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THEORY OF PHASE TRANSITIONS BY THE COLLECTIVE VARIABLES METHOD: ACCURATE CRITICAL INDICES AND DETAILED COMPARISON WITH THE OTHER APPROACHES

JA. M. ILNYTSKYI, M. P. KOZLOVSKII

Institute for Condensed Matter Physics of the Ukrainian National Academy of Sciences, 1 Svientsitskii Str., UA-290011 Lviv, Ukraine

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We discuss the collective variables method which was developed in the theory of phase transitions by Yukhnovskii. Here we present an effective combination of this approach with the Wilson's renormalization group scheme. Accurate critical indices were obtained in the case of n-component model using ϵ-expansion procedure. We made the detailed comparison between these two approaches based on comparing of two correspondent diagrammatic sets and discuss stability of the results.

1. Phase transitions theory and the collective variables method

Althought the "golden age" of the phase transitions theories lies in 1970-80s the questions of capability and accuracy of different theoretical approaches in this field are of a great interest till now. The reason lies both in the universality of critical behaviour (that is why theory can predict or describe phase transitions in newly founded physical systems) and in increasing accuracy of intermediate methods like Monte-Carlo simulations. A lot of methods based on renormalization group (RG) realization (i.e. different Wilson's type approaches [1–4], massless [5,6] and massive [7,8] field theories) are capable to provide a precise calculations of critical indices and other universal characteristics of phase transitions and cannot obtain critical temperature and other nonuniversal characteristics. On the other hand the precise calculations of critical temperature is made in within the Monte-Carlo RG approach [9], for example.

In spite of successes in calculations of fundamental properties describing system near phase transition point the main problem of phase transition theory remains still unsolved. The explicit expressions for thermodynamic functions, valid in the vicinity of T_c as the functions of temperature, magnetic field and microscopic parameters of Hamiltonian, are not calculated. These problems are solved in part within the collective variables (CV) method proposed firstly for the systems of charged particles [10,11]. This method has been applied to the theory of second order phase transitions owing to great efforts of Yukhnovskii and collaborators [12,13]. The main

purpose of these efforts is to develop the microscopic theory of the second order phase transitions which will provide not only the universal characteristics of the model, but also the possibility to obtain explicit expressions for thermodynamic and structure functions of 3D spin systems near the phase transition point. The method of layerwise integration of partition function, developed by Yukhnovskii in the end of 1970s turned to be an original and powerful mathematical tool for investigating phase transitions of different type.

This method has several distinct features in comparison with the other RG approaches. The correspondent RG transformation developed here is formulated in general case without applying any types of perturbation theory (but does not exclude such possibility) and thus it is free of asymptotically divergent series. It allows to obtain complete expressions for thermodynamic functions (specific heat, susceptibility, etc.) in the vicinity of T_c [13,14]. The prise for such level of generality are some approximations having to be done here. This results in slight inaccuracy and some instability of critical indices and other final results due to details of the RG transformation.

The aim of this report is to combine advantages of CV method with the more accurate calculational techniques developed in other RG approaches to obtain stable and correct values for critical indices and to make a detailed comparison of RG schemes used in both cases.

Firstly we shall stress due to Yukhnovskii [13] that the statistical description of phase transition process is to be performed in the appropriate phase space for a certain physical model. Among the independent variables of this space there must be the ones connected with order parameters, their non-zero average values appearing below the critical point. This phase space is formed of a CV set. For the certain considered model each CV represents the mode of density vibrations of the appropriate quantity like spin moment for magnets, Fourier-transform of the one-particle distribution function for many simple fluids, and so on. Thus CV is the general name of the variables class specific for every given physical system. For the case of magnetic system collective variables are connected with the fluctuations of the spin density [12,13]:

$$\hat{\rho}^{c}_{\vec{k}} = \frac{1}{\sqrt{N}} \sum_{\vec{l}} \cos(\vec{k}\vec{l}) \hat{s}_{\vec{l}}, \quad \hat{\rho}^{s}_{\vec{k}} = \frac{1}{\sqrt{N}} \sum_{\vec{l}} \sin(\vec{k}\vec{l}) \hat{s}_{\vec{l}}, \quad \hat{\rho}_0 = \frac{1}{\sqrt{N}} \sum_{\vec{l}} \hat{s}_{\vec{l}}, \tag{1.1}$$

where $\hat{\rho}_{\vec{k}}$ are the CV operators at the momentum $\vec{k}$ and $\hat{s}_{\vec{l}}$ is the spin operator in the $\vec{l}$ site. We shall use the complex CV for the convinience:

$$\hat{\rho}_{\vec{k}} = \hat{\rho}^{c}_{\vec{k}} - i\hat{\rho}^{s}_{\vec{k}}. \tag{1.2}$$

Transition from the initial configurational space to CV phase space corresponds to non-unitary transformation and we need to know the transition Jacobian. Due to this partition function in CV representation can be written in the following form [12,13]:

$$Z = \int \exp(-\beta \mathcal{H}(\rho)) \mathcal{J}(\rho) (d\rho)^N. \tag{1.3}$$

The Jacobian $\mathcal{J}(\rho)$ was found in [12,13]. Partition function in the CV representation is represented as an integral over CV. Under integral stands the exponent from the infinite series in products of CV. The expansion

on the interaction potential Fourier-transform in powers of k reduces $\mathcal{H}(\rho)$ to the form similar to Ginzburg-Landau Hamiltonian and Ornstein-Zernike formulae. The investigation of Euler equation corresponding to this form fulfilled by Yukhnovskii [13] proves that there exists a variable in the CV set which is connected with the order parameter. It was shown, that the basic measure density exists in the critical region which involves the second and the fourth powers of CV in the exponent [13]. Thus the partition function is represented in the form of functional integral with the quartic measure density. In the Ising-like case we have [12,13]:

$$Z = C \int \exp \left[-\frac{1}{2} \sum_{k<B} d(k) \rho_{\vec{k}} \rho_{-\vec{k}} - \frac{a_4}{4!N} \sum_{k_i<B}{}' \rho_{\vec{k}_1} \cdots \rho_{\vec{k}_4} \right.$$
$$\left. - \frac{a_6}{6!N^2} \sum_{k_i<B}{}' \rho_{\vec{k}_1} \cdots \rho_{\vec{k}_6} - \cdots \right] (d\rho)^N, \qquad (1.4)$$

where $d(k) = a_2 - \beta\tilde{\Phi}(k)$, $\beta\tilde{\Phi}(k)$ is the Fourier-transform of the interaction potential, prime $'$ over the sums denotes the momentum conserving $\vec{k}_1 + \cdots + \vec{k}_{2l} = 0$, and $\{a_{2n}\}$ are the constants of the transition Jacobian:

$$\begin{aligned} C &= \tfrac{1}{\pi} \approx 0.318310, & a_2 &= \left(\tfrac{\pi}{2}\right)^2 - 2 \approx 0.467401, \\ a_4 &= 2\left[\left(\tfrac{\pi}{2}\right)^4 - 6\right] \approx 0.176136, & a_6 &= 16\left[\left(\tfrac{\pi}{2}\right)^6 - 15\right] \approx 0.347298. \end{aligned} \qquad (1.5)$$

