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MODIFIED KINETIC THEORY WITH CONSIDERATION FOR SLOW HYDRODYNAMICAL PROCESSES

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One approach of unified description of kinetic and hydrodynamic fluctuations for classical systems is proposed. The coupled system of equations for nonequilibrium oneparticle distribution function and distribution function of hydrodynamic variables: mass density, momentum and energy for the description of kinetic and hydrodynamic processes in classical liquids is obtained.

1. Introduction

Construction of kinetic equations with consideration for slow hydrodynamic processes [1-4] is an important problem of the theory of transport processes in fluids. For instance, this problem arises in attempting to describe low-frequency anomalies in kinetic equations and related "long tails" of correlation functions [5], as well as in consistent consideration of collective effects in plasma [4,6]. The main point of the problem consists in that kinetic and hydrodynamic processes are coupled and have to be treated simultaneously.

In papers [7-9] an approach was suggested to describe consistently kinetic and hydrodynamic transport processes in dense gases and liquids on the basis of nonequilibrium statistical operator method developed by D.N.Zubarev [10,11]. In particular, this approach was used to derive form the BBGKY hierarchy the kinetic equation of revised Enskog theory [8,12] for hard spheres and Enskog-Landau kinetic equation for one-component charged hard sphere system [8]. In paper [9] generalized transport equations were obtained for hydrodynamic variables (densities of particle number, momentum and total energy) coupled with the kinetic equation for nonequilibrium one-particle distribution function. These equations were used then to investigate time correlation functions and the spectrum of collective modes for slightly nonequilibrium systems [13].

Obviously, the approach [7-9] can be used to describe both slightly and strongly nonequilibrium systems. But to describe consistently kinetic processes and nonlinear hydrodynamic fluctuations, it is convenient to reformulate the theory in order to obtain the coupled set of equations for nonequilibrium one-particle distribution function and the distribution functional for hydrodynamic variables: densities of particle number, momentum, and energy. The idea of such an approach was formulated in paper [14].

2. Nonequilibrium distribution function

To describe consistently kinetics and hydrodynamic fluctuations in one-component classic fluid, one has to choose parameters describing in a natural way both one-particle and collective processes. We take as these parameters, the nonequilibrium one-particle distribution function $f_1(x;t) = \langle \hat{n}_1(x) \rangle^t$ and the distribution function for hydrodynamic variables $f(a;t) = \langle \delta(\hat{a}-a) \rangle^t$. Here the phase function

$$\hat{n}_1(x) = \sum_{j=1}^{N} \delta(x - x_j) = \sum_{j=1}^{N} \delta(\boldsymbol{p} - \boldsymbol{p}_j)\delta(\boldsymbol{r} - \boldsymbol{r}_j) \tag{2.1}$$

represents the microscopic density of particle number, and $x_j = \{\boldsymbol{p}_j, \boldsymbol{r}_j\}$ is the aggregate of momenta and coordinates. The microscopic phase distribution for hydrodynamic fields is given by

$$\delta(\hat{a} - a) = \prod_{m=1}^{3} \prod_{|\boldsymbol{k}|<k_0} \delta(\hat{a}_{m\boldsymbol{k}} - a_{m\boldsymbol{k}}), \tag{2.2}$$

where the phase functions $\hat{a}_{1\boldsymbol{k}} = \hat{n}_{\boldsymbol{k}}$, $\hat{a}_{2\boldsymbol{k}} = \hat{\boldsymbol{J}}_{\boldsymbol{k}}$, $\hat{a}_{3\boldsymbol{k}} = \hat{\mathcal{E}}_{\boldsymbol{k}}$, are the Fourier transforms of densities of particle number, momentum and energy:

$$\hat{n}_{\boldsymbol{k}} = \sum_{j=1}^{N} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_j}, \tag{2.3}$$

$$\hat{\boldsymbol{J}}_{\boldsymbol{k}} = \sum_{j=1}^{N} \boldsymbol{p}_j e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_j}, \tag{2.4}$$

$$\hat{\mathcal{E}}_{\boldsymbol{k}} = \sum_{j=1}^{N} \left(\frac{p_j^2}{2m} + \frac{1}{2} \sum_{\substack{l=1 \\ l \neq j}}^{N} \Phi(|\boldsymbol{r}_{lj}|) \right) e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_j}, \tag{2.5}$$

and $a_{m\boldsymbol{k}} = n_{\boldsymbol{k}}$, $\boldsymbol{J}_{\boldsymbol{k}}$, $\mathcal{E}_{\boldsymbol{k}}$ are the corresponding collective variables. $\phi(|\mathbf{r}_{lj}|) = \phi(|\mathbf{r}_l - \mathbf{r}_j|)$ is a pair potential of interaction between particles. Mean values $\langle \hat{n}_1(x) \rangle^t$, $\langle \delta(\hat{a} - a) \rangle^t$ are evaluated with the N-particle distribution function $\varrho\,(x^N;t)$ which satisfies the Liouville equation and, in accordance with the idea of reduced description of nonequilibrium states, is a functional

$$\varrho\,(x^N;t) = \varrho\Big(\ldots f_1(x;t), f(a;t) \ldots \Big). \tag{2.6}$$

