

CALCULATION METHOD FOR THE THREE-DIMENSIONAL ISING FERROMAGNET THERMODYNAMICS WITHIN THE FRAMES OF ρ^6 MODEL

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Calculation of thermodynamic functions of the three-dimensional Ising ferromagnet above and below critical temperature is performed in the approximation of sixfold basis distribution (ρ^6 model). Comparison with the results for the ρ^4 model indicates that dependence of the thermodynamic functions on the renormalization group parameter s becomes weaker. The optimal interval of the renormalization group parameter values is determined.

Introduction

Significant results in the description of the system thermodynamic properties in the vicinity of the transition point have been obtained by means of the collective variables (CV) approach. The method of deriving explicit expressions for the thermodynamic and correlation functions of the three-dimensional Ising model at temperatures both above and below critical temperature T_c has been suggested within this approach. The calculations are performed with a non-Gaussian measure density. The measure is represented as an exponential function of CV, the argument of which contains, along with the quadratic term, higher powers of the variable with the corresponding interaction constants. The simplest non-Gaussian measure density is the quartic one (ρ^4 model) with the second and the fourth powers of the variable in the exponent. Then the sixfold measure goes containing the sixth power of the variable (ρ^6 model), and so forth.

The results of the theory depend on the renormalization group (RG) parameter s due to an approximation of the Ising model partition function calculation using the non-Gaussian measure densities. This dependence decreases essentially if the non-Gaussian measure density becomes more complicated. Calculations of the correlation length critical exponent ν within the ρ^{2m} models with $m = 2, 3, 4, 5$ confirm this statement [1-3]. It has been established that the ρ^6 model provides an adequate description of the Ising model critical behaviour, in particular, the critical exponents, at the RG parameter values in the interval $2 \leq s \leq 4$.

Investigation of the ρ^6 model within the numerical realization of the CV method has been performed in [4]. Analytical derivation of the explicit expressions for the ρ^6 model thermodynamic functions is the subject of the present paper. The foundations for such kind of investigations have

been developed in [5-9], where the quartic distribution was used as a basis measure.

1. General relations

The partition function of the three-dimensional Ising model within the six-fold measure density is given by

$$Z = 2^N \int \exp\left[\frac{1}{2} \sum_{\mathbf{k} \leq B} \beta \tilde{\Phi}(\mathbf{k}) \rho_{\mathbf{k}} \rho_{-\mathbf{k}} + 2\pi i \sum_{\mathbf{k} \leq B} \omega_{\mathbf{k}} \rho_{\mathbf{k}} + \sum_{n=1}^3 (2\pi i)^{2n} \times \right. \\ \left. \times N^{1-n} \frac{1}{(2n)!} \sum_{\mathbf{k}_1, \dots, \mathbf{k}_{2n} \leq B} \mathcal{M}_{2n} \omega_{\mathbf{k}_1} \cdots \omega_{\mathbf{k}_{2n}} \delta_{\mathbf{k}_1 + \dots + \mathbf{k}_{2n}}\right] (d\omega)^N (d\rho)^N, \quad (1.1)$$

where $\mathcal{M}_2=1$, $\mathcal{M}_4=-2$, $\mathcal{M}_6=16$, $\tilde{\Phi}(\mathbf{k}) = \tilde{\Phi}(0)(1 - 2b^2 k^2)$, $\beta = (kT)^{-1}$ is the inverse temperature, b is the effective interaction radius of the potential $\Phi(r) = A \exp(-r/b)$, $\tilde{\Phi}(0) = 8\pi A(b/c)^3$. Integrating (1.1) over $\rho_{\mathbf{k}}$ and $\omega_{\mathbf{k}}$ with the indices $B' < |\mathbf{k}| \leq B$ ($B = \pi/c$, c is the simple cubic lattice constant), we get an expression for the partition function of the ρ^6 model:

$$Z = 2^N 2^{\frac{N'-1}{2}} e^{a' s_0 N'} \int \exp\left[-\frac{1}{2} \sum_{\mathbf{k} \leq B'} d'(\mathbf{k}) \rho_{\mathbf{k}} \rho_{-\mathbf{k}} - \sum_{l=2}^3 \frac{1}{(2l)!} (N')^{1-l} \sum_{\mathbf{k}_1, \dots, \mathbf{k}_{2l} \leq B'} a'_{2l} \rho_{\mathbf{k}_1} \cdots \rho_{\mathbf{k}_{2l}} \delta_{\mathbf{k}_1 + \dots + \mathbf{k}_{2l}}\right] (d\rho)^{N'}. \quad (1.2)$$

Here $N' = N s_0^{-3}$, $s_0 = B/B' = \pi\sqrt{2}b/c$,

$$d'(\mathbf{k}) = a'_2 - \beta \tilde{\Phi}(\mathbf{k}). \quad (1.3)$$

Coefficients a'_{2l} depend on the ratio b/c and are given by the relations

$$\begin{aligned} a'_0 &= \ln Q(\mathcal{M}), \quad Q(\mathcal{M}) = (12s_0^3)^{1/4} \pi^{-1} I_0(\eta', \xi'), \\ a'_2 &= (12s_0^3)^{1/2} \mathcal{F}_2(\eta', \xi'), \\ a'_4 &= 12s_0^3 C(\eta', \xi'), \\ a'_6 &= (12s_0^3)^{3/2} N(\eta', \xi'), \end{aligned} \quad (1.4)$$

where the quantities $\eta' = \sqrt{3}s_0^{3/2}$, $\xi' = \frac{8\sqrt{3}}{15s_0^{3/2}}$ are the arguments, and special functions $C(\eta', \xi')$ and $N(\eta', \xi')$ read

$$\begin{aligned} C(\eta', \xi') &= -\mathcal{F}_4(\eta', \xi') + 3\mathcal{F}_2^2(\eta', \xi'), \\ N(\eta', \xi') &= \mathcal{F}_6(\eta', \xi') - 15\mathcal{F}_4(\eta', \xi')\mathcal{F}_2(\eta', \xi') + 30\mathcal{F}_2^3(\eta', \xi'). \end{aligned} \quad (1.5)$$

Here $\mathcal{F}_{2l}(\eta', \xi') = \frac{I_{2l}(\eta', \xi')}{I_0(\eta', \xi')}$, $I_{2l}(\eta', \xi') = \int_0^\infty t^{2l} e^{-\eta' t^2 - t^4 - \xi' t^6} dt$.

Using the method of layer-by-layer integration of the partition function in the phase space of CV, developed in [10], one can reduce (1.2) to the form:

$$Z = 2^N 2^{\frac{N_{n+1}-1}{2}} Z_0 Z_1 \dots Z_n (Q(P_n))^{N_{n+1}} \int w_6^{(n+1)}(\rho) (d\rho)^{N_{n+1}}, \quad (1.6)$$

where $N_n = N's^{-3n}$,

$$\begin{aligned} Z_0 &= [Q(\mathcal{M})Q(d)]^{N'}, \quad Z_1 = [Q(P)Q(d_1)]^{N_1}, \quad \dots, \\ Z_n &= [Q(P_{n-1})Q(d_n)]^{N_n}, \\ Q(P_n) &= \frac{1}{\pi} (s^3 \frac{a_4^{(n)}}{C(h_n, \alpha_n)})^{1/4} I_0(\eta_n, \xi_n), \\ Q(d_n) &= 2(24/a_4^{(n)})^{1/4} I_0(h_n, \alpha_n). \end{aligned} \quad (1.7)$$

Hereafter the arguments h_n, α_n are called basic:

$$h_n = d_n(B_{n+1}, B_n)(6/a_4^n)^{1/2}, \quad \alpha_n = \frac{\sqrt{6}}{15} a_6^{(n)} / (a_4^{(n)})^{3/2}. \quad (1.8)$$

The effective measure density of the n -th phase layer $w_6^{(n)}(\rho)$ has the form:

$$\begin{aligned} w_6^{(n)}(\rho) &= \exp[-\frac{1}{2} \sum_{k \leq B_n} d_n(k) \rho_k \rho_{-k} - \\ &\quad - \sum_{l=2}^3 \frac{1}{(2l)!} N_n^{1-l} \sum_{k_1, \dots, k_{2l} \leq B_n} a_{2l}^{(n)} \rho_{k_1} \dots \rho_{k_{2l}} \delta_{k_1 + \dots + k_{2l}}]. \end{aligned} \quad (1.9)$$

Here $B_n = B's^{-n}$. The intermediate variables η_n, ξ_n are functions of h_n and α_n :

$$\begin{aligned} \eta_n &= \sqrt{6} s^{3/2} \mathcal{F}_2(h_n, \alpha_n) [C(h_n, \alpha_n)]^{-1/2}, \\ \xi_n &= \frac{\sqrt{6}}{15} s^{-3/2} N(h_n, \alpha_n) [C(h_n, \alpha_n)]^{-3/2}. \end{aligned} \quad (1.10)$$

The form of the special functions $C(h_n, \alpha_n), N(h_n, \alpha_n)$ is given by (1.5).

Coefficients $d_n(B_{n+1}, B_n), a_4^{(n)}, a_6^{(n)}$ are related to the coefficients of the $(n+1)$ th layer by recurrent relations (RR) [11-13]. The solutions of these relations [13] are used in the calculation of the system thermodynamic characteristics.

2. Thermodynamic functions of the ρ^6 model in the regions of critical and limit Gaussian regimes (CR and LGR) above T_c

It is convenient to rewrite the model partition function as [14]

$$Z = 2^N Z_{CR} Z_{LGR}. \quad (2.1)$$

Let us consider Z_{CR} given by

$$Z_{CR} = \prod_{n=0}^{m_\tau} \left[\frac{2}{\pi} \left(\frac{24}{C(\eta_{n-1}, \xi_{n-1})} \right)^{1/4} I_0(h_n, \alpha_n) I_0(\eta_{n-1}, \xi_{n-1}) \right]^{N_n}. \quad (2.2)$$

It should be mentioned that in (2.2) $\eta_{-1} \equiv \eta', \xi_{-1} \equiv \xi'$ at $n = 0$. We represent the right-hand side (RHS) of (2.2) in the form of an explicit dependence on the phase layer number n in order to calculate Z_{CR} .

In the CR region, the basic h_n, α_n and intermediate η_n, ξ_n arguments are close to their values at the fixed point. Therefore, functions of these arguments can be written as power series of deviations of basic arguments from their values at the fixed point (see [15,16]). Using the obtained representations for $I_0(h_n, \alpha_n)$, $I_0(\eta_{n-1}, \xi_{n-1})$, $C(\eta_{n-1}, \xi_{n-1})$, we determine from (2.2) the partial free energy corresponding to the n -th phase layer:

$$\begin{aligned}
F_n &= -kTN_n \{ f_{CR}^{(0)} + \varphi_1(h_{n-1} - h^{(0)}) + \varphi_2(\alpha_{n-1} - \alpha^{(0)}) + \\
&\quad + \varphi_3(h_n - h^{(0)}) + \varphi_4(\alpha_n - \alpha^{(0)}) + \varphi'_1(h_{n-1} - h^{(0)})^2 + \\
&\quad + \varphi'_2(\alpha_{n-1} - \alpha^{(0)})^2 + \varphi'_3(h_n - h^{(0)})^2 + \varphi'_4(\alpha_n - \alpha^{(0)})^2 + \\
&\quad + \varphi'_5(h_{n-1} - h^{(0)})(\alpha_{n-1} - \alpha^{(0)}) + \varphi'_6(h_n - h^{(0)})(\alpha_n - \alpha^{(0)}) \}, \\
f_{CR}^{(0)} &= \ln\left(\frac{2(24)^{1/4}}{\pi}\right) - \frac{1}{4} \ln \mathcal{P}_{40} + \ln I_0^* + \ln I_0^{**}, \\
\varphi_m &= b_m + \mathcal{P}_{4m}/4, \quad m = 1, 2, \\
\varphi_3 &= -\mathcal{F}_2^*, \quad \varphi_4 = -\mathcal{F}_6^*, \\
\varphi'_m &= b'_m - \frac{1}{2}b_m^2 - \mathcal{P}'_{4m}/4 + \mathcal{P}_{4m}^2/8, \quad \varphi'_3 = \mathcal{F}_4^*/2 - \mathcal{F}_2^{*2}/2, \\
\varphi'_4 &= \mathcal{F}_{12}^*/2 - \mathcal{F}_6^{*2}/2, \quad \varphi'_5 = b'_3 - b_1b_2 - \mathcal{P}'_{43}/4 + \mathcal{P}_{41}\mathcal{P}_{42}/4, \\
\varphi'_6 &= \mathcal{F}_8^* - \mathcal{F}_2^*\mathcal{F}_6^*.
\end{aligned} \tag{2.3}$$

Expressions for the quantities occurring in $f_{CR}^{(0)}$, φ_i , φ'_j are given in [15,16].

Hence, the partial free energy of the n -th phase layer F_n is written as a power series of deviations of basic arguments from their fixed point values. The linear approximation for F_n was used in [14]. In the present paper, as well as in the calculations within the ρ^4 model [5,6], the quadratics of the deviations are also taken into account. It allows one to compare the results of the calculations for ρ^4 and ρ^6 models. Let us note that quadratic terms of the RR do not contribute to the elements of the matrix of the RR linearization in the vicinity of the fixed point, and the eigenvalues E_l of this matrix and the critical exponent of the correlation length are the same as within the linear approximation for the RR.

Let us find an explicit dependence of F_n on the layer number n . Using the solutions of the RR, we get for h_n and α_n :

$$\begin{aligned}
h_n &= h^{(0)} + c_1 H_1(u^{(0)})^{-1/2} E_1^n + c_2 H_2(u^{(0)})^{-1} E_2^n + \\
&\quad + c_3 H_3(u^{(0)})^{-3/2} E_3^n + c_1 c_2 H_4(u^{(0)})^{-3/2} E_1^n E_2^n + \\
&\quad + c_1 c_2^2 H_5(u^{(0)})^{-5/2} E_1^n E_2^{2n} + c_2^2 H_6(u^{(0)})^{-2} E_2^{2n} + \\
&\quad + c_1^2 H_7(u^{(0)})^{-1} E_1^{2n} + c_1^2 c_2 H_8(u^{(0)})^{-2} E_1^{2n} E_2^n + \\
&\quad + c_1^2 c_2^2 H_9(u^{(0)})^{-3} E_1^{2n} E_2^{2n}, \\
\alpha_n &= \alpha^{(0)} + c_1 L_1(u^{(0)})^{-1/2} E_1^n + c_2 L_2(u^{(0)})^{-1} E_2^n + \\
&\quad + c_3 L_3(u^{(0)})^{-3/2} E_3^n + c_1 c_2 L_4(u^{(0)})^{-3/2} E_1^n E_2^n + \\
&\quad + c_1 c_2^2 L_5(u^{(0)})^{-5/2} E_1^n E_2^{2n} + c_2^2 L_6(u^{(0)})^{-2} E_2^{2n} + \\
&\quad + c_1^2 L_7(u^{(0)})^{-1} E_1^{2n} + c_1^2 c_2 L_8(u^{(0)})^{-2} E_1^{2n} E_2^n + \\
&\quad + c_1^2 c_2^2 L_9(u^{(0)})^{-3} E_1^{2n} E_2^{2n},
\end{aligned} \tag{2.4}$$

