} \gg 1$. By same reason it is possible to restrict the class of distribution functions $f(r,p,t)$ by condition

$$f(r,p,t) = f(r, m \mid v - u(r,t) \mid, t). \quad (4.1)$$

Then the closed system of equations for the gas-dynamic functions follows from the generalized kinetic equation (2.2), with "collision integrals" (2.3), and (2.7), without the perturbation theory for the Knudsen number.

In order to obtain the gas-dynamic equations for the functions $\rho(r,t)$, $u(r,t)$, and $T(r,t)$, we substitute distribution (4.1) into kinetic equation (2.2) and go over to equations in the relevant moments of the distribution (4.1) As a result, we come to the set of equations of hydrodynamics with due account for self-diffusion:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r} j(r,t) = 0, \quad j = \rho u - D\frac{\partial \rho}{\partial r} + \frac{b}{m}F\rho, \tag{4.2}$$

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial}{\partial r}(j_j u_i) = -\frac{\partial p}{\partial r} + \rho\nu\frac{\partial^2 u_i}{\partial r^2} + \frac{\rho}{m}F, \quad p = \frac{\rho}{m}\kappa T, \tag{4.3}$$

$$\tfrac{\partial}{\partial t}\Big(\tfrac{\rho u^2}{2} + \epsilon(r,t)\Big) + \tfrac{\partial}{\partial r_i}\Big[j_i(\tfrac{\rho u^2}{2} + \epsilon) + u_i p - \rho\nu u_j \tfrac{\partial u_i}{\partial r_j} - c_v \rho\chi\tfrac{\partial T}{\partial r_i}\Big] = \tfrac{\rho}{m}Fu,$$

$$\epsilon(r,t) = \tfrac{\rho}{m}\tfrac{3}{2}\kappa T. \tag{4.4}$$

From the equation of continuity it follows that the transfer of matter is now determined by three flows: convective transfer, self-diffusion and the flow defined by the mobility of atoms. If we take the external force into account, the equation of takes the form

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}\Big(\rho u - D\frac{\partial \rho}{\partial r} + \frac{b}{m}F\rho\Big) = 0. \tag{4.5}$$

Equations (4.2)-(4.5) take into account the fact that the kinetic coefficients $D, \nu, \chi$ may be not all the same. Observe also that in this approximation the tensor of viscous stress has a different form from the conventional representation. Namely. here $\pi_{ij} = -\eta\partial u_i/\partial r_j, \eta = \rho\nu$ .

In the approximation of incompressible fluid, when $\rho$ = *const* and $F$ = *const*, and the temperature (the variance of the velocity) is zero, the distribution function $f(r,p,t)$ takes the form

$$f(r,v,t) = \delta[v - u(r,t)], \tag{4.6}$$

and the equations of hydrodynamics coincide with the Navier- Stokes equations

$$\begin{gathered}\tfrac{\partial u}{\partial t} + (u\nabla)u = -\tfrac{1}{\rho}\nabla p + \nu\Delta u + \tfrac{1}{m}F,\\ \operatorname{div} u = 0, \quad \Delta p = -\rho\tfrac{\partial u_i}{\partial R_j}\tfrac{\partial u_j}{\partial R_i}.\end{gathered} \tag{4.7}$$

The equation for the pressure follows from the equation of continuity.

We can remark here that the Navier-Stokes equations, although extremely efficient, are intrinsically contradictory, because in this approximation the entropy in a closed system does not change, and at the same time the production of entropy(3.3) is not zero.

For the class of distribution functions (4.1) in order to calculate the entropy production the solution of kinetic equation it is necessary to know. Only for the local Maxwell distribution the entropy production completely by functions $\rho(r,t)$, $u(r,t)$, and $T(r,t)$ is defined:

$$\sigma(r,t) = \frac{\kappa}{m}\Big[D\rho\Big(\frac{\nabla\rho}{\rho}\Big)^2 + \nu\rho\frac{m}{\kappa T}\Big(\frac{\partial u_i}{\partial r_j}\Big)^2 + \frac{3}{2}\chi\rho\Big(\frac{\nabla T}{T}\Big)^2\Big] \geq 0. \tag{4.8}$$

Thus, the production of entropy is non-negative also when self-diffusion is taken into account, in full compliance with the second law of thermodynamics.

Observe finally that in the incompressible fluid approximation, from (4.1),(4.8) it follows that the entropy production is nonzero while the entropy itself remains constant. Therefore, for describing thermal processes we must go beyond the incompressible fluid approximation, for instance, to the Boussinesq approximation: to solve the Navier-Stokes equations together with the heat transfer equation

$$\frac{\partial T}{\partial t} + (v\nabla)T = \chi\frac{\partial^2 T}{\partial r^2} + \frac{2}{3}\frac{m\nu}{\kappa}\Big(\frac{\partial u_i}{\partial r_j}\Big)^2. \tag{4.9}$$

We'd like to underline that the Boussinesq approximation only for the local Maxwell distribution is valid. In more general case the generalised kinetic equation it is necessary to use.

## 5. Effect of Self-Diffusion on the Spectra of Hydrodynamic Fluctuations

To calculate equilibrium hydrodynamic fluctuations we use the linearized equations of hydrodynamics (4.2)-(4.4). In the linear approximation the velocity field for the Fourier components is represented as a sum of the transverse (with respect to the wave vector $\vec{k}$) and the longitudinal parts. The inclusion of self-diffusion has no effect on the fluctuations of the transverse field of velocity, so the width of the spectrum of fluctuations is again determined by $\nu\vec{k}^2$, where $\nu$ is the kinematic viscosity.

The set of equations for the Fourier components of the density, the longitudinal velocity, the temperature and the pressure with due account for self- diffusion now has the form

$$\begin{aligned}
&(-i\omega + Dk^2)\delta\rho + \rho i(k\delta u) = 0,\\
&(-i\omega + \nu k^2)i(k\delta u)\rho = k^2\delta p,\\
&-i\omega\delta p + pDk^2\tfrac{\delta\rho}{\rho} = \tfrac{p}{T}\chi k^2\delta T + \tfrac{5}{3}pi(k\delta u),\\
&p = \tfrac{\rho}{m}\kappa T, \quad \delta p\ddot{u} = \tfrac{\kappa}{m}\rho\delta T + \tfrac{\kappa T}{m}\delta\rho.
\end{aligned} \tag{5.1}$$

In calculations of fluctuations, two regions are usually distinguished: 1. low frequencies, when (for $D = \nu = \chi$ ) $\omega, D\vec{k}^2 \ll kv_{sound}$, and 2. high frequencies, when $D\vec{k}^2 \ll \omega \sim kv_{sound}$.

