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ONE-DIMENSIONAL LATTICE GAS

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The one-dimensional lattice gas with short- and long-range interactions is considered. The thermodynamic behaviour of the system is obtained in the usual way by mapping it onto the one-dimensional Ising model, which can be exactly solved. The solution of the magnetic model is determined using the functional integral method, and a review of its critical behaviour is presented. The phase diagram of the lattice gas is obtained for various strengths of the interaction parameters, and the different critical regimes identified are compared to the ones obtained for the classical gas.

1. Introduction

In a recent series of papers Ikeda [1,2] and Kurioka and Ikeda [3,4] have considered the one-dimensional classical gas with hard core repulsive interactions, plus short-range attractive or repulsive and long-range attractive interactions. They have been able to solve exactly the model and determine the phase diagram of the system. For attractive short-range interaction they have found that the system undergoes just one transition, whereas for repulsive short-range interaction it can present two transitions. One of the main problems presented by the model is that the isotherms, in the critical region, present a non-physical behaviour, as in the case of the van der Waals model [5]. Then a Maxwell construction had to be used in order to describe correctly the physical behaviour in the critical region, and this means that further refinement has to be introduced in the model in order to get a correct description of the physical reality.

The main purpose of this paper is to look at a more refined model, namely, the lattice gas [6], considering the same types of interactions. As it is well established, the lattice gas can be mapped onto the Ising model [6] and its critical behaviour obtained from this model. Therefore, in order to obtain the phase diagram of the model, we will consider initially the one-dimensional Ising model. Although its solution is already known [7], a detailed review of the model is presented in section 2. The solution is obtained by a new method, namely, the functional integral method, and a careful discussion of the various critical behaviours is presented. The Helmholtz free energy is also discussed on the various critical regimes and the equation of state obtained for arbitrary values of the interaction parameters.

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In section 3, by using the results from section 2, we obtain the phase diagram of the lattice gas model and present explicit results. Although for repulsive long-range interactions the model does not present any transition, as a question of completeness this case is also considered. Finally in section 4 we present the conclusions and summarize the main results of the paper.

2. The Ising model

Let us consider initially the one-dimensional Ising model whose Hamiltonian is written in the form:

$$\mathrm{H} = -J\sum_{j=1}^{N}\sigma_j\sigma_{j+1} - \frac{I}{N}\sum_{j,k=1}^{N}\sigma_j\sigma_k - h\sum_{j=1}^{N}\sigma_j\,, \tag{2.1}$$

where we have assumed periodic boundary conditions and the long-range interactions properly renormalized in order to obtain the thermodynamic limit ($N \to \infty$). By using the gaussian transformation [8], we can write the partition function in terms of an integral functional as

$$\mathrm{Z}_N = \frac{e^{-\bar{I}}}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-x^2/2}Q_N(x)dx, \tag{2.2}$$

where

$$Q_N(x) = \sum_{\{\sigma\}}\exp\left[K\sum_{j=1}^{N}\sigma_j\sigma_{j+1} + h(x)\sum_{j=1}^{N}\sigma_j\right], \tag{2.3}$$

with

$$K = \beta J, \tag{2.4}$$

$$h(x) = \beta h + x\sqrt{\frac{2\beta I}{N}} \equiv \bar{h} + x\sqrt{\frac{2\bar{I}}{N}}, \tag{2.5}$$

and

$$\beta = 1/k_BT. \tag{2.6}$$

$Q_N(x)$ is easily evaluated and for arbitrary N is given by [9]:

$$Q_N(x) = \lambda_+^N(x) + \lambda_-^N(x), \tag{2.7}$$

where

$$\lambda_\pm = e^K\left\{\cosh h(x) \pm [\sinh^2 h(x) + e^{-4K}]^{1/2}\right\}. \tag{2.8}$$

In the thermodynamic limit ($N \to \infty$), Q_N is given approximately by:

$$Q_N(x) \cong \lambda_+^N(x). \tag{2.9}$$

By using this result in (2.2) and introducing the new variable $z \equiv x/\sqrt{N}$ the integral can be evaluated by the saddle point method [10]. The partition function can be written in the form:

$$\mathrm{Z}_N = \frac{e^{-\bar{I}}e^{Ng(z_0)}}{[-g''(z_0)]^{1/2}}\,, \tag{2.10}$$

where

$$g(z) = \ell n\lambda_+(z) - \frac{z^2}{2}, \tag{2.11}$$

$g''(z)$ denotes the second derivative of $g(z)$, and z_0 is obtained from the equation $g'(z_0) = 0$, namely

$$\sqrt{2\bar{I}}\frac{\sinh h(z_0)}{[\sinh^2 h(z_0) + e^{-4K}]^{1/2}} - z_0 = 0. \tag{2.12}$$

The Helmholtz free energy per spin

$$f = -\frac{\beta^{-1}}{N}\ell n Z_N \tag{2.13}$$

after some straightforward algebraic manipulations can be written in the form:

$$f = -\beta^{-1}\ell n\lambda_+(z_0) + \frac{\beta^{-1}z_0^2}{2}. \tag{2.14}$$

From this result we can obtain the magnetization per lattice site which is given by

$$\sigma \equiv \langle \sigma_j \rangle \equiv -\frac{\partial f}{\partial h} = \frac{\sinh(\bar{h} + \sqrt{2\bar{I}}z_0)}{[\sinh^2(\bar{h} + \sqrt{2\bar{I}}z_0) + e^{-4K}]^{1/2}}, \tag{2.15}$$

and by comparing it to (2.12) we conclude that $z_0 = \sqrt{2\bar{I}}\sigma$. Therefore the magnetization, as expected [11], is determined from the equation

$$\sigma - \frac{\sinh\tilde{h}}{[\sinh^2\tilde{h} + e^{-4K}]^{1/2}} = 0, \tag{2.16}$$

where

$$\tilde{h} = \bar{h} + 2\bar{I}\sigma, \tag{2.17}$$

and the free energy can be written in the form

$$f = -\beta^{-1}\ell n\lambda_+(\sigma) + \beta^{-1}\bar{I}\sigma^2. \tag{2.18}$$

The spontaneous magnetization, σ_0, is determined by considering in eq. 2.16 the limit $\bar{h} \to 0$ which gives the result:

$$\sigma_0 = \frac{\sinh(2\bar{I}\sigma_0)}{[\sinh^2(2\bar{I}\sigma_0) + e^{-4K}]^{1/2}}. \tag{2.19}$$

The critical behaviour of the system can be found by analysing (2.18) and (2.19), and it will depend on the strength of the short- and long-range interactions. Therefore the phase diagram can be constructed as a function of α ($\alpha \equiv I/J$).