where

$$\begin{aligned}
H_1 &= \sqrt{6} - \frac{h^{(0)}w_{21}^{(0)}}{2}, \quad H_2 = \sqrt{6}w_{12}^{(0)} - \frac{h^{(0)}}{2}, \\
H_3 &= \sqrt{6}w_{13}^{(0)} - \frac{h^{(0)}w_{23}^{(0)}}{2}, \\
H_4 &= \frac{3}{4}h^{(0)}w_{21}^{(0)} - \frac{\sqrt{6}}{2}(1 + w_{12}^{(0)}w_{21}^{(0)}), \\
H_5 &= \frac{3\sqrt{6}}{4}\left(\frac{1}{2} + w_{12}^{(0)}w_{21}^{(0)} - \frac{5}{4\sqrt{6}}h^{(0)}w_{21}^{(0)}\right), \\
H_6 &= \frac{1}{2}\left(\frac{3}{4}h^{(0)} - \sqrt{6}w_{12}^{(0)}\right), \quad H_7 = \frac{w_{21}^{(0)}}{2}\left(\frac{3}{4}h^{(0)}w_{21}^{(0)} - \sqrt{6}\right), \\
H_8 &= \frac{3\sqrt{6}}{4}w_{21}^{(0)}\left(1 + \frac{1}{2}w_{12}^{(0)}w_{21}^{(0)} - \frac{5}{4\sqrt{6}}h^{(0)}w_{21}^{(0)}\right), \\
H_9 &= \frac{15\sqrt{6}}{16}w_{21}^{(0)}\left(\frac{7}{4\sqrt{6}}h^{(0)}w_{21}^{(0)} - 1 - w_{12}^{(0)}w_{21}^{(0)}\right); \\
L_1 &= \frac{\sqrt{6}}{15}w_{31}^{(0)} - \frac{3\alpha^{(0)}w_{21}^{(0)}}{2}, \quad L_2 = \frac{\sqrt{6}}{15}w_{32}^{(0)} - \frac{3\alpha^{(0)}}{2}, \\
L_3 &= \frac{\sqrt{6}}{15} - \frac{3\alpha^{(0)}w_{23}^{(0)}}{2}, \\
L_4 &= \frac{15}{4}\alpha^{(0)}w_{21}^{(0)} - \frac{\sqrt{6}}{10}(w_{31}^{(0)} + w_{21}^{(0)}w_{32}^{(0)}), \\
L_5 &= \frac{\sqrt{6}}{4}\left(\frac{1}{2}w_{31}^{(0)} + w_{21}^{(0)}w_{32}^{(0)} - \frac{105}{4\sqrt{6}}\alpha^{(0)}w_{21}^{(0)}\right), \\
L_6 &= \frac{1}{2}\left(\frac{15}{4}\alpha^{(0)} - \frac{\sqrt{6}}{5}w_{32}^{(0)}\right), \\
L_7 &= \frac{w_{21}^{(0)}}{2}\left(\frac{15}{4}\alpha^{(0)}w_{21}^{(0)} - \frac{\sqrt{6}}{5}w_{31}^{(0)}\right), \\
L_8 &= \frac{\sqrt{6}}{4}w_{21}^{(0)}\left(w_{31}^{(0)} + \frac{1}{2}w_{21}^{(0)}w_{32}^{(0)} - \frac{105}{4\sqrt{6}}\alpha^{(0)}w_{21}^{(0)}\right), \\
L_9 &= \frac{7\sqrt{6}}{16}w_{21}^{(0)}\left(\frac{45}{4\sqrt{6}}\alpha^{(0)}w_{21}^{(0)} - w_{31}^{(0)} - w_{21}^{(0)}w_{32}^{(0)}\right).
\end{aligned} \tag{2.5}$$

Considering (2.4), we rewrite the partial energy of the n -th phase layer as

$$\begin{aligned}
F_n &= -kTN's^{-3n}[f_{CR}^{(0)} + f_{CR}^{(1)}(u^{(0)})^{-1/2}c_1E_1^n + f_{CR}^{(2)}(u^{(0)})^{-1}c_2E_2^n + \\
&\quad + f_{CR}^{(3)}(u^{(0)})^{-3/2}c_3E_3^n + f_{CR}^{(4)}(u^{(0)})^{-3/2}c_1c_2E_1^nE_2^n + \\
&\quad + f_{CR}^{(5)}(u^{(0)})^{-5/2}c_1c_2^2E_1^nE_2^{2n} + f_{CR}^{(6)}(u^{(0)})^{-2}c_2^2E_2^{2n} + \\
&\quad + f_{CR}^{(7)}(u^{(0)})^{-1}c_1^2E_1^{2n} + f_{CR}^{(8)}(u^{(0)})^{-2}c_1^2c_2E_1^{2n}E_2^n + \\
&\quad + f_{CR}^{(9)}(u^{(0)})^{-3}c_1^2c_2^2E_1^{2n}E_2^{2n}].
\end{aligned} \tag{2.6}$$

Here,

$$\begin{aligned}
f_{CR}^{(m)} &= H_m(\varphi_3 + \varphi_1/E_m) + L_m(\varphi_4 + \varphi_2/E_m), m = 1, 2, 3, \\
f_{CR}^{(4)} &= H_4(\varphi_3 + \varphi_1/(E_1E_2)) + L_4(\varphi_4 + \varphi_2/(E_1E_2)) + \\
&\quad + 2H_1H_2(\varphi_3' + \varphi_1'/(E_1E_2)) + 2L_1L_2(\varphi_4' + \varphi_2'/(E_1E_2)) + \\
&\quad + (H_1L_2 + H_2L_1)(\varphi_6' + \varphi_5'/(E_1E_2)), \\
f_{CR}^{(5)} &= H_5(\varphi_3 + \varphi_1/(E_1E_2^2)) + L_5(\varphi_4 + \varphi_2/(E_1E_2^2)) + \\
&\quad + 2(H_1H_6 + H_2H_4)(\varphi_3' + \varphi_1'/(E_1E_2^2)) + 2(L_1L_6 + L_2L_4) \times \\
&\quad \times (\varphi_4' + \varphi_2'/(E_1E_2^2)) + (H_1L_6 + H_6L_1 + H_2L_4 + H_4L_2) \times \\
&\quad \times (\varphi_6' + \varphi_5'/(E_1E_2^2)), \\
f_{CR}^{(6)} &= H_6(\varphi_3 + \varphi_1/E_2^2) + L_6(\varphi_4 + \varphi_2/E_2^2) + H_2^2(\varphi_3' + \varphi_1'/E_2^2) + \\
&\quad + L_2^2(\varphi_4' + \varphi_2'/E_2^2) + H_2L_2(\varphi_6' + \varphi_5'/E_2^2), \tag{2.7} \\
f_{CR}^{(7)} &= H_7(\varphi_3 + \varphi_1/E_1^2) + L_7(\varphi_4 + \varphi_2/E_1^2) + H_1^2(\varphi_3' + \varphi_1'/E_1^2) + \\
&\quad + L_1^2(\varphi_4' + \varphi_2'/E_1^2) + H_1L_1(\varphi_6' + \varphi_5'/E_1^2), \\
f_{CR}^{(8)} &= H_8(\varphi_3 + \varphi_1/(E_1^2E_2)) + L_8(\varphi_4 + \varphi_2/(E_1^2E_2)) + 2(H_1H_4 + \\
&\quad + H_2H_7)(\varphi_3' + \varphi_1'/(E_1^2E_2)) + 2(L_1L_4 + L_2L_7)(\varphi_4' + \\
&\quad + \varphi_2'/(E_1^2E_2)) + (H_1L_4 + H_4L_1 + H_2L_7 + H_7L_2) \times \\
&\quad \times (\varphi_6' + \varphi_5'/(E_1^2E_2)), \\
f_{CR}^{(9)} &= H_9(\varphi_3 + \varphi_1/(E_1E_2)^2) + L_9(\varphi_4 + \varphi_2/(E_1E_2)^2) + (2H_1H_5 + \\
&\quad + 2H_2H_8 + H_4^2 + 2H_6H_7)(\varphi_3' + \varphi_1'/(E_1E_2)^2) + (2L_1L_5 + \\
&\quad + 2L_2L_8 + L_4^2 + 2L_6L_7)(\varphi_4' + \varphi_2'/(E_1E_2)^2) + (H_1L_5 + \\
&\quad + H_5L_1 + H_2L_8 + H_8L_2 + H_4L_4 + H_6L_7 + H_7L_6) \times \\
&\quad \times (\varphi_6' + \varphi_5'/(E_1E_2)^2).
\end{aligned}$$

The quantities $u^{(0)}, w_{il}^{(0)}$ were determined in [13]. Let us note that in the expressions for h_n and α_n we can neglect a qualitatively new term proportional to E_3^n which arises within the ρ^6 model considered (E_3 is not essential as compared to E_1 or E_2).

Summing up expressions for F_n (2.6) over layers of the phase space from $n = 0$ to $n = m_\tau$, we obtain

$$\begin{aligned}
F_{CR} &= F_0' + F_{CR}', \\
F_0' &= -kTN'[\ln Q(\mathcal{M}) + \ln Q(d)], \\
F_{CR}' &= F_{CR}^{(0)} + F_{CR}^{(1)}\tau^{3\nu} + F_{CR}^{(2)}\tau^{3\nu}, \tag{2.8}
\end{aligned}$$

where

$$\begin{aligned}
F_{CR}^{(0)} &= -kTN's^{-3} \left[\frac{f_{CR}^{(0)}}{1-s^{-3}} + \frac{f_{CR}^{(1)}\varphi_0^{-1/2}\tilde{c}_1\tau E_1}{1-E_1s^{-3}} + \right. \\
&\quad \left. + \frac{f_{CR}^{(2)}\varphi_0^{-1}c_{20}E_2}{1-E_2s^{-3}} + \frac{f_{CR}^{(3)}\varphi_0^{-3/2}c_{30}E_3}{1-E_3s^{-3}} + \frac{f_{CR}^{(4)}\varphi_0^{-3/2}\tilde{c}_1\tau c_{20}E_1E_2}{1-E_1E_2s^{-3}} + \right.
\end{aligned}$$

$$\begin{aligned}
& + \frac{f_{CR}^{(5)} \varphi_0^{-5/2} \tilde{c}_1 \tau c_{20}^2 E_1 E_2^2}{1 - E_1 E_2^2 s^{-3}} + \frac{f_{CR}^{(6)} \varphi_0^{-2} c_{20}^2 E_2^2}{1 - E_2^2 s^{-3}} + \\
& + \frac{f_{CR}^{(7)} \varphi_0^{-1} \tilde{c}_1^2 \tau^2 E_1^2}{1 - E_1^2 s^{-3}} + \frac{f_{CR}^{(8)} \varphi_0^{-2} \tilde{c}_1^2 \tau^2 c_{20} E_1^2 E_2}{1 - E_1^2 E_2 s^{-3}} + \\
& + \frac{f_{CR}^{(9)} \varphi_0^{-3} \tilde{c}_1^2 \tau^2 c_{20}^2 E_1^2 E_2^2}{1 - E_1^2 E_2^2 s^{-3}}], \tag{2.9} \\
F_{CR}^{(1)} &= kTN' s^{-3m_0} \left[\frac{f_{CR}^{(0)}}{1 - s^{-3}} + \frac{f_{CR}^{(1)} \varphi_0^{-1/2} f_0}{1 - E_1 s^{-3}} + \right. \\
& \left. + \frac{f_{CR}^{(7)} \varphi_0^{-1} f_0^2}{1 - E_1^2 s^{-3}} \right], \\
F_{CR}^{(2)} &= -kTN' s^{-3m_0} \left[\frac{f_{CR}^{(2)} \varphi_0^{-1} c_{20} E_2^{m_\tau+1}}{1 - E_2 s^{-3}} + \frac{f_{CR}^{(3)} \varphi_0^{-3/2} c_{30} E_3^{m_\tau+1}}{1 - E_3 s^{-3}} + \right. \\
& + \frac{f_{CR}^{(4)} \varphi_0^{-3/2} f_0 c_{20} E_2^{m_\tau+1}}{1 - E_1 E_2 s^{-3}} + \frac{f_{CR}^{(5)} \varphi_0^{-5/2} f_0 c_{20}^2 E_2^{2(m_\tau+1)}}{1 - E_1 E_2^2 s^{-3}} + \\
& + \frac{f_{CR}^{(6)} \varphi_0^{-2} c_{20}^2 E_2^{2(m_\tau+1)}}{1 - E_2^2 s^{-3}} + \frac{f_{CR}^{(8)} \varphi_0^{-2} f_0^2 c_{20} E_2^{m_\tau+1}}{1 - E_1^2 E_2 s^{-3}} + \\
& \left. + \frac{f_{CR}^{(9)} \varphi_0^{-3} f_0^2 c_{20}^2 E_2^{2(m_\tau+1)}}{1 - E_1^2 E_2^2 s^{-3}} \right].
\end{aligned}$$

To calculate these expressions, we have used the formulas:

$$\begin{aligned}
s^{-3(m_\tau+1)} &= \tau^{3\nu} s^{-3m_0}, \tag{2.10} \\
c_1 &= \tau \tilde{c}_1 \beta \tilde{\Phi}(0), \quad c_2 = c_{20} (\beta \tilde{\Phi}(0))^2, \quad c_3 = c_{30} (\beta \tilde{\Phi}(0))^3,
\end{aligned}$$

where $m_0, \tilde{c}_1, c_{20}, c_{30}$ and f_0, φ_0 were defined in [14]. The dependence of m_0 on s for the ρ^6 model under consideration is plotted in figure 1 (solid curve). Here the dashed curve corresponds to the ρ^4 model. It is easy to notice that, when $s = s^*$ (s^* is the value at which h_n turns to zero at the fixed point; $s^*=3.5862$ for the ρ^4 model and $s^*=2.7349$ for the ρ^6 model), the values of m_0 for ρ^4 and ρ^6 models coincide.

Further, we put $F_{CR}^{(2)} = 0$ at $\tau \ll 1$, since $E_2 < 1, E_3 < 1$, and m_τ is large. Taking E_2, E_3 into account gives rise to terms characterising corrections to scaling. As a result, the free energy of the CR region takes the form:

$$\begin{aligned}
F_{CR} &= -kTN' [\gamma_0 + \delta_0 - \gamma_3^{(CR)+} \tau^{3\nu}], \\
\gamma_0 &= s^{-3} \left[\frac{f_{CR}^{(0)}}{1 - s^{-3}} + \frac{f_{CR}^{(1)} \varphi_0^{-1/2} \tilde{c}_1 \tau E_1}{1 - E_1 s^{-3}} + \frac{f_{CR}^{(2)} \varphi_0^{-1/2} c_{20} E_2}{1 - E_2 s^{-3}} + \right. \\
& + \frac{f_{CR}^{(3)} \varphi_0^{-3/2} c_{30} E_3}{1 - E_3 s^{-3}} + \frac{f_{CR}^{(4)} \varphi_0^{-3/2} \tilde{c}_1 \tau c_{20} E_1 E_2}{1 - E_1 E_2 s^{-3}} + \\
& + \frac{f_{CR}^{(5)} \varphi_0^{-5/2} \tilde{c}_1 c_{20}^2 \tau E_1 E_2^2}{1 - E_1 E_2^2 s^{-3}} + \frac{f_{CR}^{(6)} \varphi_0^{-2} c_{20}^2 E_2^2}{1 - E_2^2 s^{-3}} + \frac{f_{CR}^{(7)} \varphi_0^{-1} \tilde{c}_1^2 \tau^2 E_1^2}{1 - E_1^2 s^{-3}} + \\
& \left. + \frac{f_{CR}^{(8)} \varphi_0^{-2} \tilde{c}_1^2 c_{20} \tau^2 E_1^2 E_2}{1 - E_1^2 E_2 s^{-3}} + \frac{f_{CR}^{(9)} \varphi_0^{-3} \tilde{c}_1^2 c_{20}^2 \tau^2 E_1^2 E_2^2}{1 - E_1^2 E_2^2 s^{-3}} \right], \tag{2.11}
\end{aligned}$$

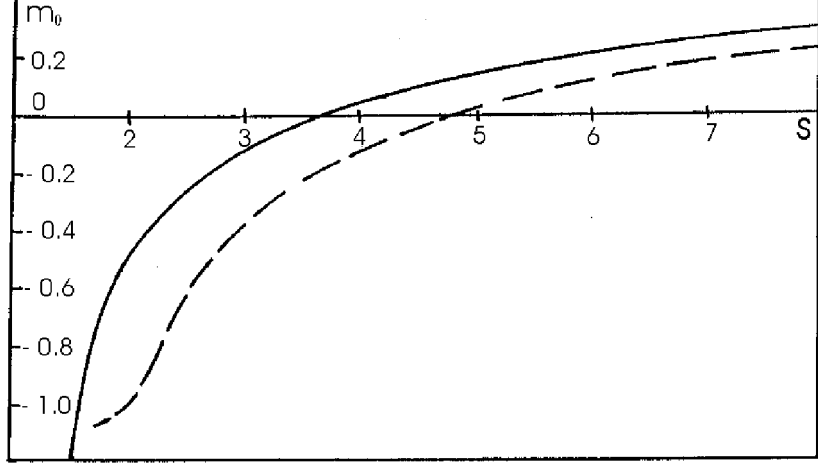


Figure 1. Dependence of the quantity m_0 on the RG parameter s for the ρ^6 model (solid curve) and the ρ^4 model (dashed curve).

$$\begin{aligned}\delta_0 &= \ln Q(\mathcal{M}) + \ln Q(d), \quad \gamma_3^{(CR)+} = c_\nu^3 \gamma^+, \quad c_\nu = \left(\frac{\tilde{c}_1^{(0)}}{f_0}\right)^\nu, \\ \gamma^+ &= \frac{f_{CR}^{(0)}}{1-s^{-3}} + \frac{f_{CR}^{(1)} \varphi_0^{-1/2} f_0}{1-E_1 s^{-3}} + \frac{f_{CR}^{(7)} \varphi_0^{-1} f_0^2}{1-E_1^2 s^{-3}}.\end{aligned}$$

Note that γ_0, δ_0 are the functions of temperature, since they are expressed in terms of $\tilde{c}_1, c_{20}, c_{30}$ and $Q(d)$. Let us extract the temperature dependence in these quantities.