Thus the problem is to find a particular solution of the Liouville equation for $\varrho\,(x^N;t)$, which has the form (2.6). To this end, let us follow Zubarev's method of the nonequilibrium statistical operator [10,11] and consider the Liouville equation with an infinitesimally small source violating the time-reversal symmetry

$$\frac{\partial}{\partial t}\varrho\left(x^N;t\right)+\imath\mathcal{L}_N\varrho\left(x^N;t\right)=-\varepsilon\Big(\varrho\left(x^N;t\right)-\varrho_q\left(x^N;t\right)\Big), \qquad (2.7)$$

where $\varepsilon\to+0$, after thermodynamical limiting transition. The source selects the retarded solutions corresponding to reduced description of nonequilibrium states of a system. The quasi-equilibrium distribution function $\varrho_q\left(x^N;t\right)$ is determined in a standard way [10,11] from the maximum of the information entropy under the supplementary conditions that the normalization be constant

$$\int d\Gamma_N\varrho_q\left(x^N;t\right)=1,$$
$$d\Gamma_N=\frac{(dx)^N}{N!}=\frac{(dx_1,...,dx_N)}{N!},\quad dx=d\boldsymbol{r}d\boldsymbol{p}, \qquad (2.8)$$

and the conditions that the parameters of reduced description

$$f_1(x;t) = \langle\hat{n}_1(x)\rangle^t, \qquad (2.9)$$
$$f(a;t) = \langle\delta(\hat{a}-a)\rangle^t \qquad (2.10)$$

are fixed. Then the quasi-equilibrium distribution function can be written as

$$\varrho_q\left(x^N;t\right)=\exp\left\{-\Phi(t)-\int dx\;\gamma(x;t)\hat{n}_1(x)-\int da\;\mathcal{F}(a;t)\hat{f}(a)\right\}, \qquad (2.11)$$

where

$$da=\{dn_{\boldsymbol{k}},\;d\boldsymbol{J}_{\boldsymbol{k}},\;d\mathcal{E}_{\boldsymbol{k}}\},$$

The Massier-Planck functional $\Phi(t)$ is determined from the normalization condition for the quasi-equilibrium distribution:

$$\Phi(t)=\ln\int d\Gamma_N\;\exp\left\{-\int dx\;\gamma(x;t)\hat{n}_1(x)-\int da\;\mathcal{F}(a;t)\hat{f}(a)\right\}. \qquad (2.12)$$

The functions $\gamma(x;t)$ and $\mathcal{F}(a;t)$ play the role of Lagrange multipliers and have to be determined from the consistency conditions

$$f_1(x;t)=\langle\hat{n}_1(x)\rangle^t=\langle\hat{n}_1(x)\rangle^t_q, \qquad (2.13)$$
$$f(a;t)=\langle\delta(\hat{a}-a)\rangle^t=\langle\delta(\hat{a}-a)\rangle^t_q, \qquad (2.14)$$
$$\langle\ldots\rangle^t=\int d\Gamma_N\;\ldots\varrho\left(x^N;t\right),$$
$$\langle\ldots\rangle^t_q=\int d\Gamma_N\;\ldots\varrho_q\left(x^N;t\right).$$

It is convenient to rewrite the quasi-equilibrium distribution function (2.11) in the form

$$\varrho_q\left(x^N;t\right)=\exp\left\{-\Phi(t)-\int dx\;\gamma(x;t)\hat{n}_1(x)\right\}\int da\;\exp\left\{-\mathcal{F}(a;t))\hat{f}(a)\right\}. \qquad (2.15)$$