2. Renormalization group transformation

The renormalization group (RG) transformation proposed by Yukhnovskii (we will refer it as RGYu) foots on the same concept of eliminating the short-wave spin fluctuations as one of the Wilson's approaches (the approach with the recursion relations) [1]. It lies in step-by-step integration of the partition function over the layers of CV $\rho_{\vec{k}}, k \in (B^{(n+1)}, B^{(n)}], B^{(n)}/B^{(n+1)} = s$ (s is the RG parameter) starting from the variables with $k \sim B$ (B is the first Brillouin zone boundary). But in contrast to the last one it is formulated in general case without any variants of perturbation theory. This technique was developed in the series of papers previously by Yukhnovskii [13] and then by Kozlovskii and Pylyuk [14,15]. Both the critical indices and critical amplitudes as well as the complete expressions for the thermodynamic functions are obtained.

Some approximations (as the averaging of the $d(k)$ coefficient in every layer of CV) enable one to avoid the perturbation theory. It is clear that in the infinitesimal limit $s \to 1$ the influence of this approximation vanishes (except of $\eta = 0$ effect, where η is the small critical index). But in the case of $s > 1$ some calculational errors do occur (which caused the dependence of the final results on s). The analysis of numerical values for the critical indices [14,16] indicates a weak dependence on s in the region of $s \in [2.5, 4]$ where RGYu works relatively well. The dependence outside this interval is much more essential. This shows that on one hand the CV method developed in [13] is a powerful tool which gives the possibility to calculate the complete thermodynamics in the vicinity of the critical point, and on the other hand the approximations made in this approach cause the instability of the final results in some limiting cases of RG parameter s.

The aim of this report is to make a background analysis of the CV method and especially the RGYu and to correct its inaccuracy. As a result we will obtain relatively accurate values for the critical indices and provide a detailed comparison with the other approaches.

The rest of this paragraph includes a short review of the RGYu which was developed and described in details in [13]. Firstly the averaging of $d(k)$ is performed:

$$\sum_{k<B} d(k)\rho_{\vec{k}}\rho_{-\vec{k}} \approx \sum_{k<B^{(1)}} d(k)\rho_{\vec{k}}\rho_{-\vec{k}} + d^{(0)} \sum_{k\in\Delta^{(0)}} \rho_{\vec{k}}\rho_{-\vec{k}}\,, \qquad (2.6)$$

where $B^{(0)} = B$, $B^{(1)} = B/s$. Then the integration over CV $\rho_{\vec{k}}$ with $k \in (B^{(1)}, B^{(0)}]$ is performed. For this the original trick is used [13]. The $\rho_{\vec{k}}$ variables with $k \in (B^{(1)}, B^{(0)}]$ are redenoted as $\eta_{\vec{k}}$ and the sums over $k \in (0, B^{(1)}$ with the $\eta_{\vec{k}}$ products are completed to the interval $k \in (0, B^{(0)}$. To avoid integration over the superfluous variables $\eta_{\vec{k}}$ the cutting δ-function is inserted:

$$Z = C \int \exp\left[-\frac{1}{2}\sum_{k<B^{(1)}} (d(k) - d^{(0)})\rho_{\vec{k}}\rho_{-\vec{k}}\right] \prod_{k<B^{(1)}} \delta(\eta_{\vec{k}} - \rho_{\vec{k}}) \qquad (2.7)$$

$$\exp\left[-\frac{1}{2}d^{(0)}\sum_{k<B}\eta_{\vec{k}}\eta_{-\vec{k}} - \frac{a_4}{4!N}{\sum_{k_i<B}}'\eta_{\vec{k}_1}\ldots\eta_{\vec{k}_4} - \cdots\right](d\eta)^N(d\rho)^{N^{(1)}}.$$

Using the Fourier-transformation for the cutting δ-function the integration over $\eta_{\vec{k}}$ variables can be easily performed:

$$\prod_{\vec{l}} \int \exp\left[-\frac{1}{2}d^{(0)}\tilde{\eta}_{\vec{l}}^{\,2} - \frac{a_4}{4!N}\tilde{\eta}_{\vec{l}}^{\,4} - \frac{a_6}{6!N}\tilde{\eta}_{\vec{l}}^{\,6} - \cdots + 2\pi i\tilde{\eta}_{\vec{l}}\tilde{\nu}_{\vec{l}}\right] d\tilde{\eta}_{\vec{l}}$$

$$= (Q(d^{(0)}))^N \exp\left[-\frac{S_2}{2}\sum_{\vec{l}}\tilde{\nu}_{\vec{l}}^{\,2} - \frac{S_4}{4!}\sum_{\vec{l}}\tilde{\nu}_{\vec{l}}^{\,4} - \frac{S_6}{6!}\sum_{\vec{l}}\tilde{\nu}_{\vec{l}}^{\,6} - \cdots\right] \qquad (2.8)$$

$$= (Q(d^{(0)}))^N \exp\left[-\frac{P_2}{2}\sum_{k<B^{(1)}}\nu_{\vec{k}}\nu_{-\vec{k}} - \frac{P_4}{4!N^{(1)}}{\sum_{k_i<B^{(1)}}}'\nu_{\vec{k}_1}\ldots\nu_{\vec{k}_4}\right.$$

$$\left. - \frac{P_6}{6!(N^{(1)})^2}{\sum_{k_i<B^{(1)}}}'\nu_{\vec{k}_1}\ldots\nu_{\vec{k}_6} - \cdots\right](d\nu)^{N^{(1)}}, \qquad (2.9)$$

where

$$\tilde{\eta}_{\vec{l}} = \frac{1}{\sqrt{N}}\sum_{k<B}\eta_{\vec{k}}\, e^{i\vec{k}\vec{l}}, \qquad \tilde{\nu}_{\vec{l}} = \frac{1}{\sqrt{N}}\sum_{k<B}\nu_{\vec{k}}\, e^{i\vec{k}\vec{l}}. \qquad (2.10)$$