Near T_c we have for \tilde{c}_1

$$\begin{aligned}\tilde{c}_1 &= \tilde{c}_1^{(0)} + \tilde{c}_1^{(1)} \tau, \\ \tilde{c}_1^{(0)} &= V_1 [1 - f_0 + v_{12}^{(0)} \varphi_0^{1/2} + v_{13}^{(0)} \psi_0 \varphi_0^{-1} + \frac{a_4' v_{12}^{(0)} \varphi_0^{-1/2}}{(\beta_c \tilde{\Phi}(0))^2} + \\ &\quad + \frac{2a_6' v_{13}^{(0)} \varphi_0^{-1}}{(\beta_c \tilde{\Phi}(0))^3}], \\ \tilde{c}_1^{(1)} &= V_1 [\frac{a_4' v_{12}^{(0)} \varphi_0^{-1/2}}{(\beta_c \tilde{\Phi}(0))^2} + \frac{3a_6' v_{13}^{(0)} \varphi_0^{-1}}{(\beta_c \tilde{\Phi}(0))^3}],\end{aligned}\tag{2.12}$$

for c_{20} and c_{30} we get, respectively,

$$\begin{aligned}c_{20} &= c_{20}^{(0)} + c_{20}^{(1)} \tau + c_{20}^{(2)} \tau^2, \\ c_{20}^{(0)} &= V_2 [-\varphi_0 - v_{21}^{(0)} (1 - f_0) \varphi_0^{1/2} - v_{23}^{(0)} \psi_0 \varphi_0^{-1/2} + \frac{a_2' v_{21}^{(0)} \varphi_0^{1/2}}{\beta_c \tilde{\Phi}(0)} +\end{aligned}$$

$$\begin{aligned}
& + \frac{a_4'}{(\beta_c \tilde{\Phi}(0))^2} + \frac{a_6' v_{23}^{(0)} \varphi_0^{-1/2}}{(\beta_c \tilde{\Phi}(0))^3}, \\
c_{20}^{(1)} &= V_2 \left[\frac{a_2' v_{21}^{(0)} \varphi_0^{1/2}}{\beta_c \tilde{\Phi}(0)} + \frac{2a_4'}{(\beta_c \tilde{\Phi}(0))^2} + \frac{3a_6' v_{23}^{(0)} \varphi_0^{-1/2}}{(\beta_c \tilde{\Phi}(0))^3} \right], \\
c_{20}^{(2)} &= V_2 \left[\frac{a_4'}{(\beta_c \tilde{\Phi}(0))^2} + \frac{3a_6' v_{23}^{(0)} \varphi_0^{-1/2}}{(\beta_c \tilde{\Phi}(0))^3} \right];
\end{aligned} \tag{2.13}$$

$$\begin{aligned}
c_{30} &= c_{30}^{(0)} + c_{30}^{(1)} \tau + c_{30}^{(2)} \tau^2, \\
c_{30}^{(0)} &= V_3 [-\psi_0 - \varphi_0^{3/2} v_{32}^{(0)} - \varphi_0 (1 - f_0) v_{31}^{(0)} + \frac{a_2' v_{31}^{(0)} \varphi_0}{\beta_c \tilde{\Phi}(0)} + \\
& + \frac{a_4' v_{32}^{(0)} \varphi_0^{1/2}}{(\beta_c \tilde{\Phi}(0))^2} + \frac{a_6'}{(\beta_c \tilde{\Phi}(0))^3}], \\
c_{30}^{(1)} &= V_3 \left[\frac{a_2' v_{31}^{(0)} \varphi_0}{\beta_c \tilde{\Phi}(0)} + \frac{2a_4' v_{32}^{(0)} \varphi_0^{1/2}}{(\beta_c \tilde{\Phi}(0))^2} + \frac{3a_6'}{(\beta_c \tilde{\Phi}(0))^3} \right], \\
c_{30}^{(2)} &= V_3 \left[\frac{a_4' v_{32}^{(0)} \varphi_0^{1/2}}{(\beta_c \tilde{\Phi}(0))^2} + \frac{3a_6'}{(\beta_c \tilde{\Phi}(0))^3} \right].
\end{aligned} \tag{2.14}$$

The values of $\beta_c \tilde{\Phi}(0)$, the correlation length critical exponent $\nu = \ln s / \ln E_1$, exponents of the corrections to scaling $\Delta_1 = -\ln E_2 / \ln E_1$, $\Delta_2 = -\ln E_3 / \ln E_1$, and coefficients of the expressions for $\tilde{c}_1, c_{20}, c_{30}$ are given in tables 1,2. In the present paper, the numerical calculations are performed at $b/c = 1$ and arithmetically averaged Fourier transform of the potential.

Table 1. Values of $\beta_c \tilde{\Phi}(0), \nu, \Delta_1, \Delta_2, \tilde{c}_1^{(0)}, \tilde{c}_1^{(1)}$ for different s .

s	$\beta_c \tilde{\Phi}(0)$	ν	Δ_1	Δ_2	$\tilde{c}_1^{(0)}$	$\tilde{c}_1^{(1)}$
2	1.1204	0.619	0.653	5.061	0.7602	0.0109
2.5	1.1405	0.634	0.552	3.963	0.7637	0.0103
2.7349	1.1506	0.637	0.525	3.647	0.7641	0.0100
3	1.1628	0.640	0.503	3.379	0.7630	0.0096
3.5	1.1882	0.645	0.476	3.038	0.7570	0.0089
3.5862	1.1929	0.645	0.473	2.994	0.7555	0.0088
4	1.2165	0.648	0.460	2.821	0.7471	0.0082

Expressions for the quantities $V_1, V_2, V_3, \psi_0, v_{ij}^{(0)}$ occurring in (2.12) - (2.14) are presented in [13,14]. The coefficient γ_0 can be written as

$$\begin{aligned}
\gamma_0 &= \gamma_0^{(0)} + \gamma_0^{(1)} \tau + \gamma_0^{(2)} \tau^2, \\
\gamma_0^{(0)} &= s^{-3} \left[\frac{f_{CR}^{(0)}}{1 - s^{-3}} + \frac{f_{CR}^{(2)} \varphi_0^{-1} c_{20}^{(0)} E_2}{1 - E_2 s^{-3}} + \frac{f_{CR}^{(3)} \varphi_0^{-3/2} c_{30}^{(0)} E_3}{1 - E_3 s^{-3}} + \right.
\end{aligned}$$

Table 2. Coefficients in equations for c_{20} (2.13) and c_{30} (2.14).

s	$c_{20}^{(0)}$	$c_{20}^{(1)}$	$c_{20}^{(2)}$	$c_{30}^{(0)}$	$c_{30}^{(1)}$	$c_{30}^{(2)}$
2	-0.2302	-0.0949	0.0075	0.2401	0.0762	-0.0161
2.5	-0.3297	-0.0893	0.0080	0.3266	0.0698	-0.0147
2.7349	-0.3820	-0.0872	0.0083	0.3642	0.0654	-0.0140
3	-0.4449	-0.0851	0.0085	0.4056	0.0603	-0.0133
3.5	-0.5737	-0.0813	0.0089	0.4821	0.0516	-0.0120
3.5862	-0.5972	-0.0807	0.0089	0.4952	0.0503	-0.0118
4	-0.7142	-0.0776	0.0090	0.5581	0.0443	-0.0108

$$\begin{aligned}
& + \frac{f_{CR}^{(6)} \varphi_0^{-2} (c_{20}^{(0)})^2 E_2^2}{1 - E_2^2 s^{-3}}], \\
\gamma_0^{(1)} = & s^{-3} \left[\frac{f_{CR}^{(1)} \varphi_0^{-1/2} \tilde{c}_1^{(0)} E_1}{1 - E_1 s^{-3}} + \frac{f_{CR}^{(2)} \varphi_0^{-1} c_{20}^{(1)} E_2}{1 - E_2 s^{-3}} + \frac{f_{CR}^{(3)} \varphi_0^{-3/2} c_{30}^{(1)} E_3}{1 - E_3 s^{-3}} + \right. \\
& + \frac{f_{CR}^{(4)} \varphi_0^{-3/2} \tilde{c}_1^{(0)} c_{20}^{(0)} E_1 E_2}{1 - E_1 E_2 s^{-3}} + \frac{f_{CR}^{(5)} \varphi_0^{-5/2} \tilde{c}_1^{(0)} (c_{20}^{(0)})^2 E_1 E_2^2}{1 - E_1 E_2^2 s^{-3}} + \\
& \left. + \frac{2 f_{CR}^{(6)} \varphi_0^{-2} c_{20}^{(0)} c_{20}^{(1)} E_2^2}{1 - E_2^2 s^{-3}} \right], \tag{2.15} \\
\gamma_0^{(2)} = & s^{-3} \left[\frac{f_{CR}^{(1)} \varphi_0^{-1/2} \tilde{c}_1^{(1)} E_1}{1 - E_1 s^{-3}} + \frac{f_{CR}^{(2)} \varphi_0^{-1} c_{20}^{(2)} E_2}{1 - E_2 s^{-3}} + \frac{f_{CR}^{(3)} \varphi_0^{-3/2} c_{30}^{(2)} E_3}{1 - E_3 s^{-3}} + \right. \\
& + \frac{f_{CR}^{(4)} \varphi_0^{-3/2} (\tilde{c}_1^{(0)} c_{20}^{(1)} + \tilde{c}_1^{(1)} c_{20}^{(0)}) E_1 E_2}{1 - E_1 E_2 s^{-3}} + \\
& + \frac{f_{CR}^{(5)} \varphi_0^{-5/2} (2 \tilde{c}_1^{(0)} c_{20}^{(0)} c_{20}^{(1)} + \tilde{c}_1^{(1)} (c_{20}^{(0)})^2) E_1 E_2^2}{1 - E_1 E_2^2 s^{-3}} + \\
& + \frac{f_{CR}^{(6)} \varphi_0^{-2} ((c_{20}^{(1)})^2 + 2 c_{20}^{(0)} c_{20}^{(2)}) E_2^2}{1 - E_2^2 s^{-3}} + \frac{f_{CR}^{(7)} \varphi_0^{-1} (\tilde{c}_1^{(0)})^2 E_1^2}{1 - E_1^2 s^{-3}} + \\
& \left. + \frac{f_{CR}^{(8)} \varphi_0^{-2} (\tilde{c}_1^{(0)})^2 c_{20}^{(0)} E_1 E_2}{1 - E_1^2 E_2 s^{-3}} + \frac{f_{CR}^{(9)} \varphi_0^{-3} (\tilde{c}_1^{(0)})^2 (c_{20}^{(0)})^2 E_1^2 E_2^2}{1 - E_1^2 E_2^2 s^{-3}} \right].
\end{aligned}$$

For δ_0 we obtain

$$\begin{aligned}
\delta_0 &= \delta_0^{(0)} + \delta_0^{(1)} \tau + \delta_0^{(2)} \tau^2, \\
\delta_0^{(0)} &= \ln Q(\mathcal{M}) + \ln Q(d, T_c), \\
\delta_0^{(1)} &= -\frac{\sqrt{6}}{\sqrt{a_4}} (1 - \bar{q}) \beta_c \tilde{\Phi}(0) \mathcal{F}_2(h_c, \alpha), \\
\delta_0^{(2)} &= -\frac{3}{a_4} (1 - \bar{q})^2 (\beta_c \tilde{\Phi}(0))^2 [\mathcal{F}_2^2(h_c, \alpha) - \mathcal{F}_4(h_c, \alpha)] +
\end{aligned} \tag{2.16}$$

$$\begin{aligned}
& + \frac{\sqrt{6}}{\sqrt{a_4'}}(1 - \bar{q})\beta_c \tilde{\Phi}(0)\mathcal{F}_2(h_c, \alpha), \\
h_c & = \frac{\sqrt{6}}{\sqrt{a_4'}}[a_2' - \beta_c \tilde{\Phi}(0)(1 - \bar{q})], \quad \alpha = \frac{\sqrt{6}}{15} \frac{a_6'}{(a_4')^{3/2}}, \quad \bar{q} = \frac{1 + s^{-2}}{2}.
\end{aligned}$$

Coefficients of the expressions for γ_0 (2.15), δ_0 (2.16) at different values of the RG parameter s are given in table 3.

Hence, the free energy of the CR region reads

$$\begin{aligned}
F_{CR} & = -kTN'[\gamma_0^{(CR)} + \gamma_1\tau + \gamma_2\tau^2 + \gamma_3^{(CR)+}\tau^{3\nu}], \\
\gamma_0^{(CR)} & = \gamma_0^{(0)} + \delta_0^{(0)}, \\
\gamma_1 & = \gamma_0^{(1)} + \delta_0^{(1)}, \\
\gamma_2 & = \gamma_0^{(2)} + \delta_0^{(2)}.
\end{aligned} \tag{2.17}$$

The numerical values of coefficients $\gamma_0^{(CR)}$, γ_1, γ_2 and quantities $\gamma^+, c_\nu = (\tilde{c}_1^{(0)}/f_0)^\nu$ occurring in $\gamma_3^{(CR)+}$ are given in table 4.