The critical temperature for the second order transitions as a function of α is obtained from (2.19) by considering the limit $\sigma_0 \to 0$ as $T \to T_c$, and we get:

$$e^{-2K_c} = 2\alpha K_c, \tag{2.20}$$

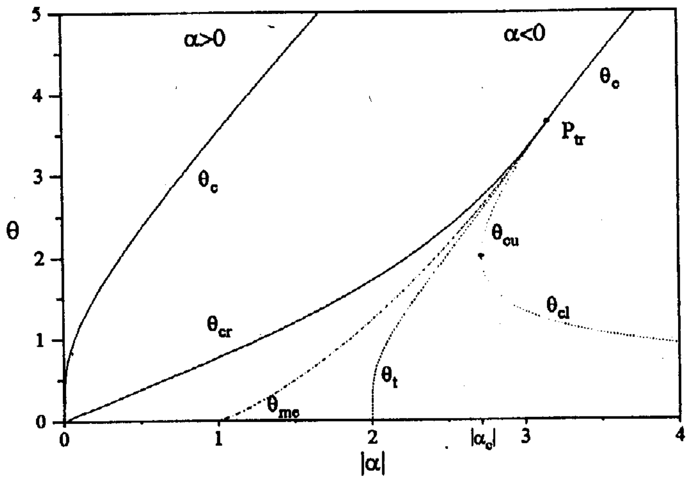

Figure 1. The continuous line, for $\alpha > 0$ or $\alpha < \alpha_{tr}$, represents the second order renormalized critical temperature θ_c ($\theta \equiv k_B T/|J|$) and, for $\alpha_{tr} <$ $< \alpha < 0$, the nonzero field critical temperature θ_{cr} as a function of α ($\alpha \equiv$ $\equiv I/J, I > 0$). The dashed line shows the zero field first order renormalized transition temperature θ_t and the dotted line represents the pseudo (θ_{cu} and θ_{cl}) second order transition line. P_{tr} identifies the tricritical point and the dashed-dotted line the temperature θ_{me} where a metastable nonzero magnetization state disappears.

which is a known result [11]. For $I < 0$, irrespective of the sign of J, as expected the system does not order since it is fully frustrated. Consequently we will restrict initially our analysis to $I > 0$. In this case the solution of the previous equation is shown in fig. 1, where $\alpha_c = -e$. For $\alpha > 0$ we have second order transitions only, however for $\alpha < 0$ first order transitions are allowed.

The first order transition temperature is determined by imposing the well known conditions on the free energy [5], namely,

$$\left.\frac{\partial f}{\partial \sigma}\right|_{\sigma=\sigma_t} = 0, \tag{2.21}$$

$$f(\sigma_t) = f(0), \tag{2.22}$$

where σ_t is the value of the magnetization at the transition. The first order transition line is also shown in fig. 1 and as can be seen it meets the second order transition line at the tricritical point [12], and it ends at $\alpha = -2$.

The tricritical point can be determined by imposing the condition that the $\sigma_t = 0$ root of the free energy is four-fold degenerate. This is achieved by considering the expansion of eq. 2.21 for $\sigma_t \to 0$ and making the coefficient of σ_t^3 equal to zero. We have to impose this condition because the coefficient of σ_t is zero, since (2.20) is also satisfied at this point. Therefore we get the known result

$$K_{tr} = -\frac{1}{4}\ell n3 = -0.274653072167... \tag{2.23}$$

$$\alpha_{tr} = -\frac{2\sqrt{3}}{\ell n3} = -3.15316117512... \tag{2.24}$$

which was first determined by Nagle [7].

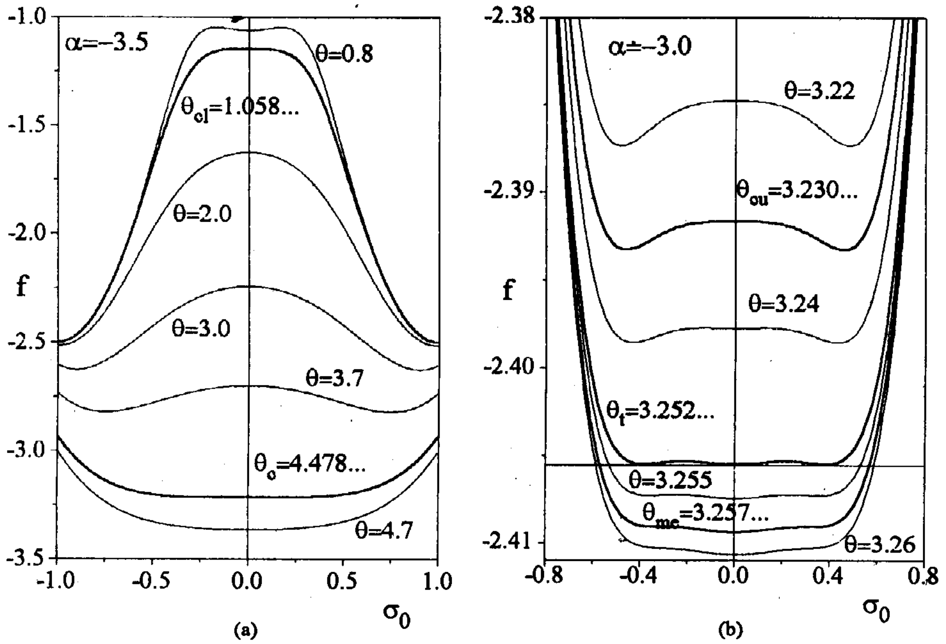

Figure 2. Renormalized free energy f (f $\equiv f/|J|$) as a function of σ for $\gamma = 0$ ($\gamma \equiv h/|J|$), $\alpha = -3.5$ (a) and $\alpha = -3.0$ (b) ($\alpha \equiv I/J$), and various renormalized temperatures θ ($\theta \equiv k_B T/|J|$)

By analysing the extremes of the free energy at a given temperature, (2.18), we conclude that reentrant behaviour shown in the solution of (2.20) is not physically acceptable, since for a given α the lower solution corresponds to a maximum of the free energy. This is shown in fig. 2(a) where the free energy for different temperatures is presented as a function of σ for $\alpha = -3.5$. It should be noted that for this value of α ($\alpha < \alpha_{tr}$), as also shown in this figure, the upper solution (T_c) corresponds to a second order transition temperature, since we have a minimum in the free energy at T_c for $\sigma = 0$.