S_{2n} and P_{2n} are represented via the functions $U(x, x_1, \ldots)$, $\phi(x, x_1, \ldots)$, $\phi_n(x, x_1, \ldots)$:

$$P_2 \;=\; S_2 = (2\pi)^2\left(\frac{3}{a_4}\right)^{1/2} U(x, x_1, \ldots),$$

$$
\begin{aligned}
P_4 &= s^{-d}S_4 = s^{-d}(2\pi)^4\left(\frac{3}{a_4}\right)\phi(x,x_1,\ldots),\\
P_6 &= s^{-2d}S_6 = s^{-2d}(2\pi)^6\left(\frac{3}{a_4}\right)^{3/2}\phi_1(x,x_1,\ldots),\\
P_8 &= s^{-3d}S_8 = s^{-3d}(2\pi)^8\left(\frac{3}{a_4}\right)^{2}\phi_2(x,x_1,\ldots),\\
P_{10} &= s^{-4d}S_{10} = s^{-4d}(2\pi)^{10}\left(\frac{3}{a_4}\right)^{5/2}\phi_3(x,x_1,\ldots),
\end{aligned}
\tag{2.11}
$$

for which we obtained the following expressions:

$$
\begin{aligned}
U(x,x_1,\ldots) &= u_1,\\
\phi(x,x_1,\ldots) &= 3(-u_2+u_1^2),\\
\phi_1(x,x_1,\ldots) &= 5!!(u_3-3u_1u_2+2u_1^3),\\
\phi_2(x,x_1,\ldots) &= 7!!(-u_4+4u_3u_1-12u_1^2u_2+3u_2^2+6u_1^4),\\
\phi_3(x,x_1,\ldots) &= 9!!(u_5-5u_4u_1+20u_3u_1^2-60u_1^3u_2+24u_1^5\\
&\quad +30u_2^2u_1-10u_2u_3).
\end{aligned}
\tag{2.12}
$$

Here $u_a(t,t_1,\ldots)$ are the correspondent moments:

$$
u_a(t,t_1,\ldots)=\frac{\Gamma(1/2)}{\Gamma(a+1/2)}\frac{\int_0^\infty \xi^{2a}\exp(-t\xi^2-\xi^4/2-t_1\xi^6-t_2\xi^8-\cdots)d\xi}{\int_0^\infty \exp(-t\xi^2-\xi^4/2-t_1\xi^6-t_2\xi^8-\cdots)d\xi}.
\tag{2.13}
$$

After this the additional integration over the $\nu_{\vec{k}}$ is to be performed. This was done in the same manner resulting in the expression:

$$
Z = C(Q(d^{(0)}))^N\,(Q(P^{(0)}))^{N^{(1)}}\,Z^{(1)},
\tag{2.14}
$$

where $(Q(d^{(0)}))$ is the result of integration over the $\eta_{\vec{k}}$ variables and $(Q(P^{(0)}))$ corresponds to the result of integration over $\nu_{\vec{k}}$ variables. $Z^{(1)}$ has the same form as the initial expression (1.4) but with the short-wave $\rho_{\vec{k}}$ variables eliminated:

$$
Z^{(1)} = C\int \exp\left[-\frac{1}{2}\sum_{k<B^{(1)}} d^{(1)}(k)\rho_{\vec{k}}\rho_{-\vec{k}} - \frac{a_4{}^{(1)}}{4!N^{(1)}}\sum_{k_i<B^{(1)}}{}' \rho_{\vec{k}_1}\cdots\rho_{\vec{k}_4} - \frac{a_6{}^{(1)}}{6!(N^{(1)})^2}\sum_{k_i<B^{(1)}}{}' \rho_{\vec{k}_1}\cdots\rho_{\vec{k}_6} - \cdots\right](d\rho)^{N^{(1)}}.
$$

Thus the RGYu is much close to the Wilson's recursion relations [1] but it is formulated in general without any perturbations. The corresponding recursion relations (RR) in this approach take the following form:

$$
\begin{aligned}
r^{(n+1)} &= (s^{(n)})^2((r^{(n)}+q^{(n)})\mathcal{N}^{(n)}-q^{(n)}),\\
u^{(n+1)} &= u^{(n)}(s^{(n)})^{4-d}\mathcal{E}^{(n)},\\
w^{(n+1)} &= w^{(n)}(s^{(n)})^{6-2d}\mathcal{K}^{(n)},
\end{aligned}
\tag{2.15}
$$

where $r^{(n)}, u^{(n)}, w^{(n)}$ are the scaled couplings:

$$d^{(n)} = (r^{(n)}+q^{(n)})s^{-2n}, \quad a_4{}^{(n)} = u^{(n)}s^{-4n}, \quad a_6{}^{(n)} = w^{(n)}s^{-6n}, \tag{2.16}$$

and $\mathcal{N}^{(n)}, \mathcal{E}^{(n)}, \mathcal{K}^{(n)}$ are the special functions represented as the combinations of corresponding integrals:

$$\mathcal{N}^{(n)} = \frac{y^{(n)}U(y^{(n)}, y_1{}^{(n)}, \ldots)}{x^{(n)}U(x^{(n)}, x_1{}^{(n)}, \ldots)}, \quad \mathcal{E}^{(n)} = (s^{(n)})^{2d}\,\frac{\varphi(y^{(n)}, y_1{}^{(n)}, \ldots)}{\varphi(x^{(n)}, x_1{}^{(n)}, \ldots)}, \tag{2.17}$$

$$\mathcal{K}^{(n)} = (s^{(n)})^{4d}\,\frac{y_1{}^{(n)}\phi_1(y^{(n)}, y_1{}^{(n)}, \ldots)}{x_1{}^{(n)}\phi_1(x^{(n)}, x_1{}^{(n)}, \ldots)},$$

where the special functions are given in (2.12).

The RR in general form (2.15) were discussed intensively both numerically and analitically in [13–15]. The advantage of the CV method is that final calculations can be reduced to evaluation of one-fold integrals of (2.13) type which allows to consider a higher couplings (up to ρ^{10}) without a dramatic increasing of calculational efforts [16]. Although the numeric values for the critical indices obtained within this technique have the interval of stability on RG parameter s quite wide (about $s \in [2.5, 4]$) the problems in the vicinity of $s = 1$ does occur. The reason is that the combination of integrals (2.13) cannot describe the limit $s \to 1$ correctly while evaluated in the general form. The perturbation theory was applied to analyse the RR (2.15) is capable to avoid the problem directly at $s \to 1$ as it was shown in [17]. On the other hand the perturbation theory is a good chance to compare RGYu with other RG approaches, for example with the Wilson's RG scheme in a form of recursion relations [1].