Using next the condition (2.14) we find the function $\mathcal{F}(a;t)$:

$$\exp\{-\mathcal{F}(a;t)\} = \frac{f(a;t)}{W(a;t)}, \tag{2.16}$$

where

$$W(a;t) = \int d\Gamma_N \exp\left\{-\Phi(t) - \int dx\ \gamma(x;t)\hat{n}_1(x)\right\} \hat{f}(a) \tag{2.17}$$

is the structural function of hydrodynamic fluctuations, which be also considered as the Jacobian [15] $\hat{f}(a)$ for the transformation into the space of collective variables $n_{\boldsymbol{k}}$, $\boldsymbol{J}_{\boldsymbol{k}}$, $\mathcal{E}_{\boldsymbol{k}}$, averaged with the "kinetic" quasi-equilibrium distribution function

$$\varrho_q^{kin}(x^N;t) = \exp\left\{-\Phi(t) - \int dx\ \gamma(x;t)\hat{n}_1(x)\right\}. \tag{2.18}$$

Thus we can write

$$W(a;t) = \int d\Gamma_N\ \varrho_q^{kin}(x^N;t)\,\hat{f}(a). \tag{2.19}$$

Taking then relations (2.18) and (2.19) into account, the distribution function (2.11) can be presented in the form

$$\varrho_q(x^N;t) = \varrho_q^{kin}(x^N;t)\left.\frac{f(a;t)}{W(a;t)}\right|_{a=\hat{a}}. \tag{2.20}$$

To the quasi-equilibrium distribution (2.20), there corresponds the entropy

$$\begin{aligned} S(t) &= -\langle \ln \varrho_q\rangle_q^t \\ &= \Phi(t) + \int dx\ \gamma(x;t)\langle\hat{n}_1(x)\rangle_t + \int da\ f(a;t)\ln\frac{f(a;t)}{W(a;t)}. \end{aligned} \tag{2.21}$$

In combination with consistency conditions (2.9) and (2.10), it can be considered as the entropy of a nonequilibrium state.

Having quasi-equilibrium distribution (2.20), we rewrite the Liouville equation (2.7) for the function $\Delta\varrho(x^N;t) = \varrho(x^N;t) - \varrho_q(x^N;t)$ which tends to zero as $t \to -\infty$:

$$\left(\frac{\partial}{\partial t} + i\mathcal{L}_N + \varepsilon\right)\Delta\varrho(x^N;t) = \left(\frac{\partial}{\partial t} + i\mathcal{L}_N\right)\varrho_q(x^N;t). \tag{2.22}$$

The time-derivative in the right side of this equation can be expressed in terms of the Kawasaki-Gunton projection operator $\mathcal{P}_q(t)$ [11]:

$$\frac{\partial}{\partial t}\varrho_q(x^N;t) = -\mathcal{P}_q(t)i\mathcal{L}_N\varrho(x^N;t). \tag{2.23}$$

In our case the projection operator $\mathcal{P}_q(t)$ acts on any phase function ϱ' according the rule

$$\mathcal{P}_q(t)\varrho' = \varrho_q\left(x^N;t\right)\int d\Gamma_N\ \varrho' + \tag{2.24}$$
$$+\int dx\ \frac{\partial\varrho_q\left(x^N;t\right)}{\partial\langle\hat{n}_1(x)\rangle_t}\left[\int d\Gamma_N\ \hat{n}_1(x)\varrho' - \langle\hat{n}_1(x)\rangle_t\int d\Gamma_N\ \varrho'\right] +$$
$$+\int da\ \frac{\partial\varrho_q\left(x^N;t\right)}{\partial(f/W)}\frac{1}{W(a;t)}\left[\int d\Gamma_N\ \hat{f}(a)\varrho' - f(a;t)\int d\Gamma_N\ \varrho'\right] +$$
$$+\int da\int dx\ \frac{\partial\varrho_q\left(x^N;t\right)}{\partial(f/W)}\frac{f(a;t)}{W(a;t)}\frac{\partial\ln W(a;t)}{\partial\langle\hat{n}_1(x)\rangle_t}\times$$
$$\times\left[\int d\Gamma_N\ \hat{n}_1(x)\varrho' - \langle\hat{n}_1(x)\rangle_t\int d\Gamma_N\ \varrho'\right].$$

Taking relation (2.23) into account, we rewrite equation (2.22) in the form

$$\left(\frac{\partial}{\partial t} + \left(1-\mathcal{P}_q(t)\right)\imath\mathcal{L}_N + \varepsilon\right)\Delta\varrho\left(x^N;t\right) = -\left(1-\mathcal{P}_q(t)\right)\imath\mathcal{L}_N\varrho_q\left(x^N;t\right). \tag{2.25}$$