Knowledge of F_{CR} allows one to calculate other thermodynamic functions of the system in the CR region at $T > T_c$. For the entropy S_{CR} , internal energy U_{CR} and specific heat C_{CR} we get

$$\begin{aligned}
S_{CR} & = kN'[s^{(0)(CR)} + c_0\tau + u_3^{(CR)+}\tau^{1-\alpha}], \\
U_{CR} & = kTN'[\gamma_1 + u_1\tau + u_3^{(CR)+}\tau^{1-\alpha}], \\
C_{CR} & = kN'[c_0 + c_3^{(CR)+}\tau^{-\alpha}], \\
s^{(0)(CR)} & = \gamma_0^{(CR)} + \gamma_1, \quad c_0 = 2(\gamma_1 + \gamma_2), \quad u_3^{(CR)+} = -3\nu\gamma_3^{(CR)+}, \\
u_1 & = 2\gamma_2 + \gamma_1, \quad c_3^{(CR)+} = -3\nu(3\nu - 1)\gamma_3^{(CR)+}, \quad \alpha = 2 - 3\nu.
\end{aligned} \tag{2.18}$$

In the region of LGR, the expression for the part Z_{LGR} of the partition function (2.1) reads

$$Z_{LGR} = \int \exp\left\{-\frac{1}{2} \sum_{k \leq B_{m_\tau+1}} [d_{m_\tau}(k) - d_{m_\tau}(B_{m_\tau+1}, B_{m_\tau})]\rho_{\mathbf{k}}\rho_{-\mathbf{k}} - \right.$$

Table 3. Values of coefficients in equations for γ_0 (2.15) and δ_0 (2.16).

s	$\gamma_0^{(0)}$	$\gamma_0^{(1)}$	$\gamma_0^{(2)}$	$\delta_0^{(0)}$	$\delta_0^{(1)}$	$\delta_0^{(2)}$
2	0.1205	-0.3160	-2.5291	0.2711	-0.3574	0.4810
2.5	0.0757	-0.2343	-2.9320	0.3237	-0.4510	0.6464
2.7349	0.0624	-0.2062	-2.9801	0.3423	-0.4862	0.7128
3	0.0510	-0.1798	-2.9798	0.3606	-0.5222	0.7827
3.5	0.0362	-0.1415	-2.9314	0.3905	-0.5836	0.9075
3.5862	0.0343	-0.1360	-2.9236	0.3953	-0.5938	0.9287
4	0.0267	-0.1135	-2.9039	0.4175	-0.6420	1.0319

Table 4. Coefficients $\gamma_0^{(CR)}$, γ_1 , γ_2 and quantities c_ν , γ^+ , γ^- contained in $\gamma_3^{(CR)\pm}$ for different values of the parameter s .

s	$\gamma_0^{(CR)}$	γ_1	γ_2	c_ν	γ^+	γ^-
2	0.3915	-0.6734	-2.0482	1.4412	-0.3170	0.7382
2.5	0.3994	-0.6852	-2.2856	1.2722	-0.7757	0.5244
2.7349	0.4047	-0.6924	-2.2672	1.2097	-0.9831	0.4188
3	0.4116	-0.7020	-2.1971	1.1462	-1.2229	0.2899
3.5	0.4267	-0.7251	-2.0239	1.0414	-1.7450	-0.0277
3.5862	0.4295	-0.7298	-1.9949	1.0250	-1.8496	-0.0973
4	0.4442	-0.7555	-1.8720	0.9517	-2.4345	-0.5148

$$\begin{aligned}
& - 2\pi i \sum_{k \leq B_{m_\tau+1}} \omega_{\mathbf{k}} \rho_{\mathbf{k}} - \frac{(2\pi)^2}{2} \sum_{k \leq B_{m_\tau+1}} P_2^{(m_\tau)} \omega_{\mathbf{k}} \omega_{-\mathbf{k}} - \\
& - \sum_{l=2}^3 \frac{(2\pi)^{2l}}{(2l)!} N_{m_\tau+1}^{1-l} \sum_{k_1, \dots, k_{2l} \leq B_{m_\tau+1}} P_{2l}^{(m_\tau)} \omega_{\mathbf{k}_1} \dots \omega_{\mathbf{k}_{2l}} \times \\
& \times \delta_{\mathbf{k}_1 + \dots + \mathbf{k}_{2l}} \} (d\rho)^{N_{m_\tau+1}} (d\omega)^{N_{m_\tau+1}}.
\end{aligned} \tag{2.19}$$

To calculate Z_{LGR} it is convenient to select two regions of the wave vector values [14]. The first, transition region, corresponds to the values of k close to B_{m_τ} ; the second, Gaussian region, corresponds to small values of the wave vector ($k \rightarrow 0$). Hence, we have

$$Z_{LGR} = Z_{LGR}^{(1)} Z_{LGR}^{(2)}. \tag{2.20}$$

The contribution to free energy from the phase space layers following the point of exit from the CR region is

$$\begin{aligned}
F_{LGR}^{(1)} &= -kT N' f_{TR} \tau^{3\nu}, \\
f_{TR} &= c_\nu^3 \bar{f}_{TR}, \quad \bar{f}_{TR} = \sum_{m=0}^{\tilde{m}_0} s^{-3m} f_{LGR_1}(m), \\
f_{LGR_1}(m) &= \ln \frac{2}{\pi} + \frac{1}{4} \ln 24 - \frac{1}{4} \ln C(\eta_{m_\tau+m}, \xi_{m_\tau+m}) + \\
&+ \ln I_0(h_{m_\tau+m+1}, \alpha_{m_\tau+m+1}) + \ln I_0(\eta_{m_\tau+m}, \xi_{m_\tau+m}),
\end{aligned} \tag{2.21}$$

where \tilde{m}_0 is the nearest integer to \tilde{m}'_0 . The quantities \tilde{m}'_0 , $\eta_{m_\tau+m}$, $\xi_{m_\tau+m}$, $h_{m_\tau+m+1}$, $\alpha_{m_\tau+m+1}$ were determined in [14]. The plots of $\tilde{m}'_0(s)$ and analogous dependence $m_0''(s)$ (ρ^4 model) are represented in figure 2.

Introducing an infinitely small external magnetic field $h = \mu_B \mathcal{H}$ (μ_B is the Bohr magneton), we can write the part of the free energy corresponding to $Z_{LGR}^{(2)}$ as

$$F_{LGR}^{(2)} = -kT N' f' \tau^{3\nu} - \beta N \gamma_4^+ h^2 \tau^{-2\nu}, \quad f' = c_\nu^3 \bar{f}',$$

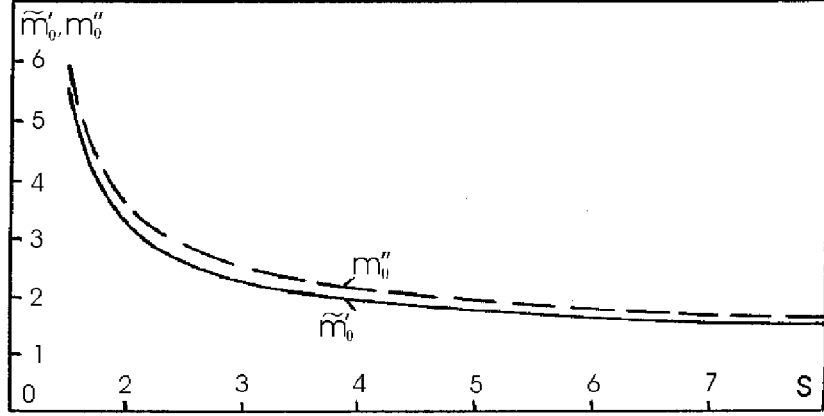


Figure 2. Behaviour of \tilde{m}'_0 (ρ^6 model) and m''_0 (ρ^4 model) with the increase of the parameter s .

$$\begin{aligned}
 \bar{f}' &= s^{-3(\tilde{m}_0+1)} f_{LGR_2}, \\
 f_{LGR_2} &= -\frac{1}{4} \ln 24 + \frac{1}{3} + \frac{1}{4} \ln \bar{u}_{m_{\tau'}-1} - \frac{1}{2} \ln \left(\tilde{G} + \frac{1}{s^2} \right) - \\
 &\quad - \frac{1}{2} \ln \mathcal{F}_2(h_{m_{\tau'}-1}, \alpha_{m_{\tau'}-1}) - \tilde{G}s^2 + (\tilde{G}s^2)^{3/2} \arctan[(\tilde{G}s^2)^{-1/2}], \\
 \gamma_4^+ &= c_\nu^{-2} \bar{\gamma}_4^+ / (\beta \tilde{\Phi}(0)), \quad \bar{\gamma}_4^+ = s^{2\tilde{m}_0} / (2\tilde{G}).
 \end{aligned} \tag{2.22}$$

Here,

$$\begin{aligned}
 m'_{\tau} &= m_{\tau} + \tilde{m}_0 + 2, \quad \bar{u}_{m'_{\tau}-1} = u_{m'_{\tau}-1} (\beta \tilde{\Phi}(0))^{-2}, \\
 \tilde{G} &= (\bar{u}_{m'_{\tau}-1} / 24)^{1/2} [\mathcal{F}_2(h_{m'_{\tau}-1}, \alpha_{m'_{\tau}-1})]^{-1} - \bar{q}.
 \end{aligned}$$

A general expression describing the contribution of long-wave fluctuations to the free energy (LGR region) reads

$$\begin{aligned}
 F_{LGR} &= -kTN' f_{LGR} \tau^{3\nu} - \beta N \gamma_4^+ h^2 \tau^{-2\nu}, \\
 f_{LGR} &= c_\nu^3 \bar{f}_{LGR}, \quad \bar{f}_{LGR} = \bar{f}_{TR} + \bar{f}'.
 \end{aligned} \tag{2.23}$$

The values of \bar{f}_{TR} , \bar{f}' , $\bar{\gamma}_4^+$ are given in table 5.

The entropy, internal energy, specific heat corresponding to the LGR region are defined by the relations

$$\begin{aligned}
 S_{LGR} &= kN' u_3^{(LGR)} \tau^{1-\alpha}, \\
 U_{LGR} &= kTN' u_3^{(LGR)} \tau^{1-\alpha}, \\
 C_{LGR} &= kN' c_3^{LGR} \tau^{-\alpha}, \\
 u_3^{(LGR)} &= 3\nu f_{LGR}, \\
 c_3^{(LGR)} &= 3\nu(3\nu - 1) f_{LGR}.
 \end{aligned} \tag{2.24}$$

Table 5. Values of \bar{f}_{TR} , \bar{f}' , $\bar{\gamma}_4^+$.

s	\bar{f}_{TR}	$\bar{f}' \times 10^5$	$\bar{\gamma}_4^+$
2	0.6529	0.4749	2.7055
2.5	0.8142	0.0102	3.0870
2.7349	0.8824	0.2155	2.1737
3	0.9541	0.0659	2.1841
3.5	1.0756	0.0091	2.2058
3.5862	1.0950	0.0067	2.2098
4	1.1822	0.0016	2.2307

3. Contributions to the thermodynamic functions of the model from the critical and inverse Gaussian regime (CR and IGR) regions below T_c

The free energy at $T < T_c$ can be written as [6,10]

$$F = F_0 + F_{CR} + F_{IGR}, \quad (3.1)$$

where $F_0 = -kTN \ln 2$ is the free energy of the system of N non-interacting spins, F_{CR} is the contribution to the free energy from the short-wave fluctuation phases of the spin moment density (CR region), and F_{IGR} is the contribution from the long-wave phases of the fluctuations (IGR region).

The number μ_τ of the CV phase space layer, separating the short-wave and long-wave phases of fluctuations, is an important characteristic of the system. It is determined from the equation

$$\frac{r_{\mu_\tau+1} - r^{(0)}}{r^{(0)}} = \delta. \quad (3.2)$$

Here $r^{(0)}$ corresponds to the fixed point of the RR, $r_{\mu_\tau+1}$ is determined from the solutions of the RR equations (see, for example, [13,14]), δ is a constant ($\delta \leq 1$). In the present paper we put $\delta = 1$ (see [8]). Let us write an equation for μ_τ

$$|\tau| \tilde{c}_1 E_1^{\mu_\tau+1} = f_0, \quad (3.3)$$

the solution of which is

$$\mu_\tau = -\frac{\ln |\tau|}{\ln E_1} + \mu_0 - 1, \quad \mu_0 = \frac{\ln f_0 - \ln \tilde{c}_1^{(0)}}{\ln E_1}. \quad (3.4)$$

We need to sum the partial free energies over the layers of the CV phase space to calculate F_{CR} . Extracting an explicit dependence on the layer number, using relations (3.3) and

$$s^{-3(\mu_\tau+1)} = |\tau|^{3\nu} s^{-3\mu_0}, \quad s^{-3\mu_0} = c_\nu^3, \quad (3.5)$$

we obtain

$$F_{CR} = -kTN'[\gamma_0^{(CR)} - \gamma_1 |\tau| + \gamma_2 |\tau|^2 - \gamma_3^{(CR)-} |\tau|^{3\nu}]. \quad (3.6)$$

Coefficients $\gamma_0^{(CR)}$, γ_1 , γ_2 are determined in (2.17),

$$\begin{aligned} \gamma_3^{(CR)-} &= c_\nu^3 \gamma^-, \\ \gamma^- &= \frac{f_{CR}^{(0)}}{1-s^{-3}} - \frac{f_{CR}^{(1)} \varphi_0^{-1/2} f_0}{1-E_1 s^{-3}} + \frac{f_{CR}^{(7)} \varphi_0^{-1} f_0^2}{1-E_1^2 s^{-3}}. \end{aligned} \quad (3.7)$$

The value of γ^- is given in table 4.

The entropy, internal energy and specific heat of the system corresponding to the CR region read

$$\begin{aligned} S_{CR} &= kN'[s^{(0)(CR)} - c_0 |\tau| + u_3^{(CR)-} |\tau|^{1-\alpha}], \\ U_{CR} &= kTN'[\gamma_1 - u_1 |\tau| + u_3^{(CR)-} |\tau|^{1-\alpha}], \\ C_{CR} &= kN'[c_0 - c_3^{(CR)-} |\tau|^{-\alpha}], \\ u_3^{(CR)-} &= 3\nu \gamma_3^{(CR)-}, \\ c_3^{(CR)-} &= 3\nu(3\nu - 1) \gamma_3^{(CR)-}. \end{aligned} \quad (3.8)$$

Let us calculate now the contribution to the free energy from the IGR region

$$F_{IGR} = -kTN' s^{-3(\mu_\tau+1)} \ln[\sqrt{2}Q(P_{\mu_\tau})] - kT \ln Z_{\mu_\tau+1}, \quad (3.9)$$

where

$$\begin{aligned} \sqrt{2}Q(P_{\mu_\tau}) &= \left(\frac{4s^3 a_4^{(\mu_\tau)}}{\pi^4 C(h_{\mu_\tau}, \alpha_{\mu_\tau})} \right)^{1/4} I_0(\eta_{\mu_\tau}, \xi_{\mu_\tau}), \\ Z_{\mu_\tau+1} &= \int \exp\left\{ -\frac{1}{2} \sum_{k \leq B_{\mu_\tau+1}} d_{\mu_\tau+1}(k) \rho_{\mathbf{k}} \rho_{-\mathbf{k}} - \right. \\ &\quad \left. - \sum_{l=2}^3 \frac{a_{2l}^{(\mu_\tau+1)}}{(2l)!} N_{\mu_\tau+1}^{1-l} \sum_{k_1, \dots, k_{2l} \leq B_{\mu_\tau+1}} \rho_{\mathbf{k}_1} \cdots \rho_{\mathbf{k}_{2l}} \delta_{\mathbf{k}_1 + \dots + \mathbf{k}_{2l}} \right\} (d\rho)^{N_{\mu_\tau+1}}. \end{aligned}$$

Consider the first term on the RHS of (3.9). Making use of the relations

$$\begin{aligned} r_{\mu_\tau} &= -\bar{r}_{\mu_\tau} \beta \tilde{\Phi}(0), \quad \bar{r}_{\mu_\tau} = f_0(1 + E_1^{-1}), \\ u_{\mu_\tau} &= \bar{u}_{\mu_\tau} (\beta \tilde{\Phi}(0))^2, \quad \bar{u}_{\mu_\tau} = \varphi_0 - f_0 \varphi_0^{1/2} w_{21}^{(0)} E_1^{-1}, \\ w_{\mu_\tau} &= \bar{w}_{\mu_\tau} (\beta \tilde{\Phi}(0))^3, \quad \bar{w}_{\mu_\tau} = \psi_0 - f_0 \varphi_0 w_{31}^{(0)} E_1^{-1}, \end{aligned} \quad (3.10)$$

we find

$$\begin{aligned} h_{\mu_\tau} &= \sqrt{6} \frac{\bar{q} - \bar{r}_{\mu_\tau}}{\sqrt{\bar{u}_{\mu_\tau}}}, \quad \alpha_{\mu_\tau} = \frac{\sqrt{6}}{15} \frac{\bar{w}_{\mu_\tau}}{(\bar{u}_{\mu_\tau})^{3/2}}, \\ \eta_{\mu_\tau} &= \sqrt{6} s^{3/2} \mathcal{F}_2(h_{\mu_\tau}, \alpha_{\mu_\tau}) [C(h_{\mu_\tau}, \alpha_{\mu_\tau})]^{-1/2}, \\ \xi_{\mu_\tau} &= \frac{\sqrt{6}}{15} s^{-3/2} N(h_{\mu_\tau}, \alpha_{\mu_\tau}) [C(h_{\mu_\tau}, \alpha_{\mu_\tau})]^{-3/2}. \end{aligned} \quad (3.11)$$

The first term on the RHS of (3.9) is equal to

$$s^{-3(\mu_\tau+1)} \ln[\sqrt{2}Q(P_{\mu_\tau})] = \gamma_g | \tau |^{3\nu} + s^{-3(\mu_\tau+1)} \ln \frac{\sqrt{\beta\tilde{\Phi}(0)}}{s^{\mu_\tau+1}}, \quad (3.12)$$

$$\gamma_g = c_\nu^3 \bar{\gamma}_g, \quad \bar{\gamma}_g = \ln[(\frac{4s^7 \bar{u}_{\mu_\tau}}{\pi^4 C(h_{\mu_\tau}, \alpha_{\mu_\tau})})^{1/4} I_0(\eta_{\mu_\tau}, \xi_{\mu_\tau})].$$