3. Perturbation theory within the collective variables method

As one can see the RGYu consists of two steps: integration over the $\eta_{\vec{k}}$ variables (2.8), and integration over the N/s $\nu_{\vec{k}}$ variables appearing due to the Fourier-transform of the cutting δ-function (2.7). The perturbation theory being applied here means the evaluation of these integrals in the perturbation form. This can be easy done in the diagrammatic form, for example:

$$I(\nu) = \int \exp\left(-\frac{d}{2}\sum_{k<B}\eta_{\vec{k}}\eta_{-\vec{k}} + 2\pi i\sum_{k<B}\eta_{\vec{k}}\tilde{\nu}_{\vec{k}}\right) \times \tag{3.18}$$

$$\times\left\{1 - \frac{a_4}{4!N}\sum_{k_i<B}{}' \eta_{\vec{k}_1}\ldots\eta_{\vec{k}_4} + \frac{1}{2}\left(\frac{a_4}{4!N}\sum_{k_i<B}{}' \eta_{\vec{k}_1}\ldots\eta_{\vec{k}_4}\right)^2 + \cdots\right\}(d\eta)^N,$$

where the extended $\nu_{\vec{k}}$ variables are introduced:

$$\tilde{\nu}_{\vec{k}} = \nu_{\vec{k}}\,\theta(B^{(1)} - k) = \begin{cases} \nu_{\vec{k}}, & k < B^{(1)}, \\ 0, & k \in [B^{(1)}, B]. \end{cases} \tag{3.19}$$

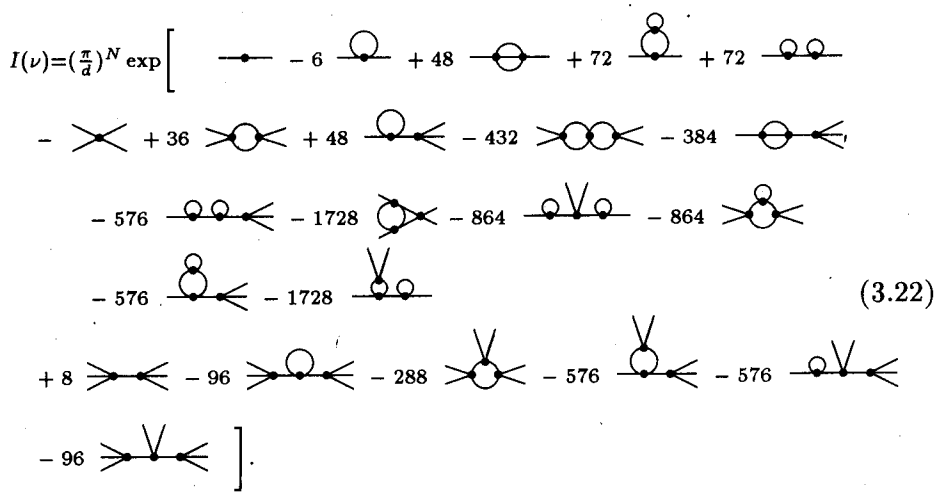
For the products of $\nu_{\vec{k}}$ we can write:

$$\int \eta_{\vec{k}_1} \dots \eta_{\vec{k}_{2l}} \exp\left(-\frac{d}{2}\sum_{k<B}\eta_{\vec{k}}\eta_{-\vec{k}} + 2\pi i \sum_{k<B}\eta_{\vec{k}}\tilde{\nu}_{\vec{k}}\right)(d\eta^N) \qquad (3.20)$$
$$= \frac{1}{(2\pi i)^{2l}} \frac{\partial}{\partial\tilde{\nu}_{\vec{k}_1}} \dots \frac{\partial}{\partial\tilde{\nu}_{\vec{k}_{2l}}} I_0(\nu),$$

where

$$I_0(\nu) = \int \exp\left(-\frac{d}{2}\sum_{k<B}\eta_{\vec{k}}\eta_{-\vec{k}} + 2\pi i \sum_{k<B}\eta_{\vec{k}}\tilde{\nu}_{\vec{k}}\right)(d\eta)^N \qquad (3.21)$$
$$= (\pi/d)^N \exp\left(\frac{(2\pi i)^2}{2d}\sum_{k<B}\tilde{\nu}_{\vec{k}}\tilde{\nu}_{-\vec{k}}\right)$$

is the nonperturbative result. This immeaditely yields the diagrammatic expansion for $I(\nu)$ (we will show here only contributions from the ρ^4 vertex for simplicity):



(3.22)

Here for each vertex stands $a_4/(4!d^2) = g_4^{(n)}/4$, and for each extenal leg variable $\tilde{\nu}_{\vec{k}}$: $(2\pi i/\sqrt{d})\tilde{\nu}_{\vec{k}}$. Because of the potential averaging (2.6) each loop is empty: $\frac{1}{N}\sum_{k<B} 1 = 1$ and they are drawn rather formally. The expansion (3.22) as compared with standard perturbation theory expansion [1,3] contains a lot of superfluous diagrams. Performing the second-step integrating over the $\nu_{\vec{k}}$ (which corresponds to the cutting δ-function) eliminates all these digrams and forms a loop-integral-like terms near the rest of diagrams. This mean pairing of all terms in (3.22) again. For example when we pair again the digrams set corresponding to P_4 (2.8) we obtain:

$(\frac{(2\pi)^2}{dP_2})^2 s^{-d}$ [-6  $+144$  $+216$  $+144$ 

$-$  $+36$  $+48$  -432  -384 

-576  -1728  -864  -864 

-576  -1728 ].

Pairing of diagrams set $P_4 \cdot P_4$ yields:

$(\frac{(2\pi)^2}{dP_2})^4 s^{-2d}$ [48  $+72$  $+72$ 

$+36$  $+48$  -864  -1152 

-1728  -1728  -1728  -1728 

-1728  -3456 ].

Here for each vertex stands again $a_4/(4!d^2) = g_4{}^{(n)}/4$, for external legs $\rho_{\vec{k}}$: $(2\pi i/\sqrt{P_2})\rho_{\vec{k}}$, each new formed loop is again empty. We obtain after gathering the similar terms and rewriting in the more usual form (in the two-loop(2L) approximation) following RR representation (see (3.26)). The pointed loops do not contain the loop integrals inside. These RR (3.26) may be easy compared with the same RR obtained in the Wilson's RR scheme [1,3]. The result is the following correspondence between two diagrammatic sets:

$$I_l = \frac{1}{N} \sum_{\vec{q} \in \Omega}^{*} \frac{1}{(d(\vec{q}))^l} \rightarrow \frac{\theta}{< d >^l} \tag{3.23}$$