Its formal solution is given by

$$\Delta\varrho\left(x^N;t\right) = -\int\limits_{t'}^{t} dt'\ e^{\varepsilon(t'-t)}T(t;t')\left(1-\mathcal{P}_q(t')\right)\imath\mathcal{L}_N\varrho_q\left(x^N;t'\right), \tag{2.26}$$

where

$$T(t;t') = \exp_+\left\{-\int\limits_{t'}^{t}\left(1-\mathcal{P}_q(t')\right)\imath\mathcal{L}_N dt'\right\} \tag{2.27}$$

is the generalized time evolution operator taking into account projection. From (2.26) we find the nonequilibrium distribution function

$$\varrho\left(x^N;t\right) = \varrho_q\left(x^N;t\right) - \int\limits_{t'}^{t} dt'\ e^{\varepsilon(t'-t)}T(t;t')\left(1-\mathcal{P}_g(t')\right)\imath\mathcal{L}_N\varrho_q\left(x^N;t'\right). \tag{2.28}$$

Let us consider an action of the Liouville operator $\imath\mathcal{L}_N$ on the quasi-equilibrium distribution function in (2.28). We have

$$\begin{aligned}\imath\mathcal{L}_N\varrho_q\left(x^N;t\right) &= -\int dx\ \gamma(x;t)\dot{\hat{n}}_1(x)\varrho_{q.}\left(x^N;t\right) + \\ &+\left[\imath\mathcal{L}_N\frac{f(a;t)}{W(a;t)}\Big|_{a=\hat{a}}\right]\varrho_q^{kin}\left(x^N;t\right),\end{aligned} \tag{2.29}$$

where $\dot{\hat{n}}_1(x) = \imath\mathcal{L}_N\hat{n}_1(x)$. Using then the relation

$$\imath\mathcal{L}_N\hat{f}(a) = \imath\mathcal{L}_N\hat{f}(n_{\boldsymbol{k}},\boldsymbol{J}_{\boldsymbol{k}},\mathcal{E}_{\boldsymbol{k}}) =$$

$$= -\sum_{k}\left(\frac{\partial}{\partial n_k}\hat{f}(a)\dot{\hat{n}}_k + \frac{\partial}{\partial \boldsymbol{J}_k}\hat{f}(a)\dot{\hat{\boldsymbol{J}}}_k + \frac{\partial}{\partial \mathcal{E}_k}\hat{f}(a)\dot{\hat{\mathcal{E}}}_k\right), \tag{2.30}$$

$$\dot{\hat{\boldsymbol{J}}}_k = i\mathcal{L}_N\hat{\boldsymbol{J}}_k, \quad \dot{\hat{\mathcal{E}}}_k = i\mathcal{L}_N\hat{\mathcal{E}}_k,$$

the last term in (2.29) may be represented in the following form:

$$\left[i\mathcal{L}_N\frac{f(a;t)}{W(a;t)}\bigg|_{a=\hat{a}}\right]\varrho_q^{kin}\left(x^N;t\right) = \int da\ \sum_{k}'\left(\dot{\hat{n}}_k W(a;t)\frac{\partial}{\partial n_k}\frac{f(a;t)}{W(a;t)} + \right.$$
$$\left. + \dot{\hat{\boldsymbol{J}}}_k W(a;t)\frac{\partial}{\partial \boldsymbol{J}_k}(\frac{f(a;t)}{W(a;t)} + \dot{\hat{\mathcal{E}}}_k W(a;t)\frac{\partial}{\partial \mathcal{E}_k}\frac{f(a;t)}{W(a;t)}\right)\varrho_{\mathcal{L}}\left(x^N,a;t\right). \tag{2.31}$$

Here we have introduced the new quasi-equilibrium distribution function $\varrho_{\mathcal{L}}\left(x^N;t\right)$ with microcanonical distribution for large-scaled collective variables:

$$\varrho_{\mathcal{L}}\left(x^N,a;t\right) = \varrho_q^{kin}\left(x^N;t\right)\frac{\hat{f}(a)}{W(a;t)}. \tag{2.32}$$

This quasi-equilibrium distribution function is related to that in (2.20) $\varrho_q\left(x^N;t\right)$ by

$$\varrho_q\left(x^N;t\right) = \int da\ f(a;t)\varrho_{\mathcal{L}}\left(x^N,a;t\right), \tag{2.33}$$

and is normalized:

$$\int d\Gamma_N\ \varrho_{\mathcal{L}}\left(x^N;t\right) = 1. \tag{2.34}$$

Using the relation (2.33), it is convenient to represent averaging with quasi-equilibrium distribution function in the form

$$\langle\ldots\rangle_q^t = \int da\ \langle\ldots\rangle_{\mathcal{L}}^t f(a;t), \tag{2.35}$$

where

$$\langle\ldots\rangle_{\mathcal{L}}^t = \int d\Gamma_N\ \ldots\varrho_{\mathcal{L}}\left(x^N;t\right). \tag{2.36}$$