To find the second term on the RHS of (3.9), we need to calculate $Z_{\mu_\tau+1}$. The coefficients occurring in it equal

$$\begin{aligned} d_{\mu_\tau+1}(k) &= r_{\mu_\tau+1} s^{-2(\mu_\tau+1)} + \tilde{q} k^2, \quad \tilde{q} = 2\beta\tilde{\Phi}(0)b^2, \\ a_4^{(\mu_\tau+1)} &= u_{\mu_\tau+1} s^{-4(\mu_\tau+1)}, \quad a_6^{(\mu_\tau+1)} = w_{\mu_\tau+1} s^{-6(\mu_\tau+1)}, \\ r_{\mu_\tau+1} &= -\bar{r}_{\mu_\tau+1} \beta\tilde{\Phi}(0), \quad \bar{r}_{\mu_\tau+1} = 2f_0, \\ u_{\mu_\tau+1} &= \bar{u}_{\mu_\tau+1} (\beta\tilde{\Phi}(0))^2, \quad \bar{u}_{\mu_\tau+1} = \varphi_0 - f_0 \varphi_0^{-1/2} w_{21}^{(0)}, \\ w_{\mu_\tau+1} &= \bar{w}_{\mu_\tau+1} (\beta\tilde{\Phi}(0))^3, \quad \bar{w}_{\mu_\tau+1} = \psi_0 - f_0 \varphi_0 w_{31}^{(0)}. \end{aligned} \quad (3.13)$$

Let us perform the change of variables

$$\rho_{\mathbf{k}} = \rho'_{\mathbf{k}} + \sqrt{N} \langle \bar{\sigma} \rangle \delta_{\mathbf{k}} \quad (3.14)$$

in the expression for $Z_{\mu_\tau+1}$ in order to extract the free energy related to the ordering that has appeared in the system. Here $\langle \bar{\sigma} \rangle$ is determined from the extremum condition [17,10]

$$\begin{aligned} \langle \bar{\sigma} \rangle^2 &= 10 \frac{a_4^{(\mu_\tau+1)}}{a_6^{(\mu_\tau+1)}} \frac{N_{\mu_\tau+1}}{N} (-1 + b_2), \\ b_2 &= \sqrt{1 + \frac{6a_6^{(\mu_\tau+1)} |d_{\mu_\tau+1}(0)|}{5(a_4^{(\mu_\tau+1)})^2}}. \end{aligned} \quad (3.15)$$

Simultaneously, we include in the treatment a constant external field $h = \mu_B \mathcal{H}$, which sustains the separated average moment, and also separate from the sums over k the terms with $k = 0$. We obtain

$$\begin{aligned} Z_{\mu_\tau+1} &= \exp(-\beta F_\sigma + \beta F_h) \int d\rho_0 \exp\{\beta \sqrt{N} h \rho_0 - \frac{1}{2} \bar{d}_{\mu_\tau+1}(0) \rho_0^2 - \\ &- \frac{b_3^{(\mu_\tau+1)}}{3! \sqrt{N_{\mu_\tau+1}}} \rho_0^3 - \frac{b_4^{(\mu_\tau+1)}}{4! N_{\mu_\tau+1}} \rho_0^4 - \frac{b_5^{(\mu_\tau+1)}}{5! N_{\mu_\tau+1} \sqrt{N_{\mu_\tau+1}}} \rho_0^5 - \frac{a_6^{(\mu_\tau+1)}}{6! N_{\mu_\tau+1}^2} \rho_0^6\} \times \\ &\times \int \exp\{-\frac{1}{2} \sum'_{k \leq B_{\mu_\tau+1}} \bar{d}_{\mu_\tau+1}(k) \rho_{\mathbf{k}} \rho_{-\mathbf{k}}\} \exp\{p_0 + p_1 \rho_0 + p_2 \rho_0^2 + \\ &+ p_3 \rho_0^3 + p_4 \rho_0^4\} (d\rho)^{N_{\mu_\tau+1}-1}. \end{aligned} \quad (3.16)$$

The prime on the sum over k means that $k \neq 0$,

$$-\beta F_\sigma = \frac{10}{3} |d_{\mu_\tau+1}(0)| \frac{a_4^{(\mu_\tau+1)}}{a_6^{(\mu_\tau+1)}} N_{\mu_\tau+1} (-1 + b_2) -$$

$$-\frac{25(a_4^{(\mu_\tau+1)})^3}{18(a_6^{(\mu_\tau+1)})^2}N_{\mu_\tau+1}(-1+b_2)^2, \quad (3.17)$$

$$\beta F_h = \beta\sqrt{N}h\frac{\sqrt{10a_4^{(\mu_\tau+1)}}\sqrt{N_{\mu_\tau+1}}}{\sqrt{a_6^{(\mu_\tau+1)}}}(-1+b_2)^{1/2}.$$

We have for the integrand coefficients of (3.16)

$$\begin{aligned} \bar{d}_{\mu_\tau+1}(k) &= 4 |d_{\mu_\tau+1}(0)| - \frac{10(a_4^{(\mu_\tau+1)})^2}{3a_6^{(\mu_\tau+1)}}(-1+b_2) + \tilde{q}k^2 = \\ &= c_\nu^2 |\tau|^{2\nu} \beta \tilde{\Phi}(0)[4\bar{r}_{\mu_\tau+1} - \frac{10}{3}\frac{\bar{u}_{\mu_\tau+1}^2}{\bar{w}_{\mu_\tau+1}}(-1+b_2)] + \tilde{q}k^2, \quad (3.18) \\ b_3^{(\mu_\tau+1)} &= \frac{\sqrt{10}a_4^{(\mu_\tau+1)}\sqrt{a_4^{(\mu_\tau+1)}}}{\sqrt{a_6^{(\mu_\tau+1)}}}(-1+b_2)^{1/2}[1 + \frac{5}{3}(-1+b_2)], \\ b_4^{(\mu_\tau+1)} &= a_4^{(\mu_\tau+1)}[1 + 5(-1+b_2)], \\ b_5^{(\mu_\tau+1)} &= \sqrt{10}\sqrt{a_4^{(\mu_\tau+1)}a_6^{(\mu_\tau+1)}}(-1+b_2)^{1/2}. \end{aligned}$$

The quantities p_i are determined by the equations

$$\begin{aligned} p_0 &= -\frac{b_3^{(\mu_\tau+1)}}{3!\sqrt{N_{\mu_\tau+1}}} \sum'_{k_i \leq B_{\mu_\tau+1}} \rho_{\mathbf{k}_1}\rho_{\mathbf{k}_2}\rho_{\mathbf{k}_3}\delta_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3} - \frac{b_4^{(\mu_\tau+1)}}{4!N_{\mu_\tau+1}} \times \\ &\times \sum'_{k_i \leq B_{\mu_\tau+1}} \rho_{\mathbf{k}_1} \cdots \rho_{\mathbf{k}_4}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_4} - \frac{b_5^{(\mu_\tau+1)}}{5!N_{\mu_\tau+1}\sqrt{N_{\mu_\tau+1}}} \times \\ &\times \sum'_{k_i \leq B_{\mu_\tau+1}} \rho_{\mathbf{k}_1} \cdots \rho_{\mathbf{k}_5}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_5} - \frac{a_6^{(\mu_\tau+1)}}{6!N_{\mu_\tau+1}^2} \sum'_{k_i \leq B_{\mu_\tau+1}} \rho_{\mathbf{k}_1} \cdots \rho_{\mathbf{k}_6}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_6}, \\ p_1 &= -\frac{b_3^{(\mu_\tau+1)}}{2\sqrt{N_{\mu_\tau+1}}} \sum'_{k \leq B_{\mu_\tau+1}} \rho_{\mathbf{k}}\rho_{-\mathbf{k}} - \frac{b_4^{(\mu_\tau+1)}}{3!N_{\mu_\tau+1}} \sum'_{k_i \leq B_{\mu_\tau+1}} \rho_{\mathbf{k}_1}\rho_{\mathbf{k}_2}\rho_{\mathbf{k}_3} \times \\ &\times \delta_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3} - \frac{b_5^{(\mu_\tau+1)}}{4!N_{\mu_\tau+1}\sqrt{N_{\mu_\tau+1}}} \sum'_{k_i \leq B_{\mu_\tau+1}} \rho_{\mathbf{k}_1} \cdots \rho_{\mathbf{k}_4}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_4} - \\ &- \frac{a_6^{(\mu_\tau+1)}}{5!N_{\mu_\tau+1}^2} \sum'_{k_i \leq B_{\mu_\tau+1}} \rho_{\mathbf{k}_1} \cdots \rho_{\mathbf{k}_5}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_5}, \quad (3.19) \\ p_2 &= -\frac{b_4^{(\mu_\tau+1)}}{4N_{\mu_\tau+1}} \sum'_{k \leq B_{\mu_\tau+1}} \rho_{\mathbf{k}}\rho_{-\mathbf{k}} - \frac{b_5^{(\mu_\tau+1)}}{12N_{\mu_\tau+1}\sqrt{N_{\mu_\tau+1}}} \times \\ &\times \sum'_{k_i \leq B_{\mu_\tau+1}} \rho_{\mathbf{k}_1}\rho_{\mathbf{k}_2}\rho_{\mathbf{k}_3}\delta_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3} - \frac{a_6^{(\mu_\tau+1)}}{48N_{\mu_\tau+1}^2} \sum'_{k_i \leq B_{\mu_\tau+1}} \rho_{\mathbf{k}_1} \cdots \rho_{\mathbf{k}_4}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_4}, \\ p_3 &= -\frac{b_5^{(\mu_\tau+1)}}{12N_{\mu_\tau+1}\sqrt{N_{\mu_\tau+1}}} \sum'_{k \leq B_{\mu_\tau+1}} \rho_{\mathbf{k}}\rho_{-\mathbf{k}} - \frac{a_6^{(\mu_\tau+1)}}{36N_{\mu_\tau+1}^2} \sum'_{k_i \leq B_{\mu_\tau+1}} \rho_{\mathbf{k}_1}\rho_{\mathbf{k}_2}\rho_{\mathbf{k}_3} \times \end{aligned}$$

$$\times \delta_{\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3},$$

$$p_4 = -\frac{a_6^{(\mu_{\tau+1})}}{48N_{\mu_{\tau+1}}^2} \sum'_{k \leq B_{\mu_{\tau+1}}} \rho_{\mathbf{k}} \rho_{-\mathbf{k}}.$$

The dependence of the quantity $4\bar{r}_{\mu_{\tau+1}} - \frac{10}{3} \frac{\bar{u}_{\mu_{\tau+1}}^2}{\bar{w}_{\mu_{\tau+1}}}(-1 + b_2)$ on the parameter s is plotted in figure 3. The dashed line shows the dependence of the analogous quantity $4f_0$ on s for the ρ^4 model. One can see from figure 3 the

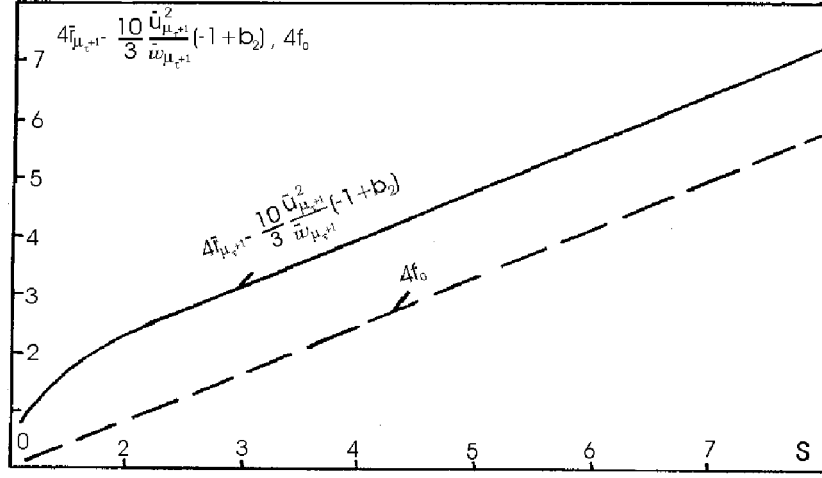


Figure 3. Dependence of the quantities $4\bar{r}_{\mu_{\tau+1}} - \frac{10}{3} \frac{\bar{u}_{\mu_{\tau+1}}^2}{\bar{w}_{\mu_{\tau+1}}}(-1 + b_2)$ (ρ^6 model) and $4f_0$ (ρ^4 model) on s .

quantity $4\bar{r}_{\mu_{\tau+1}} - \frac{10}{3} \frac{\bar{u}_{\mu_{\tau+1}}^2}{\bar{w}_{\mu_{\tau+1}}}(-1 + b_2)$ occurring in $\bar{d}_{\mu_{\tau+1}}(k)$ (3.18) (therefore, and $\bar{d}_{\mu_{\tau+1}}(k)$) is positive for all s .

Expanding $\exp\{p_0 + p_1\rho_0 + p_2\rho_0^2 + p_3\rho_0^3 + p_4\rho_0^4\}$ in series and restricting ourselves to the terms of second order, we integrate (3.16) over $\rho_{\mathbf{k}}$ with $k \neq 0$ using the Gaussian basis distribution. Gathering up the series over the averages with respect to the Gaussian distribution in the exponential, we obtain [15]

$$Z_{\mu_{\tau+1}} = \exp(-\beta F_{\sigma} + \beta F_h - \beta F_m) \prod'_{k_i \leq B_{\mu_{\tau+1}}} \left(\frac{\pi}{d_{\mu_{\tau+1}}(k)} \right)^{1/2} \int \exp(\tilde{A}_1 \rho_0 + \tilde{A}_2 \rho_0^2 + \tilde{A}_3 \rho_0^3 + \tilde{A}_4 \rho_0^4 + \tilde{A}_5 \rho_0^5 + \tilde{A}_6 \rho_0^6 + \tilde{A}_7 \rho_0^7 + \tilde{A}_8 \rho_0^8) d\rho_0, \quad (3.20)$$

where

$$-\beta F_m = \frac{1}{4} N_{\mu_{\tau+1}} \left[-\frac{1}{2} (b_4^{(\mu_{\tau+1})} + \frac{a_6^{(\mu_{\tau+1})}}{6} \mathcal{I}_1) \mathcal{I}_1^2 + \frac{b_3^{(\mu_{\tau+1})}}{3} \mathcal{I}_3 \times \right. \\ \left. \times (b_3^{(\mu_{\tau+1})} + b_5^{(\mu_{\tau+1})} \mathcal{I}_1) + \frac{(b_4^{(\mu_{\tau+1})})^2}{4} \left(\frac{\mathcal{I}_4}{3} + \mathcal{I}_1^2 \mathcal{I}_2 \right) + \frac{(b_5^{(\mu_{\tau+1})})^2}{12} \times \right]$$