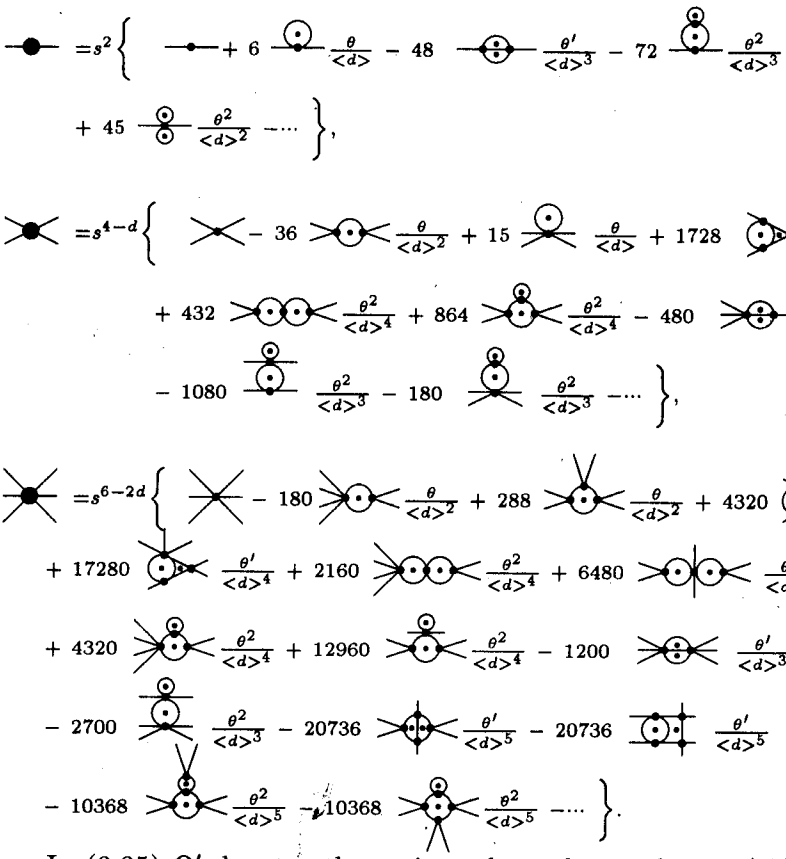
for the one-loop (1L) diagrams,

$$I_{lm} = \frac{1}{N^2} \sum_{\vec{q}_1 \in \Omega_1} \sum_{\vec{q}_2 \in \Omega_2} \frac{1}{(d(\vec{q}_1))^l (d(\vec{q}_2))^m} \rightarrow \frac{\theta^2}{< d >^{l+m}} \tag{3.24}$$

for the 2L diagrams which can be cutted along the vertex (we shall call them one-vertex reducible (1VR)), and

$$I_{lm(n)} = \frac{1}{N^2} \sum_{\vec{q}_1, \vec{q}_2 \in \Omega'} \frac{1}{(d(\vec{q}_1))^l (d(\vec{q}_2))^m (d(\vec{q}_1 + \vec{q}_2))^n} \rightarrow \frac{\theta'}{< d >^{l+m+n}} \tag{3.25}$$

for the 2L diagrams which cannot be cutted along the vertex (one-vertex irreducible (1VI)).



In (3.25) Ω' denotes the region where the conditions $|\vec{q}_1| \in (B/s, B]$, $|\vec{q}_2| \in (B/s, B]$, $|\vec{q}_1 + \vec{q}_2| \in (B/s, B]$ are satisfied. The factors

$$\theta = 1 - s^{-d}, \qquad \theta' = 1 - 3s^{-d} + 2s^{-2d} \tag{3.27}$$

correspond to the different subsets of diagrams.

Thus, the main difference of the RGYu is the original procedure of short-wave modes elimination. It is formulated in general form without using any perturbations [13] which yields nonperturbative approximate results for the critical indices and thermodynamic functions [14,15]. From the other point of view it can be verified and compared well with the other RG approaches using the perturbation technique. We performed it in a diagrammatic form which results in appearing of RR in form of (3.26). This diagrammatic expansion consists of two steps: firstly all digrams are the empty-loop ones and in addition a lot of superfluos reducible diagrams appeared (3.22); secondly all superfluos diagrams cancel and remaining diagrams acquire the loop-integral type approximate factors (3.23-3.25). As we see from (3.22) the RGYu yields the same set of diagrams as the usual perturbation technique [1]. The difference lies in correspondence between the factors (3.23-3.25). This correspondence is to be discussed more detaily.

1) The infinitesimal case ($s \to 1$).

In this case the potential averaging (2.6) effects have to disappeare. For example the exact expression for I_1 (3.23) at $d = 4$ is:

$$\begin{aligned} I_1 &= \frac{1}{N}\sum_{\vec{q}\in\Omega}\frac{1}{r+q^2} = \frac{d}{B^d}\int_{B/s}^{B}\frac{q^{d-1}\,dq}{r+q^2}\bigg|_{d=4} = \\ &= \frac{4}{B^4}\left(\frac{B^2}{2}(1-s^{-2}) - \frac{r}{2}\ln\frac{r+B^2}{r+(B/s)^2}\right) \\ &\overset{s\to 1}{\to} \frac{4\ln(s)}{r+B^2} + O(\ln^2(s)). \end{aligned} \tag{3.28}$$

The corresponding approximate expression from (3.23) reads:

$$I_1 \overset{<\ldots>_\Omega}{\to} \frac{\theta}{<d>} \overset{s\to 1}{\to} \frac{d\ln(s)}{<d>}\bigg|_{d=4} = \frac{4\ln(s)}{<d>}. \tag{3.29}$$

As we see at the infinitesimal limit both expression are the same and are equal to $4\ln(s)/<d>$. This is valid both for the 1L and 1VR 2L diagrams of (3.26). In the case of 1VI 2L diagrams we have (for example at $d = 3$):

$$\lim_{s\to 1} I_{lm(n)} \sim \begin{cases} \frac{9}{2}\ln^3(s) & \text{Wilson's approach,} \\ -3\ln(s) & \text{collective variables RG,} \end{cases} \tag{3.30}$$

that is why in the usual perturbation scheme these diagrams in infinitesimal limit disappeare (as was shown by Wegner and Houghton [4]). Due to performing of RGYu these diagrams still remain. Later we shall made an estimation of their essence while calculating the critical indices. It should be mentioned that these diagrams are neglected in principle in some approximate RG transformations (for example in the Wilson's approximate recursion formula [1]).

2) The intermediate values of s.

The approximate recursion formula [1] for example works at $s = 2$. To obtain the same results we have to neglect the 1VI 2L diagrams and then put $s = 2$ in the frames of RGYu. In fact we are able to remain these diagrams. In this case we have a good chance to work in a small interval of $s \in [1.2, 1.4]$ (fig.1) where the approximate and exact factors corresponding to 1VI 2L diagrams (3.25) are the closest. It is interesting to note that these two expressions coinside at $s \sim 2^{1/d}$ (d is dimension of space). The special role of this value for the approximate RG schemes was pointed out by Baker [18].

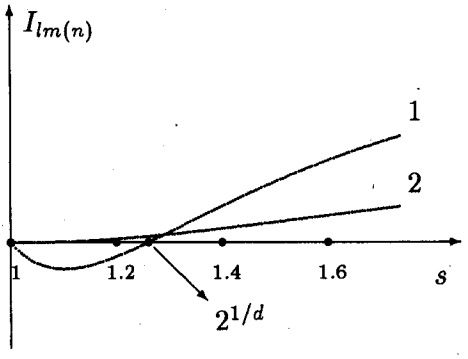


Figure 1. Dependence of 1VI 2L diagrams on RG parameter s (1: presented method, 2: Wilson's recursion relations approach)

3) Large values of s.