Calculations of equilibrium hydrodynamic fluctuations on the basis of equations of hydrodynamics can be found elsewhere [19], [4]. Here we shall only indicate the basic changes in the spectra which occur when self-diffusion is taken into account.

For low frequencies, the spectra of fluctuations of density, temperature and entropy are now given by

$$(\delta\rho\delta\rho)_{\omega,k} = \frac{2Dk^2}{\omega^2 + (Dk^2)^2}\frac{\kappa\rho}{c_p}; \quad (\delta T\delta T)_{\omega,k} = \frac{2\chi k^2}{\omega^2 + (\chi k^2)^2}\frac{\kappa T^2}{\rho c_p}; \tag{5.2}$$

$$(\delta S \delta S)_{\omega,k} = \Big[\Big(1 - \frac{c_v}{c_p}\Big)\frac{2Dk^2}{\omega^2 + (Dk^2)^2} + \frac{c_v}{c_p}\frac{2\chi k^2}{\omega^2 + (\chi k^2)^2}\Big] c_p \frac{\kappa}{\rho}. \tag{5.3}$$

When self-diffusion is neglected, the widths of all these spectra are the same, and are determined by the temperature conductivity $\chi$. This results in a rigid correlation between the fluctuations of density and temperature at all temperatures $[\delta\rho(\omega, \vec{k})/\rho = \delta T(\omega, \vec{k})/T]$. When self-diffusion is included, this condition is removed: the width of the spectrum of density fluctuations is determined by the coefficient of self-diffusion, and that of the temperature fluctuations depends on the coefficient of temperature conduction $\chi$.

Total correlation is only observed for characteristics which are integral with respect to $\ddot{u}\omega$. This correlation is dictated by the condition of local thermodynamic equilibrium.

The spectrum of fluctuations of the entropy is represented as a sum of two spectral lines, the relative contributions of which depend on the ratio between the heat capacities $c_v$ and $c_p$.

Self-diffusion also affects the spectra of fluctuations at high frequencies. The linewidth is now determined by the combination of the three dissipative characteristics $D, \nu, \chi$:

$$\gamma = \frac{1}{2}\Big[\nu + \frac{c_v}{c_p}D + \Big[1 - \frac{c_v}{c_p}\Big]\chi\Big]k^2. \tag{5.4}$$

Thus, the old problem - whether to include the contribution of self-diffusion into the equations of hydrodynamics - can also be resolved by analyzing experimental data on the spectra of molecular scattering in liquids.

The kinetic approach to the description of hydrodynamic motion described here can be extended to the case of turbulent motion.

## 6. The Kinetic Approach in the Theory of Self-Organization. Synergetics. Basic Mathematical Models

Van der Pol generators, along with the more complicated oscillators, can serve as elements of active (excitable) media. The first mathematical models of active media were proposed four decades ago in the well-known works of Wiener and Rosenbluth; Gelfand and Tsetlin (see in [13]). Landau's work (1944) on the origins of turbulence made also a fundamental contribution to the theory of excitable media.

Currently the theory of active media relies mainly on the equations of reaction-diffusion type [20–28,47]:

$$\frac{\partial X(R,t)}{\partial t} = F[X(R,t)] + \frac{\partial}{\partial R_i}\Big(D_{ij}(X)\frac{\partial X}{\partial R_j}\Big). \tag{6.1}$$

Here $X(R,t)$ is a set of macroscopic functions characterizing the system - for instance, the concentrations of chemical reactants; $F(X)$ are nonlinear functions determined by the structure of the system and the nature of the processes; $D_{ij}$ are the coefficients of spatial diffusion of the elements of the system. Here and bellow $r \equiv R$.

Specific examples of equations like (6.1) have been proposed and analyzed by Fisher (see [24]), Kolmogorov, Petrovsky, Piscunov (1937), Zel'dovich, Turing. To the same type also belong the various modifications of the Ginzburg-Landau equation, which is widely employed in the theory of equilibrium and nonequilibrium phase transitions.

Equations of reaction-diffusion type describe a broad class of physical, chemical and biological phenomena. We begin with an example of the spatial diffusion of the independent Van der Pol oscillators (generators).

If in the description of the generators we restrict ourselves to information concerning the energy of oscillations $E(R,t)$, then appropriate Kolmogorov-Petrovsky-Piskunov (KPP) equation for this function with the constant coefficient of space diffusion $D$ has the form

$$\frac{\partial E(R,t)}{\partial t} = \Big(a - bE(R,t)\Big)E(R,t) + D\frac{\partial^2 E}{\partial R^2}, \quad a = a_f - \gamma. \tag{6.2}$$

Here $\gamma$, and $b$ are linear and nonlinear friction coefficients, respectively. $a_f$ is the feedback parameter.

This equation, however, is not by itself sufficient to give a complete statistical description of the distributed system of generators, since, apart from the spatial diffusion, there is another cause that disturbs the dynamic regime of generation. This due to the effects of noise on the internal degrees of freedom of the generators with the intensity $D_E$. The combined influence of both factors is taken into account in the equation for the distribution function $f(E,R,t)(\int dE\frac{dR}{V}f = 1)$ [5,6,26]

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial E}\Big(D_{(E)}E\frac{\partial f}{\partial E}\Big) + \frac{\partial}{\partial E}[(-a + bE)Ef] + D\frac{\partial^2 f}{\partial R^2}; \tag{6.3}$$

In the special case only, when the noise acting on the internal degrees of freedom of the generator is negligibly small ($D_E \equiv 0$), then the equation (6.3) has a particular solution

$$f(E,R,t) = \delta\Big(E - E(R,t)\Big); \langle E\rangle = \int\limits_0^\infty EfdE = E(R,t), \tag{6.4}$$

which corresponds to the one-moment approximation. The function $E(R,t)$ satisfies the FKPP equation (6.2).