$$\begin{aligned}
& \times \left(\frac{\mathcal{I}_5}{5} + \mathcal{I}_1^2 \mathcal{I}_3 \right) + \frac{(a_6^{(\mu_r+1)})^2}{8} \left(\frac{\mathcal{I}_6}{45} + \frac{\mathcal{I}_1^2}{2} \left(\frac{\mathcal{I}_1^2 \mathcal{I}_2}{4} + \frac{\mathcal{I}_4}{3} \right) \right) + \\
& + \frac{b_4^{(\mu_r+1)} a_6^{(\mu_r+1)}}{4} \mathcal{I}_1 \left(\frac{\mathcal{I}_4}{3} + \frac{\mathcal{I}_1^2 \mathcal{I}_2}{2} \right) \Big], \\
\tilde{A}_1 = & \beta \sqrt{N} h + \frac{1}{2} N_{\mu_r+1}^{1/2} \left[-\mathcal{I}_1 (b_3^{(\mu_r+1)} + \frac{1}{4} b_5^{(\mu_r+1)} \mathcal{I}_1) + \right. \\
& + b_3^{(\mu_r+1)} b_4^{(\mu_r+1)} \left(\frac{\mathcal{I}_3}{3} + \frac{\mathcal{I}_1 \mathcal{I}_2}{2} \right) + \frac{b_3^{(\mu_r+1)} a_6^{(\mu_r+1)}}{2} \mathcal{I}_1 \left(\frac{\mathcal{I}_3}{3} + \frac{\mathcal{I}_1 \mathcal{I}_2}{4} \right) + \\
& + \frac{b_4^{(\mu_r+1)} b_5^{(\mu_r+1)}}{2} \left(\frac{\mathcal{I}_4}{6} + \mathcal{I}_1 \left(\frac{\mathcal{I}_3}{3} + \frac{\mathcal{I}_1 \mathcal{I}_2}{2} \right) \right) + \frac{b_5^{(\mu_r+1)} a_6^{(\mu_r+1)}}{4} \left(\frac{\mathcal{I}_5}{15} + \right. \\
& \left. \left. + \mathcal{I}_1 \left(\frac{\mathcal{I}_1 \mathcal{I}_3}{3} + \frac{\mathcal{I}_4}{6} + \frac{\mathcal{I}_1^2 \mathcal{I}_2}{4} \right) \right) \right], \\
\tilde{A}_2 = & -2 |d_{\mu_r+1}(0)| + \frac{5}{3} \frac{(a_4^{(\mu_r+1)})^2}{a_6^{(\mu_r+1)}} (-1 + b_2) + \frac{1}{4} \left[-\mathcal{I}_1 (b_4^{(\mu_r+1)} + \right. \\
& + \frac{a_6^{(\mu_r+1)}}{4} \mathcal{I}_1) + (b_3^{(\mu_r+1)})^2 \mathcal{I}_2 + (b_4^{(\mu_r+1)})^2 \left(\frac{\mathcal{I}_3}{3} + \frac{\mathcal{I}_1 \mathcal{I}_2}{2} \right) + \\
& + \frac{(b_5^{(\mu_r+1)})^2}{2} \left(\frac{\mathcal{I}_4}{6} + \mathcal{I}_1 \left(\frac{\mathcal{I}_3}{3} + \frac{\mathcal{I}_1 \mathcal{I}_2}{2} \right) \right) + \frac{(a_6^{(\mu_r+1)})^2}{4} \left(\frac{\mathcal{I}_5}{15} + \right. \\
& + \mathcal{I}_1 \left(\frac{\mathcal{I}_1 \mathcal{I}_3}{3} + \frac{\mathcal{I}_4}{6} + \frac{\mathcal{I}_1^2 \mathcal{I}_2}{4} \right) \Big) + b_3^{(\mu_r+1)} b_5^{(\mu_r+1)} \left(\frac{\mathcal{I}_3}{3} + \mathcal{I}_1 \mathcal{I}_2 \right) + \\
& \left. + b_4^{(\mu_r+1)} a_6^{(\mu_r+1)} \left(\frac{\mathcal{I}_4}{12} + \mathcal{I}_1 \left(\frac{\mathcal{I}_3}{3} + \frac{3}{8} \mathcal{I}_1 \mathcal{I}_2 \right) \right) \right], \tag{3.21} \\
\tilde{A}_3 = & \frac{1}{2 N_{\mu_r+1}^{1/2}} \left[-\frac{1}{3} (b_3^{(\mu_r+1)} + \frac{b_5^{(\mu_r+1)}}{2} \mathcal{I}_1) + \frac{b_3^{(\mu_r+1)} a_6^{(\mu_r+1)}}{2} \left(\frac{\mathcal{I}_3}{9} + \right. \right. \\
& + \frac{\mathcal{I}_1 \mathcal{I}_2}{2} \Big) + \frac{b_4^{(\mu_r+1)} b_5^{(\mu_r+1)}}{2} \left(\frac{\mathcal{I}_3}{3} + \frac{2}{3} \mathcal{I}_1 \mathcal{I}_2 \right) + \frac{b_5^{(\mu_r+1)} a_6^{(\mu_r+1)}}{12} \left(\frac{\mathcal{I}_4}{2} + \right. \\
& \left. + \mathcal{I}_1 \left(\frac{4 \mathcal{I}_3}{3} + \frac{7}{4} \mathcal{I}_1 \mathcal{I}_2 \right) \right) + \frac{b_3^{(\mu_r+1)} b_4^{(\mu_r+1)}}{2} \mathcal{I}_2 \Big], \\
\tilde{A}_4 = & \frac{1}{4 N_{\mu_r+1}} \left[-\frac{1}{6} (b_4^{(\mu_r+1)} + \frac{a_6^{(\mu_r+1)}}{2} \mathcal{I}_1) + \frac{b_4^{(\mu_r+1)}}{4} \mathcal{I}_2 (b_4^{(\mu_r+1)} + \right. \\
& + \frac{7}{6} a_6^{(\mu_r+1)} \mathcal{I}_1) + \frac{(b_5^{(\mu_r+1)})^2}{6} \left(\frac{\mathcal{I}_3}{2} + \mathcal{I}_1 \mathcal{I}_2 \right) + \frac{b_3^{(\mu_r+1)} b_5^{(\mu_r+1)}}{3} \mathcal{I}_2 + \\
& \left. + \frac{a_6^{(\mu_r+1)} b_4^{(\mu_r+1)}}{9} \mathcal{I}_3 + \frac{(a_6^{(\mu_r+1)})^2}{6} \left(\frac{\mathcal{I}_4}{8} + \mathcal{I}_1 \left(\frac{\mathcal{I}_3}{3} + \frac{7}{16} \mathcal{I}_1 \mathcal{I}_2 \right) \right) \right], \\
\tilde{A}_5 = & \frac{1}{24 N_{\mu_r+1}^{3/2}} \left[b_5^{(\mu_r+1)} \left(-\frac{1}{5} + b_4^{(\mu_r+1)} \mathcal{I}_2 \right) + \frac{a_6^{(\mu_r+1)} \mathcal{I}_2}{2} (b_3^{(\mu_r+1)} + \right. \\
& + \frac{b_5^{(\mu_r+1)}}{2} \mathcal{I}_1) + b_5^{(\mu_r+1)} a_6^{(\mu_r+1)} \left(\frac{\mathcal{I}_3}{3} + \frac{\mathcal{I}_1 \mathcal{I}_2}{3} \right) \Big], \\
\tilde{A}_6 = & \frac{1}{48 N_{\mu_r+1}^2} \left[\frac{a_6^{(\mu_r+1)}}{3} \left(-\frac{1}{5} + \frac{a_6^{(\mu_r+1)}}{3} \mathcal{I}_3 \right) + \frac{(b_5^{(\mu_r+1)})^2}{3} \mathcal{I}_2 + \right.
\end{aligned}$$

$$\begin{aligned}
& + \frac{a_6^{(\mu_r+1)}}{2} \mathcal{I}_2(b_4^{(\mu_r+1)} + \frac{a_6^{(\mu_r+1)}}{2} \mathcal{I}_1)], \\
\tilde{A}_7 &= \frac{b_5^{(\mu_r+1)} a_6^{(\mu_r+1)}}{288 N_{\mu_r+1}^{5/2}} \mathcal{I}_2, \\
\tilde{A}_8 &= \frac{(a_6^{(\mu_r+1)})^2}{2304 N_{\mu_r+1}^3} \mathcal{I}_2.
\end{aligned}$$

Expressions for \mathcal{I}_m occurring in (3.21) can be found via the transition to the spherical Brillouin zone and integration over $k \in (0, B_{\mu_r+1}]$ [17,18]. We have

$$\begin{aligned}
\mathcal{I}_1 &= \frac{1}{N_{\mu_r+1}} \sum'_{k \leq B_{\mu_r+1}} \frac{1}{d_{\mu_r+1}(k)} = \mathcal{L}[4 | d_{\mu_r+1}(0) | - \frac{10}{3} \times \\
& \times \frac{(a_4^{(\mu_r+1)})^2}{a_6^{(\mu_r+1)}} (-1 + b_2)]^{-1}
\end{aligned}$$

or

$$\mathcal{I}_1 = \frac{s^{2(\mu_r+1)}}{\beta \tilde{\Phi}(0)} \alpha_1. \quad (3.22)$$

Here,

$$\begin{aligned}
\mathcal{L} &= 3 \frac{x_r - \arctan x_r}{x_r^3}, \quad x_r = \frac{1}{\sqrt{4\bar{r}_{\mu_r+1} - \frac{10}{3} \frac{\bar{u}_{\mu_r+1}^2}{\bar{w}_{\mu_r+1}} (-1 + b_2)}}, \quad (3.23) \\
b_2 &= \sqrt{1 + \frac{6}{5} \frac{\bar{w}_{\mu_r+1} \bar{r}_{\mu_r+1}}{\bar{u}_{\mu_r+1}^2}}, \quad \alpha_1 = \frac{\mathcal{L}}{4\bar{r}_{\mu_r+1} - \frac{10}{3} \frac{\bar{u}_{\mu_r+1}^2}{\bar{w}_{\mu_r+1}} (-1 + b_2)}.
\end{aligned}$$

The quantity \mathcal{I}_2 can be represented in the form:

$$\mathcal{I}_2 = \sum_r g^2(r) = \frac{s^{4(\mu_r+1)}}{(\beta \tilde{\Phi}(0))^2} \alpha_2, \quad (3.24)$$

where

$$\begin{aligned}
g(r) &= \frac{1}{N_{\mu_r+1}} \sum'_{k \leq B_{\mu_r+1}} \frac{e^{i\mathbf{k}\mathbf{r}}}{d_{\mu_r+1}(k)} = \frac{6s^{2(\mu_r+1)}}{\beta \tilde{\Phi}(0)(B_{\mu_r+1}r)^3} \{8\bar{r}_{\mu_r+1} - \\
& - \frac{20}{3} \frac{\bar{u}_{\mu_r+1}^2}{\bar{w}_{\mu_r+1}} (-1 + b_2) + 1\}^{-1} [\sin(B_{\mu_r+1}r) - B_{\mu_r+1}r \cos(B_{\mu_r+1}r)], \\
\alpha_2 &= \alpha_1^2 + 6e_1^2(1 + e_2^2), \quad (3.25) \\
e_1 &= \frac{6}{\pi^2 [8\bar{r}_{\mu_r+1} - \frac{20}{3} \frac{\bar{u}_{\mu_r+1}^2}{\bar{w}_{\mu_r+1}} (-1 + b_2) + 1]}, \\
e_2 &= \frac{1}{2\pi} [\sin(\pi\sqrt{2}) - \pi\sqrt{2} \cos(\pi\sqrt{2})] \approx 0.034861.
\end{aligned}$$

The other $\mathcal{I}_l (l = 3, 4, 5, 6)$ are determined by analogous relations

$$\mathcal{I}_l = \sum_r g^l(r) = \frac{s^{2l(\mu_\tau+1)}}{(\beta\tilde{\Phi}(0))^l} \alpha_l \quad (3.26)$$

with

$$\alpha_l = \alpha_1^l + 6e_1^l \left(1 + \frac{e_2^l}{2^{l/2-1}}\right). \quad (3.27)$$

The values of $\alpha_m (m = 1, 2, 3, 4, 5, 6)$ are given in table 6.

Table 6. Numerical values of α_m .

s	α_1	α_2	α_3	α_4	α_5	α_6
2	0.3456	0.1893	0.0488	0.0151	0.0050	0.0017
2.5	0.2981	0.1406	0.0313	0.0083	0.0024	0.0007
2.7349	0.2821	0.1258	0.0265	0.0067	0.0018	0.0005
3	0.2664	0.1122	0.0223	0.0053	0.0014	0.0004
3.5	0.2412	0.0918	0.0165	0.0036	0.0008	0.0002
3.5862	0.2373	0.0888	0.0158	0.0033	0.0008	0.0002
4	0.2201	0.0764	0.0126	0.0025	0.0005	0.0001

Let us perform the change of the variable ρ_0 in (3.20)

$$\rho_0 = \rho_0' - \sqrt{N}\langle\bar{\sigma}\rangle, \quad (3.28)$$

which cancels terms proportional to odd powers of ρ_0 in the exponent of the integrand. We obtain

$$\begin{aligned} Z_{\mu_\tau+1} &= \exp(-\beta F'_{\mu_\tau+1}) \int \exp[\beta\sqrt{N}\rho_0 h + \tilde{B}\rho_0^2 - \frac{G}{N}\rho_0^4 - \\ &\quad - \frac{D}{N^2}\rho_0^6] d\rho_0, \end{aligned} \quad (3.29)$$

where

$$\begin{aligned} -\beta F'_{\mu_\tau+1} &= N_{\mu_\tau+1} \left\{ \frac{5}{2} \bar{r}_{\mu_\tau+1} (\alpha_1 + \frac{5}{2} \bar{r}_{\mu_\tau+1} \alpha_2) - \right. \\ &\quad - \frac{\alpha_1^2}{8} (\bar{u}_{\mu_\tau+1} + \frac{\bar{w}_{\mu_\tau+1}}{6} \alpha_1) + \frac{\bar{u}_{\mu_\tau+1}^2}{8} [\frac{\alpha_4}{6} + \alpha_1^2 \alpha_2 (\frac{1}{2} + \\ &\quad + \frac{5}{3} (-1 + b_2))] - \frac{5}{4} \bar{r}_{\mu_\tau+1} \alpha_1 \alpha_2 [\bar{u}_{\mu_\tau+1} + \frac{\bar{w}_{\mu_\tau+1}}{4} \alpha_1] + \\ &\quad + \frac{\bar{w}_{\mu_\tau+1}^2}{32} [\frac{\alpha_6}{45} + \frac{\alpha_1^2}{2} (\frac{\alpha_4}{3} + \frac{\alpha_1^2 \alpha_2}{4})] + \\ &\quad \left. + \frac{\bar{u}_{\mu_\tau+1} \bar{w}_{\mu_\tau+1} \alpha_1}{16} [\frac{\alpha_4}{3} + \frac{\alpha_1^2 \alpha_2}{2}] - \right. \end{aligned}$$

$$\begin{aligned}
& -\frac{5}{3} \frac{\bar{u}_{\mu_{\tau+1}}^2}{\bar{w}_{\mu_{\tau+1}}} [(-1+b_2)(\alpha_1 - \bar{u}_{\mu_{\tau+1}}\alpha_2(\frac{\alpha_1}{2} - \frac{10}{3} \frac{\bar{u}_{\mu_{\tau+1}}}{\bar{w}_{\mu_{\tau+1}}})) - \\
& -(7-5b_2)\bar{r}_{\mu_{\tau+1}}\alpha_2] - \frac{1}{2} \ln(\frac{\beta\tilde{\Phi}(0)}{s^{2(\mu_{\tau+1})}}) - \frac{1}{2} \ln[(1+ \\
& +4\bar{r}_{\mu_{\tau+1}} - \frac{10}{3} \frac{\bar{u}_{\mu_{\tau+1}}^2}{\bar{w}_{\mu_{\tau+1}}}(-1+b_2))\pi^{-1}] + \frac{1}{3} - \frac{\mathcal{L}}{3}, \\
\tilde{B} &= \frac{1}{2} c_\nu^2 | \tau |^{2\nu} \beta\tilde{\Phi}(0) \bar{r}_{\mu_{\tau+1}} \bar{B}, \\
G &= c_\nu | \tau |^\nu (\beta\tilde{\Phi}(0))^2 s_0^3 \bar{G}, \\
D &= (\beta\tilde{\Phi}(0))^3 s_0^6 \bar{D}, \\
\bar{B} &= 1 - \frac{\alpha_1}{2\bar{r}_{\mu_{\tau+1}}} (\bar{u}_{\mu_{\tau+1}} + \frac{\bar{w}_{\mu_{\tau+1}}}{4} \alpha_1) + \frac{\bar{u}_{\mu_{\tau+1}}^2}{\bar{r}_{\mu_{\tau+1}}} [\frac{5}{3}(-1+b_2)\alpha_2 \times \\
& \times (\frac{\alpha_1}{2} + \frac{\bar{u}_{\mu_{\tau+1}}}{\bar{w}_{\mu_{\tau+1}}}) + \frac{1}{2}(\frac{\alpha_1\alpha_2}{2} + \frac{\alpha_3}{3})] - \frac{5}{2}\alpha_2(\bar{u}_{\mu_{\tau+1}} + \\
& + \frac{\bar{w}_{\mu_{\tau+1}}\alpha_1}{2}) + \frac{\bar{w}_{\mu_{\tau+1}}^2}{8\bar{r}_{\mu_{\tau+1}}} [\frac{\alpha_5}{15} + \alpha_1(\frac{\alpha_1\alpha_3}{3} + \frac{1}{2}(\frac{\alpha_4}{3} + \frac{\alpha_1^2\alpha_2}{2}))] + \\
& + \frac{\bar{u}_{\mu_{\tau+1}}\bar{w}_{\mu_{\tau+1}}}{2\bar{r}_{\mu_{\tau+1}}} [\frac{\alpha_4}{12} + \alpha_1(\frac{3\alpha_1\alpha_2}{8} + \frac{\alpha_3}{3})], \\
\bar{G} &= \frac{1}{24} [\bar{u}_{\mu_{\tau+1}} + \frac{\bar{w}_{\mu_{\tau+1}}}{2}(\alpha_1 + 5\bar{r}_{\mu_{\tau+1}}\alpha_2)] - \frac{\bar{u}_{\mu_{\tau+1}}^2\alpha_2}{8} [\frac{1}{2} + \frac{5}{9}(-1+ \\
& +b_2)] - \frac{\bar{u}_{\mu_{\tau+1}}\bar{w}_{\mu_{\tau+1}}}{12} (\frac{7}{8}\alpha_1\alpha_2 + \frac{\alpha_3}{3}) - \frac{\bar{w}_{\mu_{\tau+1}}^2}{24} [\frac{\alpha_4}{8} + \\
& + \alpha_1(\frac{7}{16}\alpha_1\alpha_2 + \frac{\alpha_3}{3})], \\
\bar{D} &= \frac{\bar{w}_{\mu_{\tau+1}}}{48} (\frac{1}{15} - \frac{\bar{u}_{\mu_{\tau+1}}}{2}\alpha_2) - \frac{\bar{w}_{\mu_{\tau+1}}^2}{48} (\frac{\alpha_3}{9} + \frac{\alpha_1\alpha_2}{4}).
\end{aligned} \tag{3.30}$$

The quantities $\bar{B}, \bar{G}, \bar{D}$ are given in table 7.