This limit was studied by Bruce et al. [3] within the Wilson's RG scheme. It was shown that in this case the higher order couplings (ϕ^6 and higher) become irrelevant and the scheme with the RR leads to results the same as in massless field theory [19] (using $\epsilon = 4 - d$-expansion). Unfortunately the averaging of potential (2.6) is too dramatic approximation to discuss this limit within the perturbation analysis of the CV RG transformation.

4. Calculation of critical indices

In the previous section we have developed the perturbation theory technique within RGYu which is used in the CV method. We have studied as well the correspondence between the obtained diagrammatic set and the Wilson's RR approach. Here we shall obtain the relatively accurate values for the critical indices using presented in the previous section scheme.

Let us pass from the diagrammatic form of RR to the algebraic one. It should be mentioned that topological equivalence in diagrammatic set (4.26) means the same loop-integral type factor which is an approximation of the correspondent topologically dependent loop integral. Thus after gathering all diagrams of those type into the single algebraic expressions we obtain:

$$
\begin{aligned}
r^{(n+1)} &= s^2 \Big[(r^{(n)}+q)(1+3\theta g_4{}^{(n)} - 3(2\theta'+3\theta^2)(g_4{}^{(n)})^2 \\
&\qquad + \tfrac{45}{4}\theta^2 g_6{}^{(n)} + \cdots) - q\Big], \\
u^{(n+1)} &= s^{4-d} u^{(n)} \Big[1 - 9\theta g_4{}^{(n)} + \tfrac{15}{2}\theta g_6{}^{(n)}/g_4{}^{(n)} + 27(4\theta'+3\theta^2)(g_4{}^{(n)})^2 \\
&\qquad - \tfrac{15}{2}(8\theta'+21\theta^2) g_6{}^{(n)} + \cdots\Big], \qquad (4.31) \\
w^{(n+1)} &= s^{6-2d} w^{(n)} \Big[1 - 45\theta g_4{}^{(n)} + 36\theta (g_4{}^{(n)})^3/g_6{}^{(n)} \\
&\qquad + 270(5\theta'+6\theta^2)(g_4{}^{(n)})^2 - \tfrac{75}{2}(4\theta'+9\theta^2) g_6{}^{(n)} \\
&\qquad - 648(2\theta'+\theta^2)(g_4{}^{(n)})^4/g_6{}^{(n)} + \cdots\Big],
\end{aligned}
$$

where the redefined couplings read:

$$g_4{}^{(n)} = \frac{u^{(n)}}{6(r^{(n)}+q)^2}, \qquad g_6{}^{(n)} = \frac{w^{(n)}}{90(r^{(n)}+q)^3}. \tag{4.32}$$

Turning the temperature to the critical one T_c we expect the fixed point solution of the RR (4.31) $\lim_{n\to\infty} g_4{}^{(n)} = g_4^*$, $\lim_{n\to\infty} g_6{}^{(n)} = g_6^*$ which leads to the set of equations:

$$\begin{aligned} &s^{4-d}[1 - 9\theta g_4^* + \tfrac{15}{2}\theta g_6^*/g_4^* + 27(4\theta' + 3\theta^2)(g_4^*)^2 - \tfrac{15}{2}(8\theta' + 21\theta^2)g_6^*] = 1, \\ &s^{6-2d}[1 - 45\theta g_4^* + 36\theta(g_4^*)^3/g_6^* + 270(5\theta' + 6\theta^2)(g_4^*)^2 \\ &\qquad - \tfrac{75}{2}(4\theta' + 9\theta^2)g_6^* - 648(2\theta' + \theta^2)(g_4^*)^4/g_6^*] = 1. \end{aligned} \tag{4.33}$$

Recalling the 1L solution for the g_4 coupling [1]:

$$g' = g_4' = \frac{1 - s^{4-d}}{9(1 - s^{-d})} \tag{4.34}$$

we expect that taking higher orders into account will cause no change in this order. Thus when we find the g_6^* as a function of g_4^* from the second equation of (4.33):

$$g_6^* = \frac{36(1 - s^{-d})}{s^{2(d-3)} - 1}(g_4^*)^3, \quad \text{at } d \neq 3 \tag{4.35}$$

$$g_6^* = 4/5(g_4^*)^2, \quad \text{at } d = 3 \tag{4.36}$$

we see that the functional dependence (4.36) is not acceptable because it will damage the 1L solution for g_4^* being substituted into the first equation of (4.33). The main idea of solving the equation set (4.33) is to represent both g_4^* and g_6^* couplings as power series of 1L solution (4.34). It gives the power expansions of following form:

$$g_4^* = g'[1 - (\frac{30(1 - s^{-d})}{1 - s^{-2(3-d)}} + 3(4 - 11\theta))g' + \cdots], \tag{4.37}$$

$$g_6^* = -\frac{36(1 - s^{-d})}{1 - s^{-2(3-d)}}g'^3 + \cdots \tag{4.38}$$

In the infinitesimal limit $s \to 1$ we have more compact forms:

$$g_4^* = g'[1 + 3\frac{d+12}{d-3}g' + \cdots], \qquad g_6^* = \frac{18d}{d-3}g'^3 + \cdots \tag{4.39}$$

(in this case $g' = g_4' = \frac{4-d}{9d}$). Using the well known technique for linearizing of the linear RG operator [1] we obtain following expansion in powers of g_4^* for the correlation length critical index ν:

$$\nu = \frac{1}{2} + \frac{3d}{4}g_4^* + \frac{9d}{8}\cdot\frac{8 + 10d - d^2}{d-2}(g_4^*)^2 + \cdots \tag{4.40}$$

The expression for g_4^* (4.39) has to be substituted here. This results in effective reexpansion of ν in terms of 1L solution (4.34):

$$\nu = \frac{1}{2} + \frac{3d}{4}g' + \frac{9d}{8}\cdot\frac{d^3 - 11d^2 + 42d - 24}{(d-2)(d-3)}g'^2 + \cdots \qquad (4.41)$$

It is interesting to note that such reexpansion procedure leads to very interesting manipulations with the asymptotic series obtained within massive field theory and it is capable to provide a high precision calculations of critical indices without applying the Padé-Borel resummation technique (see Appendix).