For $\alpha_{tr} < \alpha < \alpha_c$, the pseudo second order transition line (T_{cu}) shown in fig. 1 (upper solution) identifies the appearance of a relative minimum in the free energy at $\sigma = 0$ since $\frac{\partial^2 f}{\partial \sigma^2}$ is zero at this temperature, and these minima correspond to metastable states when $T_{cu} < T < T_t$. This behaviour is shown in fig. 2(b) for $\alpha = -3.0$, where we also present the free energy as a function of σ for various temperatures.

We can also determine the temperature T_{me} in which the minimum of the free energy at non-zero magnetization σ_{me} disappears. This is obtained by solving the system:

$$\left.\frac{\partial f}{\partial \sigma}\right|_{\sigma=\sigma_{me}} = 0, \tag{2.25}$$

$$\left.\frac{\partial^2 f}{\partial \sigma^2}\right|_{\sigma=\sigma_{me}} = 0, \tag{2.26}$$

and these minima also correspond to new metastable states for $T_t < T < T_{me}$. This behaviour is also exemplified in fig. 2(b) for $\alpha = -3.0$. As can be seen the temperatures T_{cu} and T_{me} correspond to the limits of the hysteresis cycle which is present in first order transitions. This hysteresis cycle is shown in figs. 3(a) and 3(b) where we present the magnetization as a function of temperature for $\alpha = -3.0$. The dashed curve shown in fig. 3(a) does not represent physical states since they correspond to maxima in the free energy at nonzero magnetization as shown in fig. 2(a) for $\alpha = -3.5$. For $-2 > \alpha > -e$ T_c is no more defined, and the lower limit of the hysteresis cycle jumps to zero.

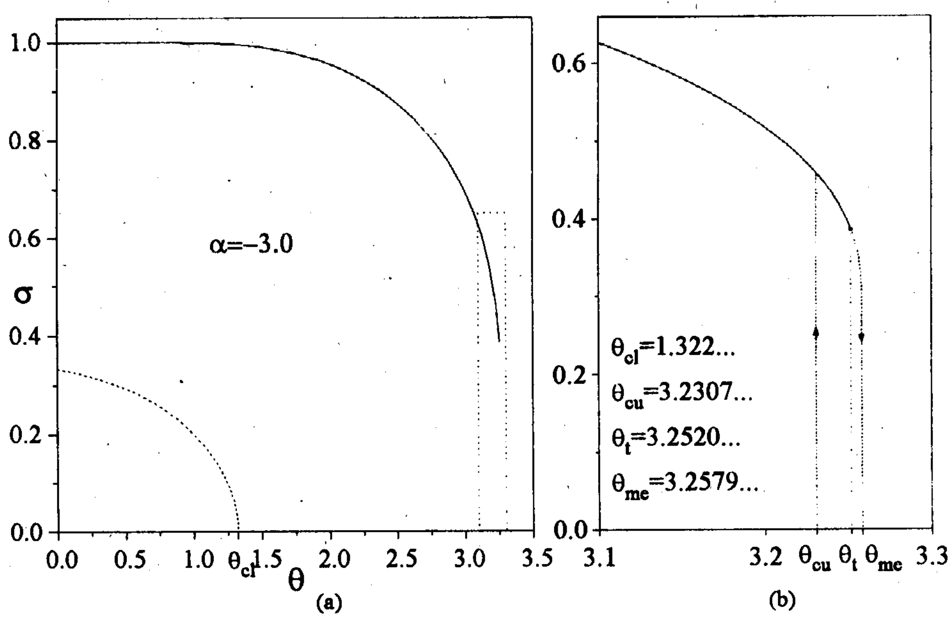

Figure 3. Magnetization as a function of temperature for $\alpha = -3.0$. In (a) the dashed line represents unstable magnetization states and in (b) the hysteresis cycle present in this first order transition, which corresponds to a magnification of the dotted rectangle in (a).

For nonzero field and $\alpha_{tr} < \alpha < 0$ the magnetization is a discontinuous function of T, for $h < h_{cr}$ (critical field). For $h \geq h_{cr}$ the magnetization is a continuous function and, in particular, T_{cr} corresponds to the limiting temperature for $h = h_{cr}$. This behaviour can also be seen by looking at the evolution of the free energy, at a fixed field, as the temperature varies. For $\alpha = -2.5$ and $\gamma < \gamma_{cr}$, $\gamma = \gamma_{cr}$ and $\gamma > \gamma_{cr}$ ($\gamma \equiv h/|J|$) the results are shown in figs. 4(a), 4(b) and 4(c) respectively. As it is shown, for $\gamma < \gamma_{cr}$ there appear two degenerate minima at σ_1 and σ_2 ($\sigma_1 < \sigma_2$), whereas for $\gamma \geq \gamma_{cr}$ we have a single minimum as the temperature varies. For $\alpha = -1.5$, which corresponds to a region where the system orders antiferromagnetically at $T = 0$ only, the discontinuity in the magnetization for $h < h_{cr}$ is also present and the behaviour is similar to the one shown in figs. 4(a)-4(c). The main consequence of these results is that the isotherms in the $h \times \sigma$ plane are discontinuous functions for $\gamma < \gamma_{cr}$, and the equation of state, obtained from (2.16), is given by:

$$\gamma|K| = \ell n \left[\frac{e^{-2K}\sigma + \sqrt{1+(e^{-4K}-1)\sigma^2}}{\sqrt{1-\sigma^2}} \right] - 2\alpha K\sigma, \quad \text{for} \begin{cases} |\sigma| < \sigma_1; \\ \text{or} \\ |\sigma| > \sigma_2; \end{cases} \tag{2.27}$$

and

$$\gamma(\sigma) = \gamma(\sigma_1) = \gamma(\sigma_2), \quad \text{for} \begin{cases} \sigma_1 < \sigma < \sigma_2; \\ \text{or} \\ -\sigma_2 < \sigma < -\sigma_1. \end{cases} \tag{2.28}$$