## 7. Kinetic and Hydrodynamic Description of the Heat Transfer in Active Medium [27,28]

Now we shall go back the generalized kinetic equation (2.2) with the collision integrals (2.3), (2.7). Let us the mean $u(r,t) = 0$ and therefore the distribution function $f = f(r, \mid v \mid, t)$ (see (4.1)), from the kinetic equation the heat transfer equation follows

$$\frac{\partial\langle E\rangle}{\partial t} = \chi\frac{\partial^2\langle E\rangle}{\partial R^2}, \langle E\rangle = \frac{3}{2}\kappa T(r,t). \tag{7.1}$$

From the kinetic equation follows and the corresponding equation for the dispersion of temperature

$$\frac{\partial}{\partial t}\langle(\delta E)^2\rangle = \frac{4}{\tau}\Big(\frac{2}{3}\langle E\rangle^2 - \langle(\delta E)^2\rangle\Big) + \chi\frac{\partial^2\langle(\delta E)^2\rangle}{\partial R^2} + 2\chi\Big(\frac{\partial\langle E\rangle}{\partial R}\Big)^2. \tag{7.2}$$

We see that the source of the temperature fluctuations is defined by the gradient of the mean temperature $T(r,t)$. Therefore, for the complete description of the heat transfer, it is necessary to use the kinetic equation for the distribution $f(R, |v|, t)$.

Let us now exists nonlinear source of the heat, then the constant $\frac{1}{\tau}$ is replaced by nonlinear dissipative coefficient $\gamma(E)$. We shall be to suppose (for the illustration only) that

$$\gamma(E) = -a_s + \frac{1}{\tau} + bE, \text{ and } D_v(E) = \Big(\frac{1}{\tau} + bE\Big)\kappa T_0. \tag{7.3}$$

Here the coefficient $a_s$ characterizes the source of heat, and $T_0$ is the thermostat's temperature. In the equilibrium state the Maxwell distribution is with the temperature $T_0$.

Now we can represent the generalized kinetic equation (2.2) in the form

$$\frac{\partial f(R,|v|,t)}{\partial t} + v\frac{\partial f}{\partial R} = \frac{\partial}{\partial v}\Big[D_{(v)}(E)\frac{\partial f}{\partial v}\Big] + \frac{\partial}{\partial v}(\gamma(E)vf) + \chi\frac{\partial^2 f}{\partial R^2}. \tag{7.4}$$

In one-moment approximation we have the equation of the nonlinear heat transfer equation

$$\frac{\partial\langle E\rangle}{\partial t} = 2\Big[\frac{3}{2}mD(\langle E\rangle) - \gamma(\langle E\rangle)\Big]\langle E\rangle + \chi\frac{\partial^2\langle E\rangle}{\partial R^2}. \tag{7.5}$$

Thus, in this approximation we obtain the reaction diffusion equation type (6.1). Here, however, not only the space diffusion $D$, but also the internal diffusion $D_{(v)}$ is taken into account. It is possible, therefore, to obtain solution for all values of the bifurcation parameter $a_s$.

In the two-moment approximation from the kinetic equation (7.4) follows the system of corresponding reaction diffusion equations.

In order to illustrate the effectiveness of the kinetic approach for description of processes in active medium it is useful to consider the stationary solution. If the function $\gamma(E)$ and $D(E)$ one are defined by the expressions (7.3), then the stationary solution we can represent in the form

$$f(v) = C\exp\Big[-\frac{mv^2}{2\kappa T_0} + \frac{a_s}{\kappa T_0 b}\ln\Big(1 + \frac{b}{\gamma}\frac{mv^2}{2}\Big)\Big]. \tag{7.6}$$

It describes the velocity distribution for all values of the rule parameter $a_s$. At $a_s = 0$ it coincides with the Maxwell distribution.

## 8. Kinetic Equation for Active Medium of Bistable Elements

We have considered some examples of the kinetic equations for active media taking into account both the spatial diffusion and the diffusion with respect to the internal variables of the elements of the medium.

Let us suppose that the nonlinear force is defined by the expression

$$F(x) = -m\omega_0^2(1 - a_f + bx^2)x \tag{8.1}$$

Here $a_f$, and $\omega_0$ are the "feedback" parameter and eigenfrequency of the linear oscillator respectively.

Let us suppose also that $f(x,v,R,t)$ is the distribution function of the internal variables $x$, $v$, and a space position $R$ of the bistable element, then we can present the generalized Fokker-Planck equation in the following form:

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial x}+\frac{F(x)}{m}\frac{\partial f}{\partial v}=\frac{\partial}{\partial v}\left(D_{(v)}\frac{\partial f}{\partial v}\right)+\frac{\partial}{\partial v}(\gamma v f)+ \\ +\frac{\partial}{\partial x}\left[D_{(x)}\frac{\partial f}{\partial x}-\frac{F(x)}{m\gamma}f\right]+D\frac{\partial^2 f}{\partial R^2}. \qquad (8.2)$$

We see that here are two internal diffusion coefficients:

$$D_{(v)}=\gamma\frac{\kappa T}{m}, \qquad D_{(x)}=D_{(0)}(1+bx^2) \quad (D_{(0)}=\frac{\kappa T}{m\gamma}) \qquad (8.3)$$

and the diffusion coefficient $D$ of elements in space $R$. For the equilibrium state the Maxwell-Boltzman distribution is the solution of the kinetic equation (8.2).

If we restrict the class of the distribution functions (like in (4.1)) by condition

$$f(x,v,t)=f(x,\mid v\mid,t), \qquad (8.4)$$

then we can obtain by integration on $v$, the corresponding generalized Einstein-Smoluchowsky equation for the function $f(x,R,t)$

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial x}\left[D_{(0)}(1+bx^2)\frac{\partial f}{\partial x}\right]-\frac{\partial}{\partial x}\left[m\omega_0^2(1-a_f+bx^2)xf\right]+D\frac{\partial^2 f}{\partial r^2}. \quad (8.5)$$

It is useful to remind that in the theory of stochastic processes the transition from the Fokker-Planck (the Kramers) equation to the Einstein-Smoluchowsky one is making by the using of the perturbation theory on the small parameter $F'/m\gamma^2$ (or $\omega_0^2/\gamma^2 \ll 1$) [29–31,6]). This method corresponds to the Hilbert, Chapman-Enskog, and Grad perturbation theory for the Boltzman equation on small Knudsen number.

We see that and in the stochastic theory it is possible to avoid the using of the perturbation theory on the corresponding small "Knudsen parameter".

It is interesting to consider some limiting cases.

1) The distribution function for the stationary homogeneous state

$$f_{St}=C\exp\left[-\frac{U_{eff}(x)}{\kappa T}\right], \quad U_{eff}=\frac{m\omega_0^2}{2}\left[x^2-\frac{a_f}{b}\ln(1+bx^2)\right]. \qquad (8.6)$$

At $a_f=0$ it coincides with the Boltzman distribution for $U=m\omega_0^2/2$.