As we see from (4.41) the resulting expression for ν will diverge at $d = 3$. This is due to the fact that the 2L contribution includes the g_6 coupling terms (having marginal dimension equal to 3). Here the dimensional regularization idea appeared naturally. While performing the expansion $\frac{1}{d-3} = \frac{1}{1-\epsilon} \approx 1 + \epsilon + \epsilon^2 + \cdots$ we obtain for the critical index ν the following ϵ-expansion:

$$\nu = \frac{1}{2} + \frac{1}{12}\epsilon + \frac{1}{18}\epsilon^2 + \cdots \qquad (4.42)$$

The ϵ-expansion obtained here can be easy generalized to the case of n-component model. It is convinient to pass to the infinitesimal limit $s \to 1$ which was provided within the Wilson's approach by Wegner and Houghton [4] and within the CV method by Vakarchuk and Rudavskỳ [20]. The ϕ^6 model was investigated in frames of last approach at $d = 3$ [17]. The differential equations of the following form were obtained [17,20]:

$$\begin{aligned}
\frac{\partial r}{\partial \xi} &= (r+q)[\frac{2r}{r+q} + (n+2)dg_4 + 2(n+2)dg_4^2], \\
\frac{\partial u}{\partial \xi} &= u[4 - d - (n+8)dg_4 + \frac{3}{2}(n+4)d(g_6/g_4) - 4(5n+22)dg_4^2 \\
&\qquad + 12(n+4)dg_6], \qquad (4.43) \\
\frac{\partial w}{\partial \xi} &= w[6 - 2d - 3(n+14)dg_4 + \frac{4}{3}(n+26)d(g_4^3/g_6) - 54(3n+22)dg_4^2 \\
&\qquad + (3n+22)dg_6],
\end{aligned}$$

where $\xi = \ln s$. The corresponding set of equations for the fixed point coordinates is:

$$\frac{\partial u}{\partial \xi} = \frac{\partial w}{\partial \xi} = 0. \qquad (4.44)$$

We will use the trick of reexpansion and then perform the dimensional regularization in the same manner as in the case of the Ising model. This results in the following expansions:

$$\nu = \frac{1}{2} + \frac{n+2}{4(n+8)}\epsilon + \frac{n+2}{8(n+8)^3}(n^2 + 31n + 76)\epsilon^2, \qquad (4.45)$$

$$\Delta = -\frac{\epsilon}{2} - \frac{n+14}{4(n+8)}\epsilon^2, \qquad (4.46)$$

$$\Delta_1 = 1 + \frac{n+14}{n+8}\epsilon + \frac{7n^3 + 306n^2 + 3156n + 8600}{2(n+8)^3}\epsilon^2, \qquad (4.47)$$

where Δ and Δ_1 are the indices of corrections to scaling. The expansion for ν (4.45) is to be compared with the same order expansion obtained within the massless field theory by Wilson [5]:

$$\nu = \frac{1}{2} + \frac{n+2}{4(n+8)}\epsilon + \frac{n+2}{8(n+8)^3}(n^2+23n+60)\epsilon^2. \qquad (4.48)$$

As we see the slight difference is present in the 2L terms. It caused by some specialities in given RG transformation and dissapeares in the spherical limit $n \to \infty$. It should be pointed again that our result (4.45) is obtained in the infinitesimal limit $s \to 1$ while the field theoretical one (4.48) corresponds to another limit $s \to \infty$. The expansion for ν (4.45) gives at $\epsilon = 1$ relatively acurate results: $\nu = 0.600, 0.639, 0.671, 0.697$ (for $n = 0, 1, 2, 3$ respectively) as compared to high-temperature series data $\nu = 0.600, 0.638, 0.670, 0.703$ [21–24] and field-theoretical results at $d = 3$ $\nu = 0.588, 0.630, 0.669, 0.705$ [7,8].

5. Discussing stability of results

Let us discuss here two points related to the stability of our results. The first one is the essence of the 1VI 2L diagrams appeared in the RR (5.26) which yields some peculiarities in calculations. Calculating the fixed point coordinates g_4^* and g_6^* we obtain from the second equation in (4.44):

$$g_6^* = \frac{2}{3}\frac{d}{d-3}(n+26)(g_4^*)^3. \qquad (5.49)$$

Substitution of this expression into the first equation in (4.44) gives the contribution:

$$+4(n+4)(n+26)dg_4^{*2} = +4(n^2+30n+104)dg_4^{*2}. \qquad (5.50)$$

It may be compared with the same order contribution which comes from the 2nd type 2L diagram initially proportional to g_4^{*2}:

$$-4(5n+22)dg_4^{*2}. \qquad (5.51)$$

There are not any other diagrams of the same order in first equation of (4.44). As we see the contribution from the 1VI 2L diagram to the fixed point coordinates is $\frac{1}{7}$ - $\frac{1}{5}$ as compared to the same order contribution which goes from the 1L diagram from the second equation of (4.44). The same estimations may be easy fulfilled for other contributions. The result is that contributions of the capricious 1VI 2L diagrams do not exceed the $\frac{1}{5}$ as compared to the main important contributions from the corresponding 1L diagrams. Thus the results obtained for the critical indices demonstrate a weak dependence on peculiarities in calculation of these diagrams. It should be noted that these considerations correspond only to the perturbation analysis of the CV RG scheme and do not refer to the general nonperturbative scheme itself. Applying the perturbation theory is only a good chance to verify this approach and to make a well established comparison with other RG schemes.

The second question appeared here is stability of the final results on choosing the value of RG parameter s. The ϵ-expansions (4.45-4.47) are

obtained within the $s \to 1$ limit. There are no problems to perform the calculations for arbitrary (not very large) value of s. In Ising-like case we obtain for the critical index ν:

$$\nu = \frac{\ln(s)}{\ln(E_1)} = \frac{1}{2} + \frac{\epsilon}{12} + \left[\frac{1}{4} + \frac{s^4 + 4s^2 + 1}{2(s^4 - 1)} \ln(s)\right] \frac{\epsilon^2}{18}, \qquad (5.52)$$

which demonstrate the relative flow of the s-dependent coefficient within the interval of $s \in [1.0, 3.0]$ not more than 0.5% Thus providing the perturbation analysis within the CV method using the reexpansion procedure and ϵ-expansion gives a possibility to obtain relatively accurate results for the critical indices of the Ising and n-component model. The numerical values obtained almost do not depend on the details of the RG transformation and are stable in the wide range of the RG parameter s.

6. Conclusions

Thus the effective combination of different methods is presented here. The usual renormalization group realizations (different Wilson's type approaches, [1–4], massless [5,6] and massive [7,8] field theories) are capable to provide a precise calculations of critical indices and other universal characteristics of phase transitions but cannot obtain critical temperature and other nonuniversal characteristics. The collective variables (CV) method developed previously for the systems of charged particles [10,11] being applied to the phase transitions phenomena [13] gives such possibility. One can obtain both values for critical indices and the complete expressions for the thermodynamic functions (like free energy, specific heat, etc.) [13–15]. One of the original aspects of this method is the use of original renormalization group transformation which is formulated in general without applying of the perturbation theory. Unfortunately the approximations made here cause some peculiarities in calculations performed for limiting cases of the renormalization parameter s.