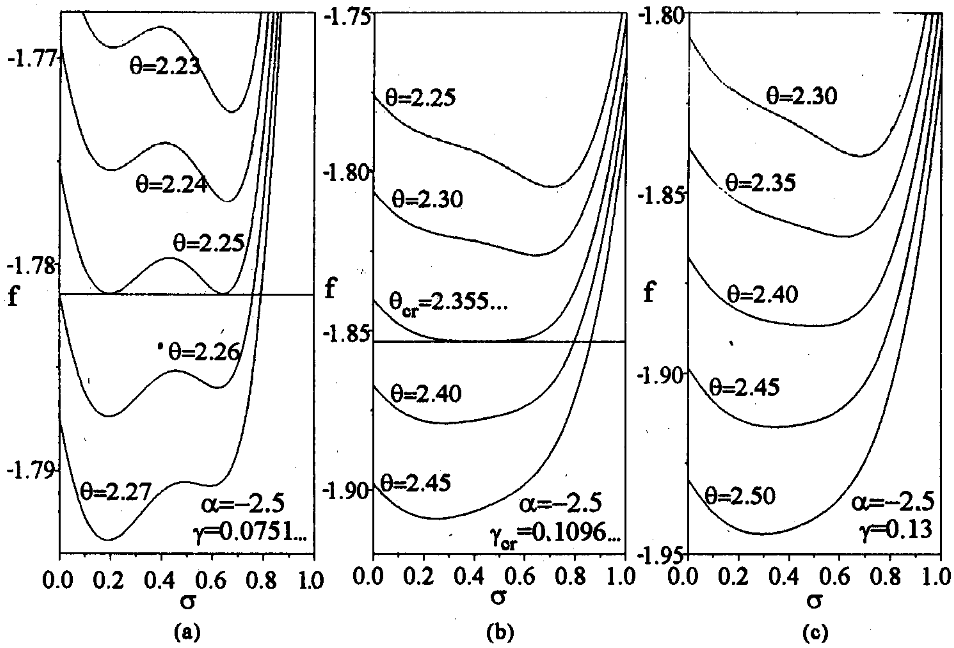

Figure 4. The same as in fig. 2 for $\alpha = -2.5$ and (a) $\gamma = 0.0751...$ ($\gamma < \gamma_{cr} = 0.1096...$), (b) $\gamma = \gamma_{cr}$ and (c) $\gamma = 0.13$ ($\gamma > \gamma_{cr}$).

This important result, as we will show later, will imply, as a main consequence, in a non-van der Waals behaviour for the lattice gas. In passing it should be noted that σ_1 and σ_2 are determined from (2.27) by using the condition shown in (2.28). At T_{cr} $\sigma_1 = \sigma_2 = \sigma_{cr}$ and γ_{cr} and K_{cr} can be immediately determined by imposing the conditions

$$\frac{\partial\gamma}{\partial\sigma} = 0, \tag{2.29}$$

$$\frac{\partial^2\gamma}{\partial\sigma^2} = 0, \tag{2.30}$$

which lead to the equation:

$$\alpha K_{cr} = \sqrt{\frac{27}{16}}(1 - e^{4K_{cr}}). \tag{2.31}$$

This critical line is also shown in fig. 1 and, as expected, it meets the first and second order critical lines at the tricritical point [12].

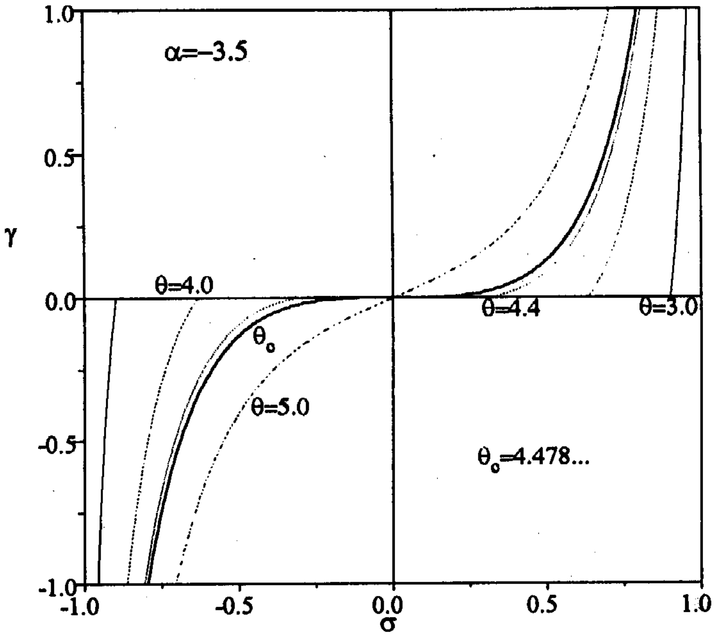

Figure 5. Isotherms for the magnetic system for $\alpha = -3.5$, $I > 0$ and $J < 0$.

From the results shown we conclude that for $I > 0$ and $\alpha < 0$ the system presents three different critical regimes which correspond to the regions $-2 < \alpha < 0$, $\alpha_{tr} < \alpha < -2$ and $\alpha < \alpha_{tr}$ respectively. The isotherms in the plane $\gamma \times \sigma$ for the three cases are shown in figs. 5-7 for α equal to -3.5, -2.5 and -1.5 respectively. For $\alpha = -3.5$, since we have second order transition the non-analytic behaviour of the free energy occurs along the σ axis. For $\alpha = -2.5$ this behaviour occurs for $T < T_t$ and for $\alpha = -1.5$, where the system orders antiferromagnetically and at $T = 0$ only, it always occurs for $\sigma \neq 0$. In this case $(-2 < \alpha < 0)$, by looking at the ground-state energy, we can show that the $T = 0$ isotherm is given by

$$\sigma = \begin{cases} 0, & \text{for } 0 \leq h < 2J + I; \\ 1, & \text{for } h \geq 2J + I. \end{cases} \tag{2.32}$$

The magnetization is discontinuous at the field $h = 2J + I$ which corresponds to $\gamma = 0.5$ for the case shown in fig. 7. For $\alpha > 0$, since there is no competition between the long- and short-range interactions, as mentioned, we just have second order transitions. In this case for any $\alpha > 0$ the isotherms present the same behaviour as the one shown in fig. 5 $(\alpha < \alpha_{tr})$, namely, that the non-analytic behaviour occurs along the σ axis.