Now in accordance with (2.31) and (2.33) we can represent the action of the Liouville operator on $\varrho_q(x^N;t)$ as

$$i\mathcal{L}_N\varrho_q\left(x^N;t\right) = -\int da\int dx\ \gamma(x;t)\hat{n}_1(x)\varrho_{\mathcal{L}}\left(x^N,a;t\right)f(a;t) + \tag{2.37}$$
$$+\int da\sum_{k}\left(\dot{\hat{n}}_k W(a;t)\frac{\partial}{\partial n_k}\frac{f(a;t)}{W(a;t)} + \dot{\hat{\boldsymbol{J}}}_k W(a;t)\frac{\partial}{\partial \boldsymbol{J}_k}\frac{f(a;t)}{W(a;t)} + \right.$$
$$\left. + \dot{\hat{\mathcal{E}}}_k W(a;t)\frac{\partial}{\partial \mathcal{E}_k}\frac{f(a;t)}{W(a;t)}\right)\varrho_{\mathcal{L}}\left(x^N;t\right).$$

Substituting this expression into (2.26), one obtains for nonequilibrium distribution function the following result:

$$\varrho\left(x^N;t\right) = \int da\; \varrho_{\mathcal{L}}\left(x^N,a;t\right) f(a;t) + \tag{2.38}$$

$$+\int da\int dx\int\limits_{-\infty}^{t} dt' e^{\varepsilon(t'-t)}T(t,t')\Big(1-\mathcal{P}_q(t')\Big)\dot{\hat{n}}_1(x)\varrho_{\mathcal{L}}\left(x^N,a;t'\right)f(a;t')\gamma(x;t')$$

$$-\int da\sum_k\int\limits_{-\infty}^{t} dt' e^{\varepsilon(t'-t)}T(t,t')\Big(1-\mathcal{P}_q(t')\Big)\left(\dot{\hat{n}}_k W(a;t')\frac{\partial}{\partial n_k}\frac{f(a;t')}{W(a;t')}+\right.$$

$$\left.+\,\dot{\hat{J}}_k W(a;t')\frac{\partial}{\partial J_k}\frac{f(a;t')}{W(a;t')}+\dot{\hat{\mathcal{E}}}_k W(a;t')\frac{\partial}{\partial \mathcal{E}_k}\frac{f(a;t')}{W(a;t')}\right)\varrho_{\mathcal{L}}\left(x^N,a;t'\right).$$

This formula gives the nonequilibrium distribution function which describes consistently both kinetic and nonlinear hydrodynamic fluctuations in a classical fluid. One can go over to the traditional scheme accepted in the kinetic theory, if hydrodynamic fluctuations are neglected. The corresponding quasi-equilibrium distribution $\varrho_q^k(x^N;t)$ is obtained from (2.33) if one puts $f(a;t)\sim\delta(a-\overline{a})$, where $\overline{a}=ea=\{\overline{a}_k\}$ is the set of mean values of hydrodynamic quantities. Then the nonequilibrium distribution (2.38) takes the form

$$\varrho^k\left(x^N;t\right) = \varrho_q^k\left(x^N;t\right) + \tag{2.39}$$

$$+\int dx\int\limits_{-\infty}^{t} dt'\; e^{\varepsilon(t'-t)}T^k(t,t')\Big(1-\mathcal{P}_q^k(t')\Big)\dot{\hat{n}}_1(x)\varrho_q^k\left(x^N;t'\right)\gamma(x;t'),$$

$$\varrho_q^k\left(x^N;t\right) = \exp\left\{-\Phi^k(t) - \int dx\; \gamma(x;t)\hat{n}_1(x)\right\}, \tag{2.40}$$

$$\Phi^k(t) = \ln\int d\Gamma_N\; \exp\left\{-\int dx\; \gamma(x;t)\hat{n}_1(x)\right\}. \tag{2.41}$$

Calculating then the Lagrange multiplier $\gamma(x;t)$ in (2.40) from the consistency condition $\langle\hat{n}_1(x)\rangle^t = \langle\hat{n}_1(x)_q^t\rangle$, we get

$$\varrho_q^k\left(x^N;t'\right) = \prod_{j=1}^{N}\frac{f_1(x_j;t)}{e}, \tag{2.42}$$

$f_1(x_j;t)$ being the nonequilibrium one-particle distribution function. It can be shown [16,17] that the Liouville equation with the source specified by quasi-equilibrium distribution (2.42) leads to the well-known BBGKY hierarchy for reduced s-particle distribution functions with Bogoliubov's condition of complete weakening of initial correlations.