2) The space distribution function $f(R,t)$. From (8.5) the self-diffusion equation follows:

$$\frac{\partial f}{\partial t}=D\frac{\partial^2 f}{\partial R^2}, \quad \int f(R,t)\frac{dR}{V}=1. \qquad (8.7)$$

3) *The one-moment approximation for incompressible medium.* If $D_{(0)}\equiv 0$, then the distribution function have the form

$$f(x,R,t)=\delta\big(x-x(R,t)\big), \quad x(R,t)=\langle x\rangle. \qquad (8.8)$$

For the "field function" $x(r,t)$ from the generalized Einstein-Smoluchowsky equation follows the FKPP type equation:

$$\frac{\partial x(R,t)}{\partial t} = \gamma\Big[a_f - 1 + bx^2(R,t)\Big]x(R,t) + D\frac{\partial^2 x(r,t)}{\partial R^2}.\gamma = \frac{\omega_0^2}{\gamma} \qquad (8.9)$$

In this approximation the internal diffusion coefficient $D_{(x)} \equiv 0$. The diffusion $D_{(x)}$ can play, however, the important role.

Let us suppose that the space diffusion is one-dimensional, in the direction $\zeta$, then there exists the following well-known stationary solution of the equation (8.9)

$$x(\zeta) = \left(\frac{a_f - 1}{b}\right)^{1/2} \mathrm{th}\left[\left(\frac{a_f^{-1}}{2D}\right)^{1/2} \zeta\right]. \qquad (8.10)$$

The relative width of the front is determined by the expression $\sqrt{\frac{2D}{(a_f-1)}}$. We see that this result does not valid in the region near the bifurcation point $a_f = 1$.

In order to obtain a more general solution for all values of the bifurcation parameter, it is necessary to use the kinetic equation [26].

## 9. Kinetic fluctuations in active media

### 9.1. The Langevin source in the kinetic equation

The "collision integrals" are defined by the small-scale (kinetic) fluctuations. We shall now consider the large-scale (kinetic) fluctuations, whose dynamics is determined by the kinetic equations themselves.

There are two well-known methods of calculating the kinetic fluctuations [1–6,19,29,32–35]. One of them is based on the approximative solution of the set of equations for the moments of random (pulsating) distribution functions. The second one is based on solving the corresponding Langevin equations for the random distribution function $\tilde{f}(x,R,t)$ and this is what we are going to use here.

In first, for example, we introduce a Langevin source $y(r,x,t)$ into the kinetic equation (8.5) for the active medium of bistable elements. In the Gaussian approximation the two moments of the Langevin source are given by the following expressions [5,26]:

$$\langle y(x,R,t)\rangle = 0, \quad \langle y(x,R,t)y(x',R',t')\rangle =$$

$$= 2\Big[D_{(x)}\tfrac{\partial^2}{\partial x\partial x'} + D\tfrac{\partial^2}{\partial R\partial R'}\Big]\delta(x-x')\tfrac{1}{n}\delta(R-R')f(x,R,t)\delta(t-t'). \quad (9.1)$$

The distribution function $f = \langle f\rangle$ is defined by the equation (8.5).

These general expressions gives possibility to find the Langevin sources in the diffusion equation for the space distribution $\tilde{f}(R,t) = \int \tilde{f}(x,R,t)dx$ and in the FKPP equation (8.9).

## 9.2. Spatial Diffusion. "Tails" in the Time Correlations

From the kinetic equation (8.5) with the Langevin source (9.1) follows the equation for the fluctuations of space distribution $\tilde{n} = n\tilde{f}(R,t)$ ($n = N/V$, $n(R,t) = \langle \tilde{n}(R,t) \rangle$)

$$\frac{\partial \delta n}{\partial t} = D\frac{\partial^2 \delta n}{\partial R^2} + y(R,t), \qquad \langle y(R,t) \rangle = 0, \tag{9.2}$$

$$\Big\langle y(R,t) y(R',t') \Big\rangle = 2D\frac{\partial^2 f}{\partial R \partial R'}\delta(R-R')n(R,t)\delta(t-t').$$

In this way, we have come to the well-known equation of spatial diffusion with the random source. The distribution $n(R,t)$ is the solution of the diffusion equation (8.7).

In the equilibrium state the spectral density of fluctuation is defined by the well-known expression

$$(\delta n \delta n)_{\omega,k} = \frac{2Dk^2}{\omega^2 + (Dk^2)^2} n. \tag{9.3}$$

Using the Fourier transformation over $\omega$ and $k$ we find the expression for the space-time correlation on the fluctuations $\delta n$:

$$\langle \delta n \delta n \rangle_{r,\tau} = \frac{n}{(4\pi D \mid \tau \mid)^{3/2}} \exp\Big(-\frac{r^2}{4D \mid \tau \mid}\Big), \tag{9.4}$$

$$r = r - r', \quad \tau = t - t'.$$

We see that at $r = 0$ the time correlations fall off by according to a power law: $\propto 1/\mid \tau \mid^{3/2}$.

It is necessary to underline, that the results presented here were obtained without taking into account the boundary conditions. In other words, they are valid only for an infinite medium. If the size of system $L$ is finite, then these results is valid only at conditions

$$\omega \gg \tau_D^{-1}, \quad \tau \ll \tau_D = L^2/D. \tag{9.5}$$

The role of the boundaries will be dealt in the section 10.

## 9.3. The Langevin Source in Reaction Diffusion (FKPP) Equation

For the incompressible medium the integral $\int f(x,R,t) = const$, therefore the correlator of the source $y(R,t)$ in the diffusion equation - the correlator (9.2), now is zero, but the moment of the Langevin source $y_{(x)}(R,t) = \int xy(x,R,t)dx$ are not zero and are defined by the expressions

$$\langle y_{(x)}(R,t) \rangle = 0, \quad \langle y_{(x)}(R,t) y_{(x)}(R',t') \rangle = \tag{9.6}$$

$$2\Big[D_{(0)} \quad (1 + b\langle x^2 \rangle_{(R,t)}) + D\frac{\partial^2}{\partial R \partial R'}\langle x^2 \rangle_{(R,t)}\Big]\frac{1}{n}\delta(R-R')\delta(t-t'). \tag{9.7}$$

In order to obtain the closed expression for this correlator it is necessary to use the solution of the generalized Einstein-Smoluchowsky equation (8.5).