For $I < 0$ where, as pointed out, the system does not present critical behaviour at finite temperature, the isotherms are shown in figs. 8 and 9 for $J > 0$ and $J < 0$ respectively. For $J > 0$ and $I = 0$ the ground state is obviously ferromagnetic. However for $I \neq 0$ it can be shown easily, by looking at the ground state energy, that the ground state is a two-domain

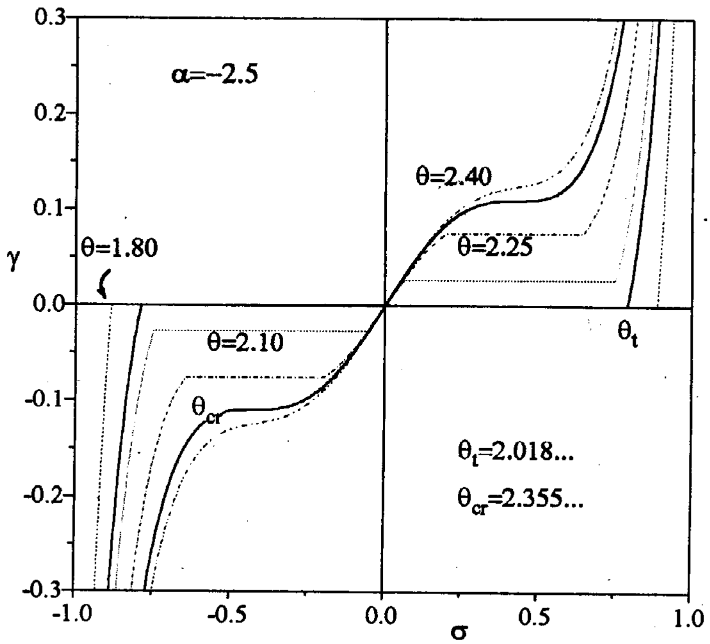

Figure 6. The same as in fig. 5 for $\alpha = -2.5$.

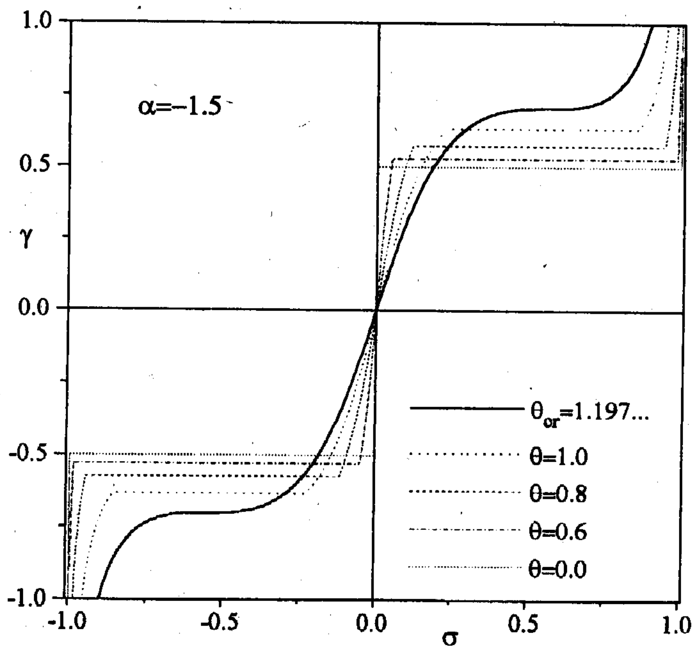

Figure 7. The same as in fig. 5 for $\alpha = -1.5$.

structure with zero magnetization. It also can be shown by analysing the ground-state energy that the $T = 0$ isotherm is given by

$$\sigma = \begin{cases} \frac{h}{2I}, & \text{for } 0 \le h \le 2I; \\ 1, & \text{for } h > 2I. \end{cases} \tag{2.33}$$

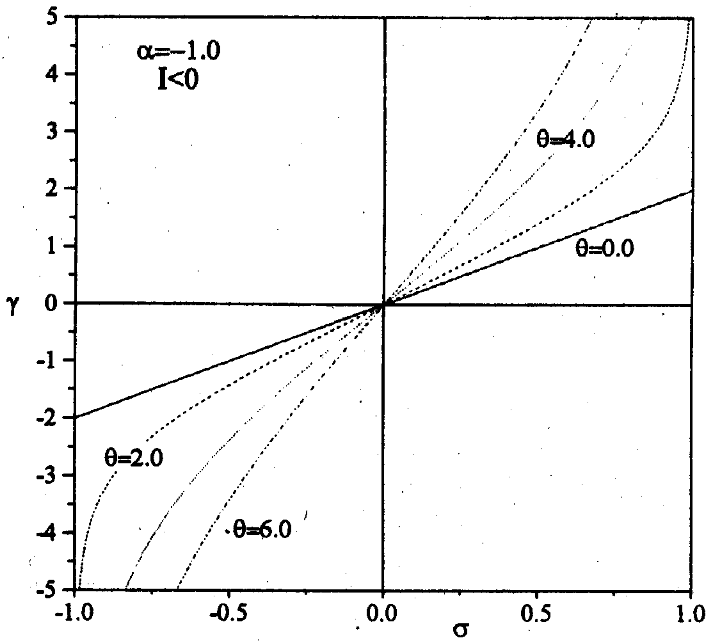

Figure 8. Isotherms for the magnetic system for $\alpha = -1.0$, $I < 0$ and $J > 0$.