The result (2.38) can be used to derive the set of equations for the one-particle distribution function $f_1(x;t)$ and the distribution function of hydrodynamic variables $f(a;t)$. We start with the obvious relations

$$\begin{aligned}\frac{\partial}{\partial t}f_1(x;t) &= \langle\dot{\hat{n}}(x)\rangle^t = \langle\dot{\hat{n}}(x)\rangle^t_q + \langle I_n(x;t)\rangle^t, \\ \frac{\partial}{\partial t}f(a;t) &= \int d\Gamma_N\, \varrho\langle x^N;t\rangle \imath\mathcal{L}\hat{f}(a).\end{aligned} \tag{2.43}$$

Then the substitution of the explicit expression (2.42) into these relations gives after simple but somewhat lengthy manipulations

$$\begin{aligned}&\frac{\partial}{\partial t}f_1(x;t) + \imath\mathcal{L}(1)f_1(x;t) + \int dx'\, \imath\mathcal{L}(1,2)g_2(x,x';t) = \\ &= \int dx' \int da \int\limits_{-\infty}^{+\infty} dt'\, \mathrm{e}^{\varepsilon(t'-t)}\phi_{nn}(x,x',a;t,t')f(a;t')\gamma(x';t') - \\ &-\sum_{\boldsymbol{k}} \int da \int dt'\, \mathrm{e}^{\varepsilon(t'-t)} \Big\{\phi_{nJ}(x,a;t,t')W(a;t')\frac{\partial}{\partial \boldsymbol{J}_{\boldsymbol{k}}} + \\ &\qquad + \phi_{n\mathcal{E}}(x,a;t,t')W(a;t')\frac{\partial}{\partial \mathcal{E}_{\boldsymbol{k}}}\Big\}\frac{f(a;t')}{W(a;t')},\end{aligned} \tag{2.44}$$

$$\begin{aligned}&\frac{\partial}{\partial t}f(a;t) + \sum_{\boldsymbol{k}}\Big\{\frac{\partial}{\partial n_{\boldsymbol{k}}}v_n(a;t)f(a;t) + \\ &\qquad + \frac{\partial}{\partial \boldsymbol{J}_{\boldsymbol{k}}}v_J(a;t)f(a;t) + \frac{\partial}{\partial \mathcal{E}_{\boldsymbol{k}}}v_{\mathcal{E}}(a;t)f(a;t)\Big\} = \\ &-\sum_{\boldsymbol{k}}\frac{\partial}{\partial \boldsymbol{J}_{\boldsymbol{k}}}\int da' \int dx' \int\limits_{-\infty}^{+\infty} dt'\, \mathrm{e}^{\varepsilon(t'-t)}\phi_{Jn}(\boldsymbol{k},x',a,a';t,t')f(a';t')\gamma(x';t') - \\ &-\sum_{\boldsymbol{k}}\frac{\partial}{\partial \mathcal{E}_{\boldsymbol{k}}}\int da' \int dx' \int\limits_{-\infty}^{+\infty} dt'\, \mathrm{e}^{\varepsilon(t'-t)}\phi_{\mathcal{E}n}(\boldsymbol{k},x',a,a';t,t')f(a';t')\gamma(x';t') + \\ &+\sum_{\boldsymbol{k},\boldsymbol{q}}\int da' \int\limits_{-\infty}^{+\infty} dt'\, \mathrm{e}^{\varepsilon(t'-t)}\frac{\partial}{\partial \boldsymbol{J}_{\boldsymbol{k}}}\phi_{JJ}(\boldsymbol{k},\boldsymbol{q},a,a';t,t')W(a';t)\frac{\partial}{\partial \boldsymbol{J}_{\boldsymbol{q}}}\left(\frac{f(a';t'}{W(a';t')}\right) \\ &+\sum_{\boldsymbol{k},\boldsymbol{q}}\int da' \int\limits_{-\infty}^{+\infty} dt'\, \mathrm{e}^{\varepsilon(t'-t)}\frac{\partial}{\partial \mathcal{E}_{\boldsymbol{k}}}\phi_{\mathcal{E}\mathcal{E}}(\boldsymbol{k},\boldsymbol{q},a,a';t,t')W(a';t)\frac{\partial}{\partial \mathcal{E}_{\boldsymbol{q}}}\left(\frac{f(a';t'}{W(a';t')}\right) \\ &+\sum_{\boldsymbol{k},\boldsymbol{q}}\int da' \int\limits_{-\infty}^{+\infty} dt'\, \mathrm{e}^{\varepsilon(t'-t)}\Big\{\frac{\partial}{\partial \boldsymbol{J}_{\boldsymbol{k}}}\phi_{J\mathcal{E}}(\boldsymbol{k},\boldsymbol{q},a,a';t,t')W(a';t)\frac{\partial}{\partial \mathcal{E}_{\boldsymbol{q}}} + \\ &\qquad + \frac{\partial}{\partial \mathcal{E}_{\boldsymbol{k}}}\phi_{\mathcal{E}J}(\boldsymbol{k},\boldsymbol{q},a,a';t,t')W(a';t)\frac{\partial}{\partial \boldsymbol{J}_{\boldsymbol{q}}}\Big\}\left(\frac{f(a';t'}{W(a';t')}\right).\end{aligned} \tag{2.45}$$