The situation is much more simple when it is possible to use the stationary solution (8.6). In this case $\langle x^2\rangle$ in (9.6) is defined for all values of the bifurcation parameter $a_f$ by the following expression:

$$\langle x^2\rangle = C\int x^2 \exp\Big[-\frac{m_0\omega_0^2}{2\kappa T}\Big(x^2 - \frac{a_f}{b}\ln(1+bx^2)\Big)\Big]dx. \tag{9.8}$$

Thus, in the stationary state (9.9) the intensity of noise is defined by the mean-square value $\langle x^2\rangle$, therefore for the calculation of the fluctuation $\delta x(R,t)$ it is possible to use the self-consistent approximation on $\langle x^2\rangle$ [3,4,26]

$$\frac{\partial\delta x(R,t)}{\partial t} + \gamma_x\delta x(R,t) - D\frac{\partial\delta x(R,t)}{\delta R^2} = y(R,t). \tag{9.9}$$

With help of this equation and the definition (9.7) for the intensity of noise we obtain the following expression for the spectral density of fluctuations

$$(\delta x\delta x)_{\omega,k} = \frac{2(\gamma_x + Dk^2)}{\omega^2 + (\gamma_x + Dk^2)^2}\frac{\langle x^2\rangle}{n}. \tag{9.10}$$

We introduced here the special definition for the halfwidth of the spectral line is determined by the internal diffusion $D_{(0)}$ in any bistable element:

$$\gamma_x = \frac{D_0}{\langle x^2\rangle}(1 + b\langle x^2\rangle). \tag{9.11}$$

This definition is valid for all values of the bifurcation parameter $a_f$.

It is necessary to underline that the results are presented in this section, as and the corresponding results of the previous section, are valid only for an infinite medium-the condition (9.5).

## 10. Natural Flicker Noise ("1/f Noise")

### 10.1. Natural Flicker Noise for Diffusion Processes [4,5,36–38]

For a fluctuative diffusion process we can distinguish two domains: $\tau_{cor} \ll \tau_D$; $\tau_{cor} \gg \tau_D$. For the first region the size of system has little effect on the fluctuations. The spectrum in this case practically coincides with the spectrum (9.3).

Natural flicker noise exists in the low-frequency region

$$\frac{1}{\tau_{obs}} \ll \omega \ll \omega_{\max} = \frac{1}{\tau_D} = \frac{D}{L^2}. \tag{10.1}$$

Thus, the upper limit of the region is determined by the diffusion time The minimum frequency is determined by the time of observation $\tau_{obs}$, which is limited only by the lifetime of the device.

From (10.1) it follows that the actual volume of diffusion $V_\omega \equiv L_\omega^3 =$ depends upon the frequency and is much larger than the volume $V$ of the sample,

$$V_\omega \equiv L_\omega^3 = (D/\omega)^{3/2} \gg V. \tag{10.2}$$

Under this condition the size of the system is not important, and in the limit the sample can be treated as a point (dimension zero).

In order to find the spectral density of the Langevin source, it is necessary to take into account that the diffusion volume $V_\omega$ for the region of flicker noise is $V_\omega/V$ times larger than the volume of the sample $V$. Accordingly, the mean concentration $n$ is replaced by the effective concentration [36]

$$n \to n_{eff} = AV_\omega \langle \delta n_V \delta n_V \rangle. \tag{10.3}$$

Here we have used $\delta n_V$ to denote the one-point correlator of fluctuations, smoothed over the volume of the sample $V$. The constant $A$ will be defined below. As a result, the expression for the spectral density of the Langevin source now takes the form

$$(y, y)_{\omega,k} \equiv 2Dk^2 n_{eff} \exp\Big(-\frac{L_\omega^2 k^2}{2}\Big). \tag{10.4}$$

The corresponding expression for the spectral density of $\delta n$:

$$(\delta n \delta n)_{\omega,k} = \frac{2Dk^2}{\omega^2 + (Dk^2)^2} AV_\omega \langle \delta n_V \delta n_V \rangle \exp\Big(-\frac{L_\omega^2 k^2}{2}\Big). \tag{10.5}$$

From these expressions, the variance of the distribution of wave vector is $\langle (\delta k)^2 \rangle = L_\omega^{-2} = \omega/2D$ and tends to zero as $\omega \to 0$. In this way, we come to the coherent distribution in the space of wave numbers.

After integrating over $k$, we find the expression for the temporal spectral density of natural flicker noise

$$(\delta n \delta n)_\omega = \frac{\pi}{\ln(\tau_{obs}/\tau_D)} \frac{\langle \delta n_V \delta n_V \rangle}{\omega}, \qquad \tau_{obs}^{-1} \ll \omega \ll \tau_D^{-1}. \tag{10.6}$$

The constant $A$ in (10.5) is defined here from the condition

$$\int\limits_{1/\tau_{obs}}^{1/\tau_D} (\delta n \delta n)_\omega \frac{d\omega}{\pi} = \langle \delta n_V \delta n_V \rangle. \tag{10.7}$$

Thus we assume that the main contribution to the integral over $\omega$ comes from the region of flicker noise. This expression can be written in the form

$$\frac{(\delta n \delta n)_\omega}{n^2} = \frac{2\pi a}{N\omega}, \qquad a = \frac{1}{2\ln(\tau_{obs}/\tau_D)} \frac{\langle \delta n_V \delta n_V \rangle}{n/V}. \tag{10.8}$$

Here we have introduced the notation a for Hoog's constant [39].

The time correlator for the region of natural flicker noise is given by the expression [36]

$$\begin{gathered} \langle \delta n \delta n \rangle_\tau = [C - \ln(\tau/\tau_D)/\ln(\tau_{obs}/\tau_D)] \langle \delta n_V \delta n_V \rangle; \\ \tau_D \ll \tau \ll \tau_{obs}, \quad C = 1 - \gamma/\ln(\tau_{obs}/\tau_D), \quad \gamma = 0.577. \end{gathered} \tag{10.9}$$

We see that the dependence on $\tau$ in the region of flicker noise is very weak (logarithmic with a large argument), and therefore we may speak of the residual time correlations.

*The Langevin equations for the function $\delta n$ the region of flicker noise.* The Langevin equation for the Fourier component $\delta n(\omega, k)$ can be presented in the form

$$(-i\omega + Dk^2)\delta n(\omega, k) = y(\omega, k). \tag{10.10}$$

The spectral density of the source is given by (10.4).