A similar analysis for $J < 0$, whose ground state is antiferromagnetic for $I = 0$, leads to the conclusion that the ground state is also antiferromagnetic for $I \neq 0$. Moreover, we can show that the $T = 0$ isotherm is given by

$$\sigma = \begin{cases} 0, & \text{for } h \le 2J; \\ \frac{h-2J}{2I}, & \text{for } 2J \le h \le 2J + 2I; \\ 1, & \text{for } h > 2I \end{cases} \tag{2.34}$$

as shown in fig. 9.

3. The Lattice Gas

The Hamiltonian of the one-dimensional lattice gas with short- and long-range interactions which we will consider is given by:

$$\mathcal{H} = -\epsilon_1 \sum_{j=1}^{N} n_j n_{j+1} - \frac{\epsilon_2}{2N} \sum_{j,k=1}^{N} n_j n_k, \tag{3.1}$$

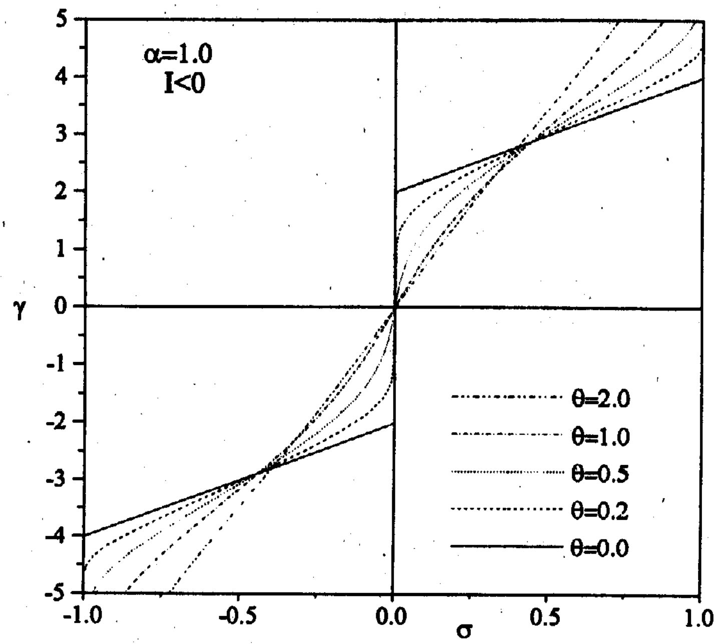

Figure 9. Isotherms for the magnetic system for $\alpha = 1.0$, $I < 0$ and $J < 0$.

where $n = 0, 1$, N is the total number of sites of the lattice and where we have assumed periodic boundary conditions. The thermodynamics of the system is obtained from the grand-canonical partition function [9],

$$\mathcal{Z} = \sum_{\{n\}} e^{-\beta(\mathcal{H} - \mu\bar{n})}, \tag{3.2}$$

where μ is the chemical potential and $\bar{n}$ is given by

$$\bar{n} = \sum_{j=1}^{N} n_j. \tag{3.3}$$

The equation of state is obtained from the equations

$$P = \frac{k_B T}{N} \ell n \mathcal{Z}, \tag{3.4}$$

$$\rho \equiv \frac{<\bar{n}>}{N} = \frac{\partial P}{\partial \mu}, \tag{3.5}$$

by eliminating the chemical potential from the above equations. The interaction parameters ϵ_1 and ϵ_2 can be written in terms of the Ising parameters by introducing the transformation

$$n_j = \frac{1 + \sigma_j}{2} \tag{3.6}$$

in (3.1), and by comparing it with (2.1) we obtain:

$$\epsilon_1 = 4J,\ \epsilon_2 = 8I,\ \mu = 2h - 4J - 4I. \tag{3.7}$$

In terms of the new variables the grand-partition function is then written in the form:

$$\mathcal{Z} = \sum_{\{\sigma\}} \exp\left\{\beta N(h - J - I) + \beta J \sum_{j=1}^{N} \sigma_j\sigma_{j+1} + \frac{\beta I}{N}\sum_{j,l=1}^{N}\sigma_j\sigma_l + \beta h \sum_{j=1}^{N}\sigma_j\right\} \tag{3.8}$$

and from (3.4) and (3.5) we can show immediately that

$$\rho \equiv \frac{1}{v} = \frac{1+\sigma}{2}, \tag{3.9}$$

$$P = h - J - I - f, \tag{3.10}$$

where f is the Helmholtz free energy of the Ising model given by (2.18). Then the phase diagram can be obtained from previous equations and the results are shown in figs. 10-12 for $I > 0$ and in fig. 13 for $I < 0$.

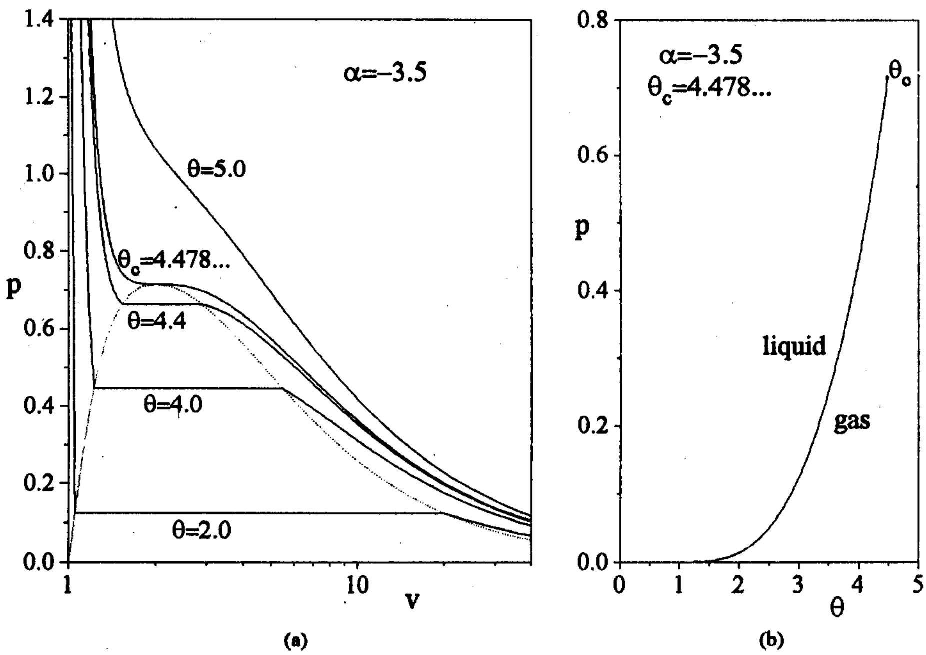

Figure 10. (a) Isotherms for the lattice gas system for $\alpha = -3.5$ ($\alpha \equiv I/J$), $I > 0$, and where p is the renormalized pressure ($p \equiv P/|J|$); (b) Phase diagram showing the gas-liquid transition and the critical temperature.