where the one-particle and two-particle Liouville operators are given by the formulas

$$\imath\mathcal{L}(1) = \frac{\boldsymbol{p}}{m}\cdot\frac{\partial}{\partial \boldsymbol{r}}, \qquad \imath\mathcal{L}(1,2) = -\frac{\partial}{\partial \boldsymbol{r}}\Phi(|\boldsymbol{r}-\boldsymbol{r}'|)\left(\frac{\partial}{\partial \boldsymbol{p}} - \frac{\partial}{\partial \boldsymbol{p}'}\right).$$

We have also introduced the binary distribution function $g_2(x_1, x_2; t)$ in the quasi-equilibrium state

$$g_2(x_1, x_2; t) = \int d\Gamma_{N-2}\, \varrho_q\left(x^N; t\right). \tag{2.46}$$

The contribution of microscopic dynamics is described by the transport kernels $\phi_{\alpha\beta}$ which are expressed in terms of the correlation functions of generalized fluxes I_α:

$$\begin{aligned}
\phi_{nn}(x, x', a; t, t') &= \langle I_n(x;t) T_q(t,t') I_n(x';t')\rangle_{\mathcal{L}}^{t'}, &(2.47)\\
\phi_{nJ}(\boldsymbol{k}, x, a; t, t') &= \langle I_n(x;t) T_q(t,t') I_J(\boldsymbol{k};t')\rangle_{\mathcal{L}}^{t'}, &(2.48)\\
\phi_{n\mathcal{E}}(\boldsymbol{k}, x, a; t, t') &= \langle I_n(x;t) T_q(t,t') I_{\mathcal{E}}(\boldsymbol{k};t')\rangle_{\mathcal{L}}^{t'}, &(2.49)\\
\phi_{Jn}(\boldsymbol{k}, x, a; t, t') &= \langle I_J(\boldsymbol{k};t) T_q(t,t') I_n(x;t')\rangle_{\mathcal{L}}^{t'}, &(2.50)\\
\phi_{\mathcal{E}n}(\boldsymbol{k}, x, a; t, t') &= \langle I_{\mathcal{E}}(\boldsymbol{k};t) T_q(t,t') I_n(x;t')\rangle_{\mathcal{L}}^{t'}, &(2.51)\\
\phi_{JJ}(\boldsymbol{k}, \boldsymbol{q}, a, a'; t, t') &= \langle I_J(\boldsymbol{k};t) T_q(t,t') I_J(\boldsymbol{q};t')\rangle_{\mathcal{L}}^{t'}, &(2.52)\\
\phi_{J\mathcal{E}}(\boldsymbol{k}, \boldsymbol{q}, a, a'; t, t') &= \langle I_J(\boldsymbol{k};t) T_q(t,t') I_{\mathcal{E}}(\boldsymbol{q};t')\rangle_{\mathcal{L}}^{t'}, &(2.53)\\
\phi_{\mathcal{E}J}(\boldsymbol{k}, \boldsymbol{q}, a, a'; t, t') &= \langle I_{\mathcal{E}}(\boldsymbol{k};t) T_q(t,t') I_J(\boldsymbol{q};t')\rangle_{\mathcal{L}}^{t'}, &(2.54)\\
\phi_{\mathcal{E}\mathcal{E}}(\boldsymbol{k}, \boldsymbol{q}, a, a'; t, t') &= \langle I_{\mathcal{E}}(\boldsymbol{k};t) T_q(t,t') I_{\mathcal{E}}(\boldsymbol{q};t')\rangle_{\mathcal{L}}^{t'}, &(2.55)
\end{aligned}$$