Since the distribution over the wave numbers in the region of flicker noise is very narrow, in (10.10) we may replace

$$k^2 \rightarrow \langle k^2 \rangle_k = L_\omega^{-2} = \omega / D \tag{10.11}$$

and use a simpler equation for the fluctuations $\delta n(\omega)$:

$$(-i\omega + \gamma_\omega)\delta n(\omega) = y(\omega); \qquad \gamma_\omega = \mid \omega \mid . \tag{10.12}$$

The expression for the spectral density of the Langevin source:

$$(yy)_\omega = 2\gamma_\omega \frac{\pi}{\ln(\tau_{obs}/\tau_D)} \langle \delta n_V \delta n_V \rangle; \qquad \gamma_\omega = \mid \omega \mid . \tag{10.13}$$

Thus, for the region of natural flicker noise the dissipative coefficient $\gamma_\omega = |\omega|$, and tends to zero as $\omega \Rightarrow 0$. This expression relates the spectral density to the dissipative coefficient and one-time correlator, and is therefore an FDR for the region of natural flicker noise.

In this way, we have formulated the principal results of the author's theory of flicker noise [36–38,4,5]. The natural flicker noise arises whenever the final stage of relaxation towards the equilibrium state is associated with spatial diffusion.

The dependence on the actual structure of the system only enters via two parameters: the time of diffusion $\tau_D$ and the one- time correlator $\langle \delta n_V \delta n_V \rangle$. Recall that we are considering the diffusion of a physical entity of any kind.

This theory has been employed elsewhere to explain the existence of natural flicker noise in music, as observed by Voos and Clark [40]. We considered the possible connection between natural flicker noise and superconductivity [38,5]. So far these two fundamental phenomena have been treated independently. We demonstrated, however, that the existence of low-frequency natural flicker noise in a system of charged Bose particles results in the vanishing of electrical resistance and thus facilitates the existence of a permanent superconducting current and the Meissner-Oxenfeld effect.

The transition from the normal to the superconducting state is a well-known example of a phase transition of the second kind, which results in the appearance of a macroscopic quantity of charged Bose particles (Cooper pairs). In order to understand the important role of natural flicker noise in a system of Cooper pairs for the existence of permanent superconductivity electrical current, it is necessary to take into account that the appearance of natural flicker noise, according to this theory,implies the creation of a coherent state in the space of wave numbers at $\omega \Rightarrow 0$. In this way, we are talking about the linkage between two coherent processes.

We can remark that the natural flicker noise exists not only in a system of Cooper's pairs. In a system of neutral Bose particles ($He^4$) the existence of natural flicker noise makes superfluidity in narrow gaps possible. The coefficient of diffusion is then of the order of Planck's constant $h$.

Naturally, there are also "technical " noise with $1/f$ spectrum, which are associated, for example, with mobile defects.

It is very interesting also to remark that after integrating over $\omega$ (not over $k$!), from (10.5) we find the spatial natural flicker noise "$1/k$". Thus, it exists not only the temporal, but also and the spatial natural flicker noise — the noise "$1/\mid k \mid$".

## 10.2. Natural Flicker Noise for Reaction-Diffusion Processes

We saw that for statistical description of processes in active media more naturally to use, instead of the reaction-diffusion equation (6.1), the corresponding generalized kinetic equations. We saw also that such approach gives possibility for the description kinetic fluctuations. The one of the methods of description of kinetic fluctuations is based on the solving the corresponding Langevin equations. For example, the moments of the Langevin source in the Einstein-Smoluchowsky equation for active media of bistable elements are defined by the expressions (9.1).

We used this general expressions in order to define the Langevin source in the generalized reaction- diffusion equation for the "field" variable $x(R,t)$. As result, we obtained the expression (9.9) for the spectral density of fluctuations. This result is valid for all values of the bifurcation parameter $a_f$.

As and for the diffusion processes (see (9.3)), this result was obtained without taking the boundary conditions into account, therefore it is valid only for an infinite medium: for the region of frequencies $\omega \gg \tau_D^{-1} = D/L^2$.

For the diffusion systems the natural flicker noise exists in the low-frequency region (10.1). For the reaction-diffusion systems exists not only dissipation is determined by diffusion, but also the additional internal dissipation. If the corresponding coefficient $\gamma_{(x)} \ll \tau_D$, then exists the low-frequency region of the flicker noise

$$\gamma_{(x)} = \omega_{\min} \ll \omega \ll \omega_{\max} = 1/\tau_D = D/L^2. \qquad (10.14)$$

but by the friction coefficient $\gamma_{(x)}$.

In order to find the corresponding spectral function, it is necessary to change the second term (9.6) in correspondence with the definition (10.4). After integration over $k$ we again obtain the expression for the temporal spectral density for the low-frequency region (10.14)

$$(\delta x \delta x)_{\omega,k} = \frac{2(\gamma_{(x)} + \mid \omega \mid)}{\omega^2 + (\gamma_{(x)} + \mid \omega \mid)^2} A \langle \delta x_v \delta x_v \rangle. \qquad (10.15)$$

The constant A is defined by the expression:

$$A = 4/\pi \int\limits_{\gamma_{(x)}}^{1/\tau_D} \frac{2(\gamma_{(x)} + \mid \omega \mid)}{\omega^2 + \left(\gamma_{(x)} + \mid \omega \mid\right)^2} d\omega. \qquad (10.16)$$

The dissipative coefficient $\gamma_{(x)} \equiv \omega_{\min}$ is defined by the expression (9.12) for all values of the bifurcation parameter $a_f$.

## 11. Criteria of Self-Organization

### 11.1. Evolution in the Space of Controlling parameters. S-Theorem

Assume that we have made choice of controlling (rule) parameters $a$, and shall be consider the evolution of the sequence of stationary states corresponding to different values of the rule parameters.

We single out a state corresponding to $a = a_0$, and another with $a = a_0 + \Delta a$. The state with the distribution function $f_0(x, a_0)$ we take as the state of physical chaos (the correctness of this assumption will have to be verified). We also introduce the distribution function $f(x, a_0 + \Delta a)$ and represent it as a effective "canonical Gibbs distribution"

$$f(x, a_0 + \Delta a) = \exp \frac{F - H(x, a_0 + \Delta a)}{D}; \qquad f_0 = f(x, a_0 + \Delta a)|_{\Delta a = 0}. \tag{11.1}$$

The distribution functions $f, f_0$ are normalized in the same way:

$$\int f \, dx = \int f_0 \, dx = 1. \tag{11.2}$$

In order to find the actual form of the distribution (11.1) we need a mathematical model of our system. However, since in many cases it is difficult to construct a mathematical model, in the next section we shall give a modified formulation of the criterion of the self-organization, which allows us to define the effective Hamiltonian function directly from experimental data, without using a mathematical model.