Table 7. Values of $\bar{B}, \bar{G}, \bar{D}, \langle \bar{\sigma} \rangle^{(0)}, \Gamma^+, \Gamma^-$ for different s .

s	\bar{B}	\bar{G}	\bar{D}	$\langle \bar{\sigma} \rangle^{(0)}$	Γ^+	Γ^-
2	0.8296	0.0153	0.0002	3.0942	2.6052	0.2970
2.5	0.8289	0.0206	0.0003	3.0031	3.8147	0.3121
2.7349	0.8277	0.0231	0.0003	2.9669	2.9709	0.3211
3	0.8267	0.0260	0.0003	2.9309	3.3248	0.3318
3.5	0.8266	0.0316	0.0003	2.8730	4.0679	0.3528
3.5862	0.8268	0.0326	0.0003	2.8640	4.2071	0.3565
4	0.8282	0.0376	0.0004	2.8238	4.9254	0.3748

Using the method of the steepest descent for $Z_{\mu_{\tau+1}}$ (3.29), we find

$$Z_{\mu_{\tau+1}} = \sqrt{\frac{2\pi}{E_0''(\bar{\rho})}} \exp[-\beta F_{\mu_{\tau+1}}' - N E_0(\bar{\rho})]. \quad (3.31)$$

Here $\bar{\rho}$ is the extremum point of the expression

$$E_0(\rho) = D\rho^6 + G\rho^4 - \tilde{B}\rho^2 - \beta h\rho \quad (3.32)$$

arising in the exponential of the integrand of (3.29) at the substitution

$$\rho_0 = \sqrt{N}\rho. \quad (3.33)$$

For $E_0(\bar{\rho})$ at $h = 0$ we obtain

$$\begin{aligned} E_0(\bar{\rho}) &= -s^{-3(\mu_{\tau+1})} s_0^{-3} \bar{E}_0, \\ \bar{E}_0 &= \frac{2\bar{G}^3}{27\bar{D}^2} (-1 + \sqrt{1 + \frac{3}{2} \frac{\bar{r}_{\mu_{\tau+1}} \bar{B} \bar{D}}{\bar{G}^2}}) + \frac{\bar{r}_{\mu_{\tau+1}} \bar{B} \bar{G}}{6\bar{D}} \times \\ &\quad \times (-1 + \frac{2}{3} \sqrt{1 + \frac{3}{2} \frac{\bar{r}_{\mu_{\tau+1}} \bar{B} \bar{D}}{\bar{G}^2}}). \end{aligned} \quad (3.34)$$

Having (3.12) and (3.31), we can calculate the contribution to the system free energy at $T < T_c$ from the long-wave phases of the spin moment density fluctuations (3.9):

$$\begin{aligned} F_{IGR} &= -kTN'(\gamma_3^{(\mu_{\tau})} + \gamma_3^{(\sigma)}) | \tau |^{3\nu}, \\ \gamma_3^{(\mu_{\tau})} &= \gamma_g + \gamma_{\rho} = c_{\nu}^3 \bar{\gamma}_3^{(\mu_{\tau})}, \quad \bar{\gamma}_3^{(\mu_{\tau})} = \bar{\gamma}_g + \bar{\gamma}_{\rho}, \\ \gamma_{\rho} &= c_{\nu}^3 \bar{\gamma}_{\rho}, \quad \bar{\gamma}_{\rho} = \frac{5}{2} \bar{r}_{\mu_{\tau+1}} (\alpha_1 + \frac{5}{2} \bar{r}_{\mu_{\tau+1}} \alpha_2) - \frac{\alpha_1^2}{8} \times \\ &\quad \times (\bar{u}_{\mu_{\tau+1}} + \frac{\bar{w}_{\mu_{\tau+1}}}{6} \alpha_1) + \frac{\bar{u}_{\mu_{\tau+1}}^2}{8} [\frac{\alpha_4}{6} + \alpha_1^2 \alpha_2 (\frac{1}{2} + \frac{5}{3} (-1 + b_2))] - \\ &\quad - \frac{5}{4} \bar{r}_{\mu_{\tau+1}} \alpha_1 \alpha_2 [\bar{u}_{\mu_{\tau+1}} + \frac{\bar{w}_{\mu_{\tau+1}}}{4} \alpha_1] + \frac{\bar{w}_{\mu_{\tau+1}}^2}{32} [\frac{\alpha_6}{45} + \frac{\alpha_1^2}{2} \times \\ &\quad \times (\frac{\alpha_4}{3} + \frac{\alpha_1^2 \alpha_2}{4})] + \frac{\bar{u}_{\mu_{\tau+1}} \bar{w}_{\mu_{\tau+1}} \alpha_1}{16} [\frac{\alpha_4}{3} + \frac{\alpha_1^2 \alpha_2}{2}] - \frac{5}{3} \frac{\bar{u}_{\mu_{\tau+1}}^2}{\bar{w}_{\mu_{\tau+1}}} \times \\ &\quad \times [(-1 + b_2)(\alpha_1 - \bar{u}_{\mu_{\tau+1}} \alpha_2 (\frac{\alpha_1}{2} - \frac{10}{3} \frac{\bar{u}_{\mu_{\tau+1}}}{\bar{w}_{\mu_{\tau+1}}})) - (7 - 5b_2) \times \\ &\quad \times \bar{r}_{\mu_{\tau+1}} \alpha_2] - \frac{1}{2} \ln [\frac{1 + 4\bar{r}_{\mu_{\tau+1}} - \frac{10}{3} \frac{\bar{u}_{\mu_{\tau+1}}^2}{\bar{w}_{\mu_{\tau+1}}} (-1 + b_2)}{\pi}] + \frac{1}{3} - \frac{\mathcal{L}}{3}, \\ \gamma_3^{(\sigma)} &= c_{\nu}^3 \bar{\gamma}_3^{(\sigma)}, \quad \bar{\gamma}_3^{(\sigma)} = \bar{E}_0. \end{aligned} \quad (3.35)$$

The quantity $\gamma_3^{\mu_{\tau}}$ determines the free energy after the exit from the CR region, and $\gamma_3^{(\sigma)}$ determines the free energy of the ordering. The values of $\bar{\gamma}_g, \bar{\gamma}_{\rho}, \bar{\gamma}_3^{(\mu_{\tau})}, \bar{\gamma}_3^{(\sigma)}$ are given in table 8.

Table 8. Values of $\bar{\gamma}_g, \bar{\gamma}_\rho, \bar{\gamma}_3^{(\mu_\tau)}, \bar{\gamma}_3^{(\sigma)}$.

s	$\bar{\gamma}_g$	$\bar{\gamma}_\rho$	$\bar{\gamma}_3^{(\mu_\tau)}$	$\bar{\gamma}_3^{<\sigma>}$
2	-0.3024	1.0386	0.7362	1.7618
2.5	-0.0869	1.0269	0.9399	2.0456
2.7349	0.0039	1.0227	1.0265	2.1572
3	0.0986	1.0179	1.1164	2.2808
3.5	0.2579	1.0076	1.2655	2.5154
3.5862	0.2832	1.0056	1.2888	2.5563
4	0.3967	0.9955	1.3922	2.7538

The entropy, internal energy and specific heat of the system corresponding to the IGR region read

$$\begin{aligned}
S_{IGR} &= S_{\mu_\tau} + S_{\langle\sigma\rangle}, \quad U_{IGR} = U_{\mu_\tau} + U_{\langle\sigma\rangle}, \quad C_{IGR} = C_{\mu_\tau} + C_{\langle\sigma\rangle}, \\
S_\eta &= -kN' |\tau|^{1-\alpha} u_3^{(\eta)}, \quad U_\eta = -kTN' |\tau|^{1-\alpha} u_3^{(\eta)}, \\
C_\eta &= kN' c_3^{(\eta)} |\tau|^{-\alpha}, \quad u_3^{(\eta)} = 3\nu\gamma_3^{(\eta)}, \quad c_3^{(\eta)} = 3\nu(3\nu - 1)\gamma_3^{(\eta)}.
\end{aligned} \tag{3.36}$$

The index η can take two values: μ_τ and $\langle\sigma\rangle$.

4. The order parameter of the system

The mean spin moment is the order parameter of the model investigated. It is related to the presence of the nonzero value of the CV ρ_0 at which the integrand of (3.29) has an extremum. Performing the substitution (3.33) in that integrand, we obtain

$$Z_{\mu_\tau+1} = e^{-\beta F'_{\mu_\tau+1}} \sqrt{N} \int e^{-N E_0(\rho)} d\rho, \tag{4.1}$$

where $E_0(\rho)$ is given in (3.32). Owing to the factor N in the exponent in (4.1), the integrand has a sharp maximum at the point $\bar{\rho}$ corresponding to the equilibrium value of the order parameter. The value of $\bar{\rho}$ can be found from the extremum condition $\frac{\partial E_0(\rho)}{\partial \rho} = 0$ or

$$6D\bar{\rho}^5 + 4G\bar{\rho}^3 - 2\tilde{B}\bar{\rho} - \beta h = 0. \tag{4.2}$$

In the case $h = 0$ we obtain a biquadratic equation, which is reduced by means of substitution

$$\bar{\rho}^2 = y \tag{4.3}$$

to the equation

$$6Dy^2 + 4Gy - 2\tilde{B} = 0. \tag{4.4}$$

Extracting the temperature dependence, we obtain an equation for the mean spin moment $\langle \sigma \rangle = \bar{\rho} = \sqrt{y}$:

$$\begin{aligned} \langle \sigma \rangle &= |\tau|^\beta \langle \sigma \rangle^{(0)}, \quad \beta = \nu/2, \\ \langle \sigma \rangle^{(0)} &= c_\nu^{1/2} (\beta \tilde{\Phi}(0))^{-1/2} s_0^{-3/2} \langle \bar{\sigma} \rangle^{(0)}, \\ \langle \bar{\sigma} \rangle^{(0)} &= \left[\frac{\bar{G}}{3\bar{D}} (-1 + \sqrt{1 + \frac{3\bar{r}_{\mu_\tau+1}\bar{B}\bar{D}}{\bar{G}^2}}) \right]^{1/2}. \end{aligned} \quad (4.5)$$

The value of $\langle \bar{\sigma} \rangle^{(0)}$ is given in table 7.

The susceptibility per particle χ can be found from equation (4.2) by differentiating it with respect to \mathcal{H} and using the relation $\chi = \mu_B \frac{\partial \langle \sigma \rangle}{\partial \mathcal{H}}$:

$$\chi = \frac{\beta \mu_B^2}{30D\bar{\rho}^4 + 12G\bar{\rho}^2 - 2\tilde{B}}. \quad (4.6)$$

Separating the temperature dependence in the coefficients D , G , \tilde{B} (see (3.30)), we obtain a final expression for the susceptibility.

5. Thermodynamics of the system in the vicinity of the phase transition point

Having calculated the contributions to the system free energy from the short-wave and long-wave modes of the spin moment density oscillations both above and below T_c , we can compute the total free energy

$$F = \begin{cases} F_0 + F_{CR} + F_{LGR}, & T > T_c, \\ F_0 + F_{CR} + F_{IGR}, & T < T_c, \end{cases}$$

the entropy, internal energy and specific heat.

We obtain the total free energy of the system at $h = 0$ taking (2.17), (2.23) and (3.6), (3.35) into account:

$$F = \begin{cases} -kTN'[\gamma_0 + \gamma_1\tau + \gamma_2\tau^2 + \gamma_3^+\tau^{3\nu}], & T > T_c, \\ -kTN'[\gamma_0 + \gamma_1|\tau| + \gamma_2|\tau|^2 + \gamma_3^-|\tau|^{3\nu}], & T < T_c, \end{cases} \quad (5.1)$$

where (see table 9)

$$\begin{aligned} \gamma_0 &= \gamma_0^{(CR)} + s_0^3 \ln 2, \\ \gamma_3^+ &= -\gamma_3^{(CR)+} + f_{LGR}, \\ \gamma_3^- &= -\gamma_3^{(CR)-} + \gamma_{IGR}, \quad \gamma_{IGR} = \gamma_3^{(\mu_\tau)} + \gamma_3^{(\sigma)}. \end{aligned} \quad (5.2)$$

The coefficients γ_3^\pm can be written as a product of the universal part $\bar{\gamma}_3^\pm$ (table 9) and non-universal factor c_ν^3 depending on the microscopic parameters of the Hamiltonian (the lattice constant c , the effective interaction radius b and the value $\tilde{\Phi}(0)$ of the interaction potential Fourier transform at $k = 0$):

$$\begin{aligned} \gamma_3^\pm &= c_\nu^3 \bar{\gamma}_3^\pm, \\ \bar{\gamma}_3^+ &= -\gamma^+ + \bar{f}_{LGR} = -\gamma_+ + \bar{f}_{HO} + \bar{f}', \\ \bar{\gamma}_3^- &= -\gamma^- + \bar{\gamma}_g + \bar{\gamma}_\rho + \bar{\gamma}_3^{(\sigma)}. \end{aligned} \quad (5.3)$$

Table 9. Coefficients γ_0 , γ_3^\pm and $\bar{\gamma}_3^\pm$.

s	γ_0	$\bar{\gamma}_3^+$	γ_3^+	$\bar{\gamma}_3^-$	γ_3^-
2	61.1798	0.9699	2.9033	1.7599	5.2680
2.5	61.1878	1.5898	3.2734	2.4612	5.0675
2.7349	61.1930	1.8654	3.3020	2.7650	4.8944
3	61.1999	2.1770	3.2783	3.1073	4.6793
3.5	61.2150	2.8206	3.1856	3.8086	4.3013
3.5862	61.2179	2.9445	3.1704	3.9423	4.2448
4	61.2325	3.6167	3.1179	4.6608	4.0180

We have for the entropy, internal energy and specific heat of the system

$$\begin{aligned}
S &= \begin{cases} kN'[s^{(0)} + c_0\tau + u_3^+\tau^{1-\alpha}], & T > T_c, \\ kN'[s^{(0)} - c_0|\tau| - u_3^-|\tau|^{1-\alpha}], & T < T_c, \end{cases} \\
U &= \begin{cases} kTN'[\gamma_1 + u_1\tau + u_3^+\tau^{1-\alpha}], & T > T_c, \\ kTN'[\gamma_1 - u_1|\tau| - u_3^-|\tau|^{1-\alpha}], & T < T_c, \end{cases} \\
C &= \begin{cases} kN'[c_0 + c_3^+\tau^{-\alpha}], & T > T_c, \\ kN'[c_0 + \bar{c}_3^-|\tau|^{-\alpha}], & T < T_c \end{cases}
\end{aligned} \tag{5.4}$$

with coefficients given by the relations

$$\begin{aligned}
s^{(0)} &= \gamma_0 + \gamma_1, \\
u_3^\pm &= c_\nu^3 \bar{u}_3^\pm, \quad \bar{u}_3^\pm = 3\nu \bar{\gamma}_3^\pm, \\
u_1 &= 2\gamma_2 + \gamma_1, \\
c_3^\pm &= c_\nu^3 \bar{c}_3^\pm, \quad \bar{c}_3^\pm = 3\nu(3\nu - 1) \bar{\gamma}_3^\pm.
\end{aligned} \tag{5.5}$$

The formula for the specific heat can also be rewritten as

$$\begin{aligned}
C/kN' &= \frac{A^\pm}{\alpha} |\tau|^{-\alpha} + B^\pm, \\
A^\pm &= c_\nu^3 \alpha \bar{c}_3^\pm, \quad B^\pm = c_0.
\end{aligned} \tag{5.6}$$

The plus and minus signs correspond to $T > T_c$ and $T < T_c$, respectively.