For $I > 0$, as in the magnetic system, we will have three different critical regimes. For $\alpha < \alpha_{tr}$ or $\alpha > 0$ we just have two phases, namely, gas and liquid phases and the general behaviour of isotherms is shown in fig. 10(a) for $\alpha = -3.5$. In this case the system behaves as a normal liquid-gas system and $P \to 0$ as $T \to 0$. The transition line is shown in fig. 10(b) and it starts at the origin of the coordinate axes. For $\alpha_{tr} < \alpha < -2$ the system presents three phases which we identify with gas, liquid and solid phases. An important point concerning these phases is that the critical isotherm is the same for both transitions. This can be seen in fig. 11(a) where we present the isotherms for $\alpha = -2.5$. Another important aspect concerning the critical behaviour is the existence of a triple point which means that the system can undergo the sublimation process. In this process $P \to 0$ as $T \to 0$ and the phase diagram is shown in fig. 11(b).

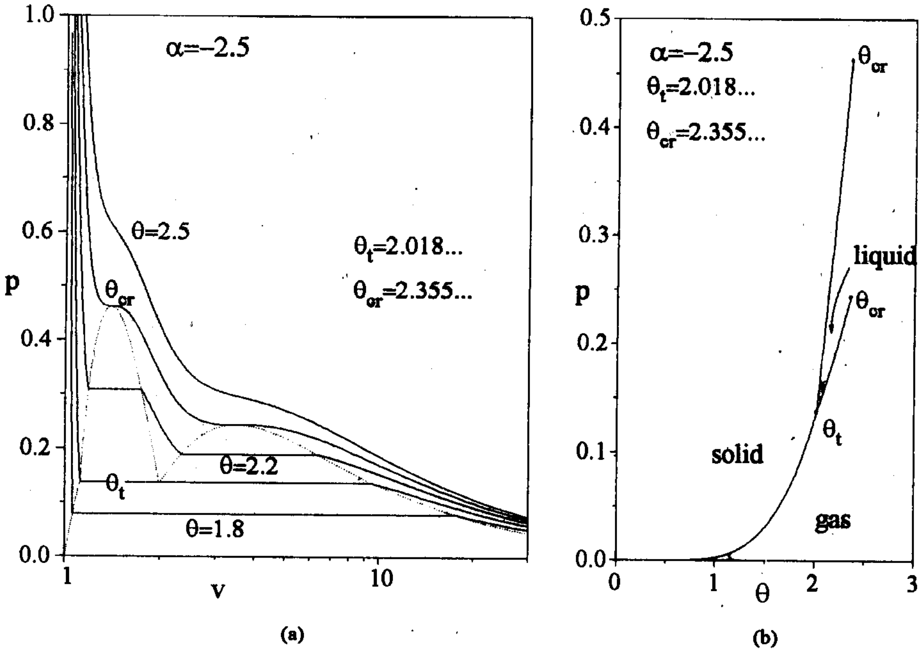

Figure 11. (a) The same as in fig. 10(a) for $\alpha = -2.5$ ($\alpha \equiv I/J$), $I > 0$; (b) Phase diagram showing the transitions gas-liquid, liquid-solid, gas-solid, the critical temperatures and the triple point.

For $-2 < \alpha < 0$ we also have three phases and the same critical temperature for both transitions. The isotherms for $\alpha = -1.5$ are shown in fig. 12(a). In this case there is no triple point and another important feature is that the isotherms can cross. In the gas-liquid transition $P \to 0$ as $T \to 0$ as the phase diagram in fig. 12(b) shows and this imply that no sublimation process is allowed.

All results for $I > 0$ agree qualitatively with the ones obtained for the classical gas [1-4], apart from the van der Waals behaviour presented by the isotherms in the coexistence region which is not present in the lattice gas. This is a direct consequence of the discontinuous behaviour of the isotherms in the Ising model shown in (2.27) and (2.28). Another important difference from the classical gas is the fact that the critical isotherm is the same for the different transitions in the three phase case.

Finally in figs.13(a) and 13(b) we present the results for $I < 0$ where we have a single phase. The results for $\alpha = -1.0$ ($I < 0$) are presented in fig. 13(a) and they show that the system behaves qualitatively as a pseudo van der Waals gas where we have a repulsive interaction between the particles. The isotherms are monotonic concave functions and in particular

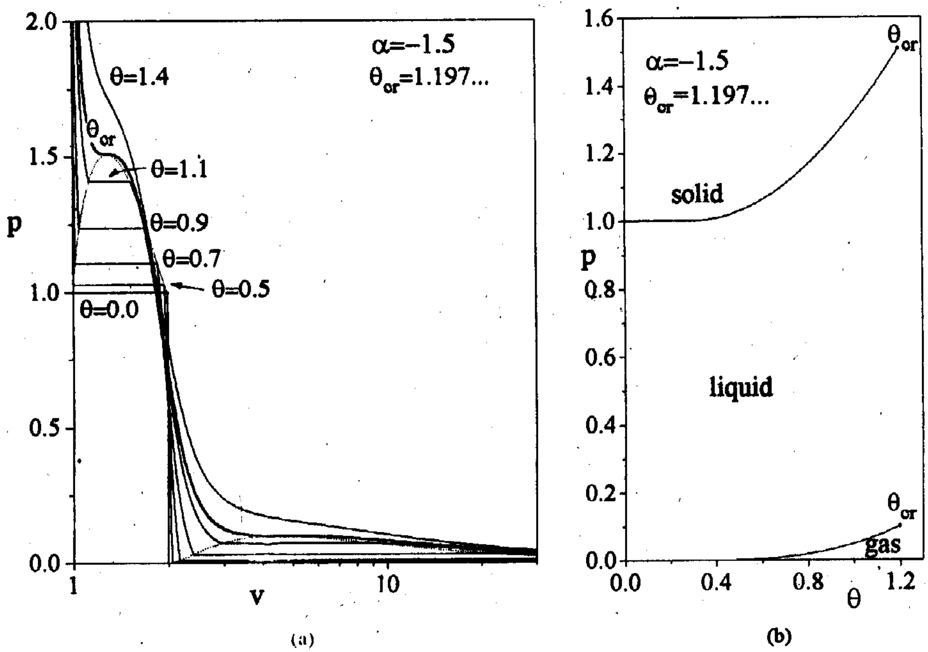

Figure 12. (a) The same as in fig. 10(a) for $\alpha = -1.5$ ($\alpha \equiv I/J$), $I > 0$; (b) The same as in fig. 11(b) without triple point.