$$\begin{aligned}
I_n(x;t) &= \Big(1 - \mathcal{P}(t)\Big)\dot{\hat{n}}_1(x), &(2.56)\\
I_J(\boldsymbol{k};t) &= \Big(1 - \mathcal{P}(t)\Big)\dot{\hat{\boldsymbol{J}}}_{\boldsymbol{k}}(x), &(2.57)\\
I_{\mathcal{E}}(\boldsymbol{k};t) &= \Big(1 - \mathcal{P}(t)\Big)\dot{\hat{\mathcal{E}}}_{\boldsymbol{k}}(x). &(2.58)
\end{aligned}$$

The Mori projection operator $\mathcal{P}(t)$ appearing in the fluxes is related to the Kawasaki-Gunton projection operator (2.24) by the equation

$$\mathcal{P}_q(t) a(x) \varrho_q\left(x^N; t\right) = \varrho_q\left(x^N; t\right) \mathcal{P}(t) a(x). \tag{2.59}$$

It should be emphasized that in formulas (2.47)–(2.55) the averages are calculated with the distribution $\varrho_{\mathcal{L}}\left(x^N, a; t\right)$ (2.32), so that the transport kernels are some functions of collective variables $a_{\boldsymbol{k}}$. In the second equation of (2.45), functions $v_n(a;t)$, $v_J(a;t)$, $v_{\mathcal{E}}(a;t)$ represent fluxes in the space of collective variables and are defined as

$$\begin{aligned}
v_n(a;t) = &\int d\Gamma_N\, \dot{\hat{n}}_{\boldsymbol{k}}\, \varrho_{\mathcal{L}}(x^N, a; t) = &\langle \dot{\hat{n}}_{\boldsymbol{k}}(a)\rangle_{\mathcal{L}}^{t},\\
v_J(a;t) = &\int d\Gamma_N\, \dot{\hat{\boldsymbol{J}}}_{\boldsymbol{k}}\, \varrho_{\mathcal{L}}(x^N, a; t) = &\langle \dot{\hat{\boldsymbol{J}}}_{\boldsymbol{k}}(a)\rangle_{\mathcal{L}}^{t}, \qquad (2.60)\\
v_{\mathcal{E}}(a;t) = &\int d\Gamma_N\, \dot{\hat{\mathcal{E}}}_{\boldsymbol{k}}\, \varrho_{\mathcal{L}}(x^N, a; t) = &\langle \dot{\hat{\mathcal{E}}}_{\boldsymbol{k}}(a)\rangle_{\mathcal{L}}^{t}.
\end{aligned}$$

Let us summarize. The set of equations (2.44) and (2.45) gives the consistent description of kinetic and hydrodynamic processes in a classical fluid with allowance made for long-living fluctuations. The transport kernel ϕ_{nn}

describes the dissipation of kinetic fluctuations, while the kernels ϕ_{nJ}, ϕ_{Jn}, $\phi_{n\mathcal{E}}$, and $\phi_{\mathcal{E}n}$ describe the dissipation of correlations between kinetic and hydrodynamic degrees of freedom. Finally, the transport kernels ϕ_{JJ}, $\phi_{J\mathcal{E}}$, $\phi_{\mathcal{E}J}$, and $\phi_{\mathcal{E}\mathcal{E}}$ correspond to dissipative processes connected with correlations of viscous and heat hydrodynamic modes. The coupled equations for kinetics and fluctuating hydrodynamics provide a basis for the calculation of low-frequency anomalies in neutral classical fluids and some other systems, say, in dense plasmas. Besides, these equations can be used to consider the influence of large-scaled fluctuations on kinetic processes in the vicinity of the critical point. These problems will be a subject of future work.

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МОДИФІКОВАНА КІНЕТИЧНА ТЕОРІЯ ДЛЯ ОПИСУ ПОВІЛЬНИХ ГІДРОДИНАМІЧНИХ ПРОЦЕСІВ

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Запропоновано один з підходів узгодженого опису кінетичних і гідродинамічних флуктуацій для класичних систем. Отримано зв'язану систему рівнянь для нерівноважної одночастинкової функції розподілу і функції розподілу гідродинамічних змінних: густини маси, імпульсу, енергії для опису кінетичних та гідродинамічних процесів в класичних рідинах.