Thus the idea to combine CV method with other renormalization techniques appear naturally. Besides of improving resulting accuracy within the CV method it gives a good possibility of making detailed comparison between it and other approaches which is of great interest even as a separate problem.

This is the subject of presented report. We performed perturbation analysis of the renormalization group transformation within the CV method. This gives us the possibility both to make a detailed comparison between this method and Wilson's recursion relations approach and to obtain relatively accurate values for the critical indices. We show that slight differences between two approaches appeared in two-loop level are caused by peculiarities in calculation of one-vertex irreducible diagrams and turn to be unessential.

The original reexpansion procedure was used for search of the fixed point coordinates and the dimension regularization is fulfilled to avoid the divergencies at $d = 3$. The numerical values of critical indices obtained here are in a good agreement with the high-temperature series analysis. The detailed verification of stability and validity of results is made. We show that applying the ϵ-expansion it is possible to avoid dependence of the final results on the detais of renormalization group scheme.

Finally we can state that combining of the CV method with the perturbation theory technique gives a fine-tuned tool in the theory of phase

transitions.

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Appendix. Application of the reexpansion procedure to the asymptotic series of massive field theory

Here we shall apply (and testify) the reexpansion procedure in the case of asymptotic series obtained within the massive field theory at fixed dimension $d = 3$ [7,8]. The critical behaviour of the system of n-component spins located in the sites of d-dimensional lattice and interacting via short-range potential corresponds to the field theory with the action [8]:

$$\mathcal{A}(\vec{\phi}) = \int \left[\frac{1}{2}(\partial_\mu \vec{\phi})^2 + \frac{1}{2}m^2\vec{\phi}^2 + \frac{1}{4!}g(\vec{\phi}^2)^2\right] d^dx. \qquad \text{(A.1)}$$

The vertex functions $\Gamma^{(L,N)}(q_i, p_j)$ satisfy the Callan-Symanzik equation [8]:

$$\left[m\frac{\partial}{\partial m} + W(g)\frac{\partial}{\partial g} - \frac{N}{2}\eta(g) - L(\nu^{-1}(g) - 2)\right]\Gamma^{(L,N)}(q_i, p_j) = m^2(2 - \eta(g))\Gamma^{(L+1,N)}(0, q_i, p_j), \qquad \text{(A.2)}$$

where g is the renormalized ϕ^4 type coupling and $W(g)$ is the RG function. Zeros of this function give the fixed point coordinates g^* [8]:

$$W(g^*) = 0. \qquad \text{(A.3)}$$

Then g^* is to be substituted into $\nu(g)$ and $\eta(g)$ which gives physical values for critical indices ν and η:

$$\nu = \nu(g^*), \qquad \eta = \eta(g^*). \qquad \text{(A.4)}$$

These two-step calculations may be performed in the different way: numerically (applying or not the Padé-Borel resummation technique [7,8]) or using some analytical methods. The reexpansion procedure could be chosen as one of such analytical methods.

Using the perturbation theory one can obtain the functions $W(g^*)$, $\nu(g^*)$, and $\eta(g^*)$ as power series on g. We shall recall here a six-loop results obtained by Baker et. al. at $d = 3$ [7]. In the Ising-like case ($n = 1$) [7]:

$$\begin{aligned}
W(g) &= g(1 - g + 0.422497g^2 - 0.351070g^3 + 0.376527g^4 \\
&\quad - 0.495548g^5 + 0.749689g^6 - \cdots), \\
\nu(g) &= 0.5 + 0.083333g - 0.001886g^2 + 0.008363g^3 \\
&\quad - 0.006093g^4 + 0.008043g^5 - 0.011164g^6 - \cdots \\
\eta(g) &= 0.010974g^2 + 0.000914g^3 + 0.001796g^4 \\
&\quad - 0.000654g^5 + 0.001388g^6 - \cdots
\end{aligned} \qquad \text{(A.5)}$$

According to reexpansion scheme we have to find the 1L solution of eq.(A.3) g' (here due to $W(g)$ normalization $g' = 1$) and then find the complete solution g^* as a power series on g'. This gives:

$$g^* = g'(1 + 0.422497g' + 0.005937g'^2 + 0.011984g'^3 - 0.041231g'^4 + 0.040135g'^5 - \cdots). \quad (A.6)$$

This series is to be substituted into the series for $\nu(g^*)$ and $\eta(g^*)$. After gathering the similar terms we obtain the reexpanded series for $\nu(g')$ and $\eta(g')$:

$$\begin{aligned} \nu &= 0.5 + 0.083333g' + 0.033322g'^2 + 0.007264g'^3 + 0.005146g'^4 \\ &\quad -0.001118g'^5 + 0.003694g'^6 - \cdots \quad (A.7) \\ \eta &= 0.010974g'^2 + 0.010187g'^3 + 0.005044g'^4 + 0.003206g'^5 \\ &\quad +0.001295g'^6 + \cdots \end{aligned}$$

It is clear that the asymptotic nature of the initial series remains. But the coefficients of reexpanded series became of the same sign and display monotonous decreasing. Second important feature is appearence of relatively small coefficients (of order 10^{-3}). This allows us to use one property of asymptotic series and to neglect the rest of the series with the error not greater than the first term omited (we marked the neglected part by small font). This immeadetily yields:

$$\nu = 0.629 \pm 0.001, \qquad \eta = 0.029 \pm 0.001, \quad (A.8)$$

which is in exellent accordance with the same result obtained via the Padé-Borel resummation method [7,8]. The situation for other dimensions of spin n is similar. In the whole interval $n \in [0, 3]$ we have observed appearence of relatively small terms which makes possible to obtain the results listed in tabl.1. It should be mentioned that accuracy of index η obtained with the help of reexpansions is even better than it accuracy while using Padé-Borel method.

This example shows the efficiency and accuracy of the reexpansion procedure used in the modified renormalization scheme within the collective variables method presented here. It has to be mentioned that this procedure is close to the pseudo-ϵ-expansion [8,21].

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МЕТОД КОЛЕКТИВНИХ ЗМІННИХ У ТЕОРІЇ ФАЗОВИХ ПЕРЕХОДІВ: ПРЕЦИЗІЙНІ КРИТИЧНІ ПОКАЗНИКИ І ДЕТАЛЬНЕ ПОРІВНЯННЯ З ІНШИМИ ПІДХОДАМИ

Я.М. Ільницький, М.П. Козловський

Обсуджується метод колективних змінних, розвинутий для випадку теорії фазових переходів І.Р.Юхновським. Пропонується ефективне поєднання цього підходу із ренормгруповою схемою К.Вільсона. В результаті застосування процедури ϵ-розкладу отримано прецизіцні значення для критичних індексів у випадку n-компонентної моделі. Виконано детальне порівняння обох підходів, яке базується на порівнянні відповідних діаграмних розвинень. Значна увага звертається на стійкість отриманих результатів.