For accessing the relative orderliness of states with different values of $a$ according to the entropy values, we must renormalize it to the given value of the energy [41-44,5,6].

We shall formulate the $S$-Theorem in two steps.

1) As an additional condition we fix the mean energy $\langle H(x, a_0) \rangle$ for the state of physical chaos and renormalize $f_0 \Rightarrow \tilde{f}_0$. The function $\tilde{f}_0$ is presented as

$$\tilde{f}_0(x, a_0, \Delta a) = \exp \frac{\tilde{F}_0 - H(x, a_0)}{\tilde{D}(\Delta a)}, \qquad \int \tilde{f}_0 \, dx = 1. \tag{11.3}$$

The additional condition has the form

$$\int H(x, a_0) \tilde{f}_0(x, a_0, \Delta a) \, dx = \int H(x, a_0) f(x, a_0 + \Delta a) \, dx. \tag{11.4}$$

From solution of this equation, we find the effective temperature $\tilde{D}$ as a function of $\Delta a$:

$$\tilde{D} = \tilde{D}(\Delta a); \qquad \tilde{D}(\Delta a)|_{\Delta a = 0} = D. \tag{11.5}$$

By $\tilde{S}_0$ we denote the entropy for the state with the distribution $\tilde{f}_0$. Then, subject to the conditions (11.2), (11.4), the difference between $\tilde{S}_0$ and $S$ is given by

$$\tilde{S}_0 - S = \int \ln \frac{f}{\tilde{f}_0} f\, dx \geq 0. \qquad (11.6)$$

So we have obtained two results, (11.5), (11.6). The change in the degree of order upon the transition $a_0 \Rightarrow a_0 + \Delta a$ is accessed from the solution (11.5). If the inequality

$$\tilde{D}(\Delta a) > D \qquad (11.7)$$

is satisfied (that is, if the effective temperature in the state with $a = a_0$ must be increased in order to make (11.4) valid), then $a \Rightarrow a_0 + \Delta a$ is the transition from a less ordered state (physical chaos) to a more ordered state. The difference in the entropies (11.6) is a quantitative measure of the increase in the degree of order.

2) If the inequality (11.7) is not satisfied, then the change in a is not a controlling one. This is an indication that we have to look for new controlling parameters. Thus, the $S$-Theorem serves as a tool for checking of the correct choice of the controlling parameters.

If there are several controlling parameters, the search for the most ordered states can be optimized (see ref. in [4,5]) The above statement was termed the $S$-Theorem, "$S$" standing for "self-organization", to emphasize that the $S$-Theorem is a criterion of self-organization.

## 11.2. The Comparison of the Relative Degree of Order of States on the Basis of the S-Theorem Using Experimental Data

Practical applications of the criterion of the relative degree of order based on the $S$-Theorem, as outlined above, they require a knowledge of the structure of the effective Hamiltonian function. It is very important to have criteria which could use the experimental data directly.

Such criteria can be based on $K$-entropy, Lyapunov indices, fractal dimensions which can be obtained from the realizations of the processes concerned. Let us show that the relative degree of order can be accessed directly from experimental data using the $S$-Theorem criterion [43].

As above, we start by choosing the controlling parameter $a$. The state with $a = a_0$ is taken as that of physical chaos, with which the state for $a = a_0 + \Delta a$ will be compared.

We use the experimental realizations $x(t, a_0), x(t, a_0 + \Delta a)$ as the chosen set of internal parameters $x$ of our process. The realizations must be sufficiently long, so that they could be used for obtaining the distribution functions

$$f_0(x, a_0), \quad f(x, a_0 + \Delta a); \quad \int f_0\, dx = \int f\, dx = 1. \qquad (11.8)$$

Using the distribution $f_0$, which by our assumption corresponds to the state of physical chaos, we find the function

$$H_{eff} = -\ln f_0, \qquad \left(f_0 = \exp(-H_{eff})\right), \qquad (11.9)$$

which in the renormalized distribution $\tilde{f}_0$ will serve as the effective Hamiltonian function. From (11.8), (11.9) it follows that we need no extra information in order to find $H_{eff}$, except the known realization $x(t, a_0)$. The mean value of the effective energy for the distributions (11.8) in general will depend on$\Delta a$. Let us now renormalize to the preset value of $\langle H_{eff} \rangle$.

We introduce the renormalized distribution $f_0$ which we represent as the canonical distribution

$$\tilde{f}_0(x, a_0, \Delta a) = \exp \frac{F(D) - H_{eff}(x, a_0)}{D(\Delta a)}, \qquad \int \tilde{f}_0 \, dx = 1. \tag{11.10}$$

The effective free energy $F$ as a function of the temperature $D$ is found from the normalization condition of the function $\tilde{f}_0$. The effective temperature $D$ as function of $\Delta a$ is derived by solving the equation

$$\int H_{eff}(x, a_0) \tilde{f}_0(x, a_0, \Delta a) \, dx = \int H_{eff} f(x, a_0 + \Delta a) \, dx, \tag{11.11}$$

that is, by requiring that the mean effective Hamiltonian function $H_{eff}$ be constant. Using the solution of this equation, we find the required function

$$D(\Delta a); \quad D(\Delta a)|_{\Delta a = 0} = 1, \qquad \Delta a \geq 0. \tag{11.12}$$

Now we use again the distributions $\tilde{f}_0$, $f$ to find the difference between the entropies:

$$\tilde{S}_0 - S = \int \ln \frac{f}{\tilde{f}_0} f \, dx \geq 0 \qquad \text{at} \qquad \langle H_{eff}(x, a_0) \rangle = const. \tag{11.13}$$

If the solution of (11.12) is such that

$$D(\Delta a) > 1; \qquad D(\Delta a)|_{\Delta a = 0} = 1. \tag{11.14}$$

then the state with $a = a_0 + \Delta a$ is more ordered than the state with $a = a_0$, which we took for the state of physical chaos. This conclusion must, however, be verified (see [4,5]).

It is useful now to remark the following.

During the time evolution to the equilibrium state we were dealing with degradation processes.

Dealing with the evolution of stationary states in the space of controlling parameters we encounter a new possibility. The fact is that the controlling parameters can be found among the parameters which characterize the stationary state. Changes in the latter may lead to the reduction of the entropy, thus resulting in self-organization. In this way, the formulation of the second law of thermodynamics is extended: the evolution of stationary states in the space of controlling parameters may be associated with both increasing and decreasing entropy. In the latter case the set of parameters which define the stationary state includes the controlling parameters.