The system susceptibility per particle at infinitely small values of the external field \mathcal{H} at $T > T_c$ ($\chi = -\frac{1}{N} \frac{\partial^2 F_{LGR}}{\partial \mathcal{H}^2}$, see (2.23)) and $T < T_c$ (see (4.6)) is given by

$$\chi = \begin{cases} \Gamma^+ \tau^{-\gamma} \frac{\mu_B^2}{\Phi(0)}, & T > T_c, \\ \Gamma^- |\tau|^{-\gamma} \frac{\mu_B^2}{\Phi(0)}, & T < T_c. \end{cases} \tag{5.7}$$

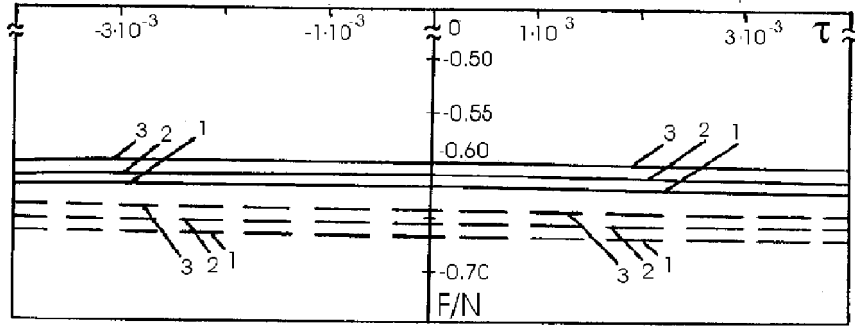


Figure 4. Temperature dependence of the free energy of the system for different values of the RG parameter s within the frames of the ρ^6 model (solid lines). For comparison we show the free energy of the system in the quartic basis distribution approximation with allowance for confluent corrections [9] (dashed lines). (1) $s=2$, (2) $s=2.5$, (3) $s=3$.

Here, (see table 7),

$$\begin{aligned}\Gamma^+ &= 2c_\nu^{-2}\bar{\gamma}_4^+, \\ \Gamma^- &= c_\nu^{-2}\left\{\frac{10}{3}\frac{\bar{G}^2}{\bar{D}}(-1 + \sqrt{1 + \frac{3}{2}\frac{\bar{r}_{\mu\tau+1}\bar{B}\bar{D}}{\bar{G}^2}})\left(\frac{1}{5} + \sqrt{1 + \frac{3}{2}\frac{\bar{r}_{\mu\tau+1}\bar{B}\bar{D}}{\bar{G}^2}}\right) - \right. \\ &\quad \left. - \bar{r}_{\mu\tau+1}\bar{B}\right\}^{-1}, \\ \gamma &= 2\nu.\end{aligned}\quad (5.8)$$

Plots of the temperature dependence of the system free energy F/N , entropy S/kN , specific heat C/kN , mean spin moment $\langle\sigma\rangle$ (4.5), susceptibility χ (5.7) at $s = 2, 2.5, 3$ are shown in figures 4-8.

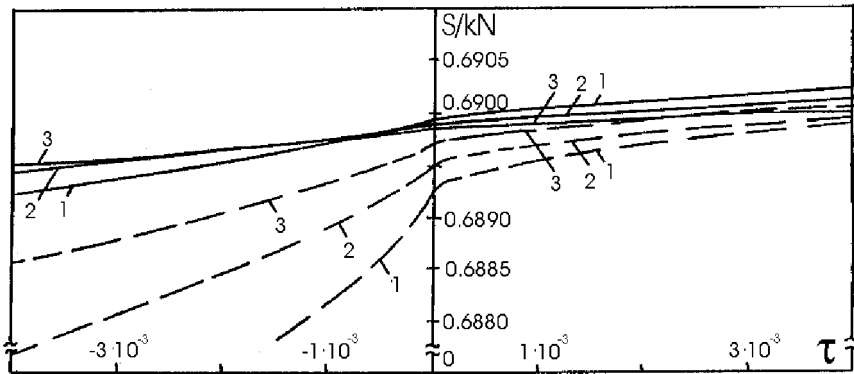


Figure 5. Dependence of the system entropy on τ (Notations are the same as in figure 4).

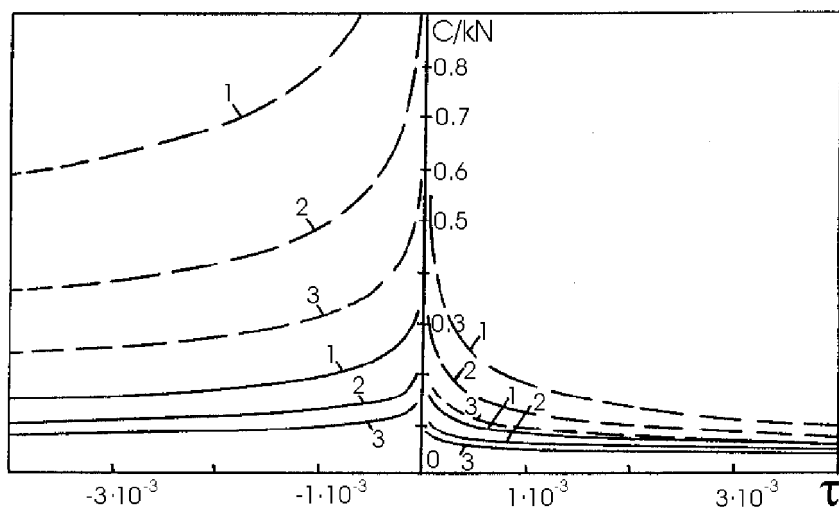


Figure 6. Specific heat of the system (Notations are the same as in figure 4).

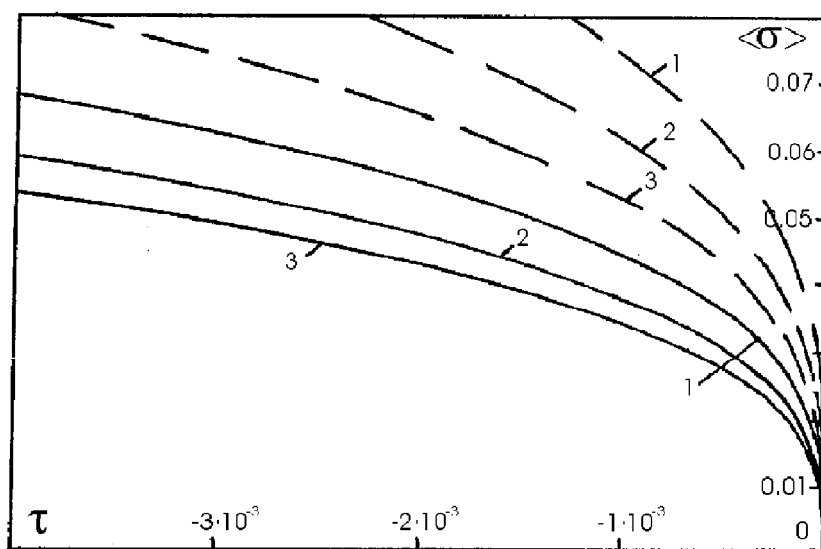


Figure 7. Mean spin moment $\langle \sigma \rangle$ at $T \leq T_c$ (Notations are the same as in figure 4).

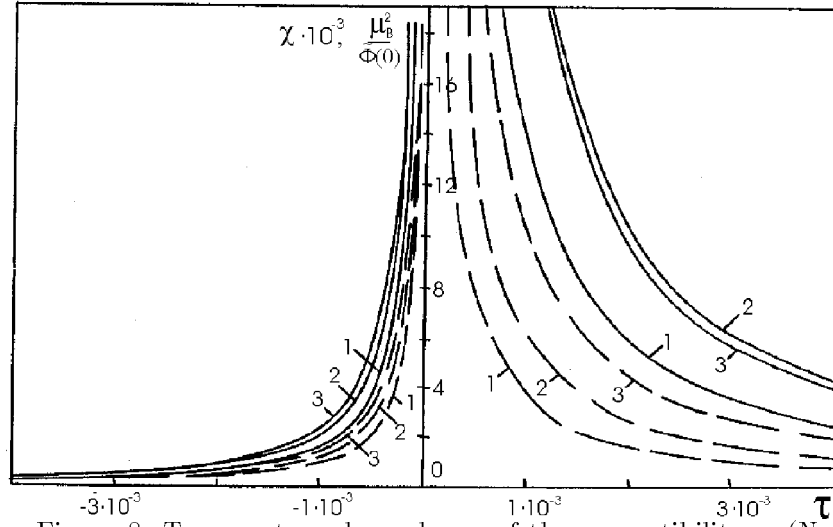


Figure 8. Temperature dependence of the susceptibility χ (Notations are the same as in figure 4).

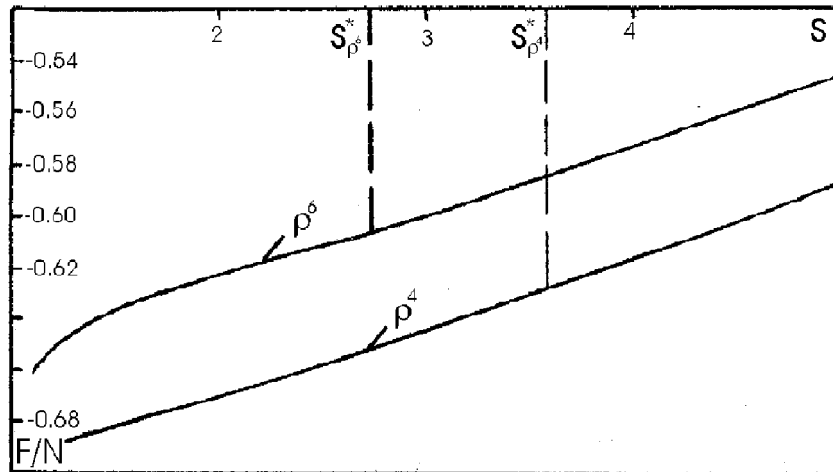


Figure 9. Behaviour of F/N as a function of s for ρ^4 and ρ^6 models.

The corresponding plots of the thermodynamic characteristics calculated within the ρ^4 model, with the confluent corrections [9] being taken into account, are also given here (dashed curves). Comparison of these plots shows that the dependence of the thermodynamic functions on the parameter s for the ρ^6 model is weaker than for the ρ^4 model. The dependence of F/N on s for these two models is represented in figure 9.

Let us note that the calculations performed are best suited for the intermediate values of s , close to the quantity s^* , at which h_n turns to zero at the fixed point ($s^*=2.7349$ for the ρ^6 model and $s^*=3.5862$ for the ρ^4 model). The use of the difference form of the RR based on a non-Gaussian measure density works especially well for this region of s . At small values of s ($s \rightarrow 1$), some complications arise when the unit element is extracted. In this limit, the RR should be represented as the perturbation series with respect to the Gaussian distribution (h_n is large at $s \rightarrow 1$, and expansions in h_n^{-2} , α_n can be used [19,20]). There also exists an upper limit for s . At large s , one must take into account the correction due to the potential averaging [10, 21], which increases with s .

The point $s \approx s^*$ corresponds to the beginning of the $\nu(s)$ curve stabilization [3]. Table 10 contains the values of the critical exponents $\nu, \alpha, \beta, \gamma$, the exponent of the scaling correction $\Delta_1 = -\ln E_2 / \ln E_1$, and the ratios of the critical amplitudes $A^+/A^-, \Gamma^+/\Gamma^-$ and their combinations $P = \frac{1}{\alpha}[1 - \frac{A^+}{A^-}]$, $R_c^+ = A^+\Gamma^+ / [s_0^3(\langle\sigma\rangle^{(0)})^2]$ at $s = s^*$ for ρ^4 and ρ^6 models. These values are in agreement with the data obtained within the field theory approach (FTA) [22-27] and high-temperature series (HTS) [28-32].

Table 10. Values of the critical exponents, ratios of the critical amplitudes and their combinations for $s = s^*$ ($s^*=2.7349$ for the ρ^6 model and $s^*=3.5862$ for the ρ^4 model) obtained by means of the CV method. Data calculated within the field theory approach (FTA) [22-27] and high-temperature series (HTS) [28-32].

Quantity	Model		Ref. data	
	ρ^4	ρ^6	FTA	HTS
ν	0.605	0.637	0.630	0.638
α	0.185	0.088	0.110	0.125
β	0.303	0.319	0.325	0.312
γ	1.210	1.275	1.241	1.250
Δ_1	0.463	0.525	0.498	0.50
A^+/A^-	0.435	0.675	0.54, 0.48	0.51
Γ^+/Γ^-	6.967	9.253	4.77, 5.12	5.07
P	3.054	3.711	3.90, 4.03, 4.2, 4.72	
R_c^+	0.098	0.162	0.059, 0.052	0.059

Conclusions

The method for the calculation of the three-dimensional Ising model thermodynamics is developed in the sixfold distribution approximation. Both temperature regions above and below the critical value of T_c are considered. The main distinguishing feature of the approach is a separate allowance for contributions from the short- and long-wave fluctuation phases of the spin moment density to the free energy of the system near T_c . Within the framework of the ρ^6 model, we obtained the explicit expressions for the critical amplitudes of the thermodynamic functions of the three-dimensional Ising ferromagnet and calculated the coefficients of the free energy, the universal characteristics (the critical exponents, the ratios of the critical amplitudes). Calculation of the free energy, entropy, specific heat, mean spin moment, susceptibility was performed for different values of s . Comparison of the results obtained within ρ^4 and ρ^6 models indicates that the dependence of thermodynamic functions on the RG parameter s is weaker for the ρ^6 model.

The values of s close to s^* are optimal for the method presented. Obtaining analytical expressions for the critical amplitudes and system thermodynamic characteristics as functions of the Hamiltonian microscopic parameters is the advantage of the method developed. The leading critical amplitudes for the specific heat and other thermodynamic characteristics are represented as a product of the universal part, independent of microscopic parameters, and the non-universal factor, which depends on these parameters.

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МЕТОД РОЗРАХУНКУ ТЕРМОДИНАМІКИ ТРИВИМІРНОГО ІЗІНГІВСЬКОГО ФЕРОМАГНЕТИКА В РАМКАХ МОДЕЛІ ρ^6

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Здійснено розрахунок термодинамічних функцій тривимірного ізінгівського феромагнетика вище і нижче критичної температури в наближенні базисного розподілу, який включає шосту степінь змінної (модель ρ^6). Порівняння з результатами для моделі ρ^4 свідчить про послаблення залежності термодинамічних функцій від параметра ренормгрупи s . Вказаний оптимальний інтервал значень параметра ренормгрупи.