the limiting isotherm, namely, at $T = 0$ is given by

$$P = \begin{cases} \infty, & \text{for } v \le 1; \\ \frac{4|\alpha|}{v^2}, & \text{for } v > 1; \end{cases} \tag{3.11}$$

which is identical to the one in the above mentioned pseudo van der Waals gas with repulsive interaction [5].

For $\alpha = 1.0$ the results are shown in fig. 13(b) and in this case the isotherms can cross for $v < 2$. This leads to an anomalous behaviour for this single phase and the isotherms in this region present two inflexion points. In particular the limiting isotherm, at $T = 0$, is given by:

$$P = \begin{cases} \infty, & \text{for } v \le 1; \\ 4\left(1 + \frac{\alpha}{v^2}\right), & \text{for } 1 < v < 2; \\ \frac{4\alpha}{v^2}, & \text{for } v > 2; \end{cases} \tag{3.12}$$

and the results show that at least qualitatively for $v > 2$ the gas has the same behaviour as the one presented for $\alpha < 0$, namely, a pseudo van der Waals gas. It also should be noted that there is a limiting temperature T^* above which the isotherms do not cross for any value of v. This temperature is a function of α, and can be determined by imposing that the limiting isotherm contains the point $(4 + \alpha, 2)$ of the $p \times v$ plane.

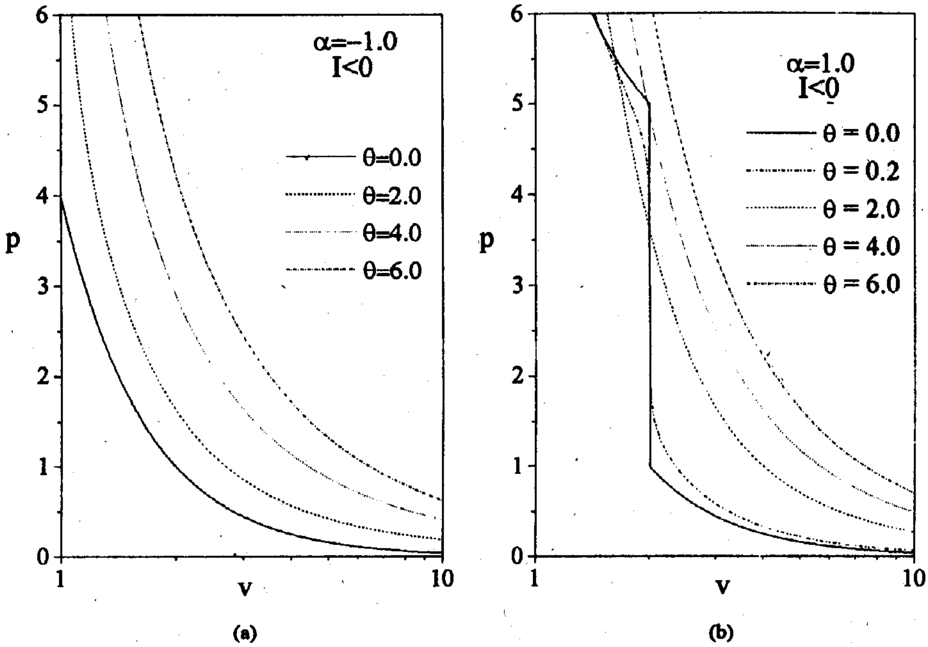

Figure 13. The same as in fig. 10(a) for $I < 0$, (a) for $\alpha = -1.0$ ($\alpha \equiv I/J$) and (b) for $\alpha = 1.0$

4. Conclusions

In this paper we have studied the lattice gas with short- and long-range interactions. For attractive long-range interaction the system presents two different critical behaviours depending on the strenght of the short-range interaction, namely, two or three phase regimes. These regimes are qualitatively identical to the ones presented by the classical gas [1-4] although no van der Waals behaviour is presented by the isotherms in the coexistence regions. Moreover we always have a unique critical isotherm in the three phase regime, which exists for repulsive short-range interaction only, whereas in the classical gas we can have two critical isotherms. This has as an immediate consequence that when there is a triple point and the two-phase regime is attained from the three-phase regime by varying continuously the short-range interaction, the coexistence regions above the triple point isotherm are simutaneously suppressed. In the three phase regime the isotherms can cross for some values of the parameters, and this anomalous behaviour resembles the one presented by water.

For repulsive long-range interaction we have just a single phase for $J < 0$ or $J > 0$. For $J < 0$ the isotherms can cross for $T < T^*$ (limiting temperature) and $v < 2$, leading to an anomalous behaviour. For $T > T^*$ and arbitrary v or $T < T^*$ and $v > 2$ the isotherms are monotonic concave functions and present the behaviour of a pseudo van der Waals gas with repulsive interaction. This is also presented by the system when $J > 0$ for any temperature, since the isotherms are monotonic concave functions.

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ОДНОВИМІРНИЙ ГРАТКОВИЙ ГАЗ

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Розглянуто одновимірний гратковий газ з близько- й далекосяжною взаємодією. Термодинамічні властивості системи отримано звичайним способом як і для моделі Ізінга, що може бути розв'язана точно. Розв'язок магнітної моделі знайдено методом функціонального інтегрування, представлено огляд критичних показників. Отримано фазові діаграми граткового газу для різної величини параметрів взаємодії. Критичні показники порівнюються з отриманими раніше для класичного газу.