These two types of evolution represent in a sense the two extremes. It would be interesting to investigate a more general case, when the parameters change during the time evolution.

## 12. Conclusion. Associative Memory and Pattern Recognition

Since the well-known work by Hopfield (1982) the problems of associative memory and pattern recognition occupy an important place in statistical physics, in particular, in the theory of spin glasses, in optical systems.

The models considered in the theory of associative memory and pattern recognition are mostly based on discrete neural networks, each neuron possessing a number of discrete states. Haken (1988) proposed treating the recognition of patterns as a process similar to the formation of dissipative structures in synergetic systems. In the proposed models the dynamics of the system is determined by a potential function which depends on the parameters. This dependence may be used for storing the information concerning the pattern to be recognized. The test patterns are introduced via the initial components of the vector of the dynamic system. Systems of this kind exhibit the phenomenon of associative memory since they are capable of restoring the complete pattern in the process of evolution towards the stationary state, starting with the incomplete information contained in the initial conditions [45,46].

Let us demonstrate that the simplest medium capable of associative memory can be constructed from active elements, which are VanderPol generators interacting via common feedback. Such a system is practically feasible.

Consider the following set of equations [26]:

$$\begin{aligned}
&\tfrac{dX_i}{dt} + \tfrac{1}{2}\sum_{j=1}^{N}(-a_j + b_jE_j)x_j = V_i, \quad i = 1,2,\ldots,N, \\
&\tfrac{dV_i}{dt} + \tfrac{1}{2}\sum_{j=1}^{N}(-a_j + b_jE_j)V_j + \omega_i^2 x_i = 0, \qquad E_i = \tfrac{1}{2}(V_i^2 + \omega_i^2 X_i^2).
\end{aligned} \tag{12.1}$$

The generators are linked with each other via the common feedback. In the general case all the coefficients $a_i, b_i$ are different. Observe that the excitation of all generators in the system only requires that $X_{0i}, V_{0i}$ should be nonzero for at least one generator.

Introducing the "dissipative potential" of a system of generators

$$U = \sum_i U_i, \qquad U_i = \frac{b_i}{2}\Big(E_i - \frac{a_i}{b_i}\Big)^2, \tag{12.2}$$

we may write these equations in the form

$$\frac{dX_i}{dt} + \frac{1}{2}\sum_j \frac{\partial U}{\partial X_j}\frac{1}{\omega_j^2} = V_i; \qquad \frac{dV_i}{dt} + \frac{1}{2}\sum_j \frac{\partial U}{\partial V_j} + \omega_i^2 X_i = 0. \tag{12.3}$$

In the stationary state the dissipative potential (5.9.2) goes to zero. The stationary solution with fixed initial phases $\phi_{0i}$ depends on $N$ parameters $a_i/b_i$. This dependence can be used for storing information about a pattern of $N$ points. For restoring the pattern it is sufficient to introduce into the initial parameters the information about at least one point of the pattern. Then in the process of evolution towards the stationary state the system will restore complete information about the pattern, installed into the stationary state via the values of the parameters $a_i/b_i$. Obviously, this model is just a very simple illustration. Possible generalizations may proceed in various directions. Of interest, for instance, is the statistical generalization, when the state of the system in the general case is characterized by the "$N$-particles" distribution function $f_N$. We then enter the domain of the statistical theory of "nonideal" active systems with interaction of different "particles",which can simulate the processes of associative memory and pattern recognition with due associative account fluctuative processes involving both the internal variables of the system's macroscopic elements and the variables which describe the motion of each element as a whole [5,6].

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# ВІД ГАМІЛЬТОНОВОЇ МЕХАНІКИ ДО НЕПЕРЕРВНОГО СЕРЕДОВИЩА. ДИСИПАТИВНІ СТРУКТУРИ. КРИТЕРІЇ САМООРГАНІЗАЦІЇ

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Ця робота задумана як представлення деяких головних ідей та результатів сучасної статистичної теорії макроскопічних відкритих систем.

Ми починаємо з демонстрації необхідності й можливості об'єднаного опису кінетики, гідродинаміки й дифузійних процесів в нелінійних макроскопічних відкритих системах на основі узагальнених кінетичних рівнянь.

Виведення узагальнених кінетичних рівнянь базується на конкретному фізичному означенні неперервного середовища. “Точка” неперервного середовища визначається за означенням фізично інфінітеземального масштабу. На цій самій основі дається означення ансамблю Гібса. Як тестові системи використовуються больцманівський газ та повністю іонізована плазма.

При переході від оборотніх рівнянь Гамільтона до узагальнених кінетичних рівнянь динамічна нестійкість руху частинок грає конструктивну роль.

Кінетичне рівняння больцманівського газу складається з двох дисипативних доданків: 1. “Інтеграл зіткнень” визначається процесами в просторі швидкостей; 2. Додатковий дисипативний доданок дифузійного типу в просторі координат. Завдяки останньому стає можливим об'єднаний опис кінетичних, гідродинамічних та дифузійних процесів для всіх значень чисел Кнудсена.

Доведено $H$-теорему для узагальненого кінетичного рівняння. Виробництво ентропії означене як сума двох незалежних додатніх доданки, що відповідають перерозподілу частинок в просторі швидкостей та координат відповідно.

Потік ентропії також складається з двох частин. Одна з них пропорційна до ентропії, інша – до її градієнта. Існування останнього доданку дозволяє дати загальне означення потоку тепла для довільних значень чисел Кнудсена. Це загальне означення при малих числах Кнудсена й постійному тиску приводить до законів Фур'є.

Рівняння газової динаміки для особливого класу функцій розподілу слідують з узагальненого кінетичного рівняння в рамках теорії збурень для чисел Кнудсена. Ці рівняння відрізняються від традиційних врахуванням процесів самодифузії.

Розглянуто узагальнене кінетичне рівняння для опису броунівського руху й автохвильових процесів в активному середовищі. Встановлено зв'язок з реакційно-дифузійним рівнянням Фішера-Колмогорова-Петровського-Піскунова та рівнянням Гінзбурга-Ландау. Обговорюється зв'язок між дифузією частинок в обмеженій системі з природнім шумом $1/f$ в пасивних та активних системах.

Представлено співставлення типу: критерій відносної степені порядку стану відкритої системи – критерій самоорганізації.