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Damping rate of a massive fermion in a hot medium*

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In a hot system every excitation acquires a finite lifetime, manifesting itself in a non-zero spectral width. Ordinary damping as well as quantum memory effects arise from this nontrivial spectral function. This report presents a new method for the self-consistent calculation of the spectral width of a fermion coupled to massless bosons of scalar, vector and pseudoscalar type. In accordance with the known procedures of Quantum Electrodynamics, the self-consistent summation of the corresponding Fock diagram eliminates all infrared divergences although the bosons are not screened at all. The solutions for the fermion damping rate are analytical in the coupling constant g, but not analytical in the temperature parameter, i.e., $\gamma \propto g^2 T + \mathcal{O}(g^4 T \log(T/M))$.

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1. Introduction

The *damping rate* of a fermion moving through a hot system, i.e., the width of its spectral function, is an interesting quantity for a wide range of physical systems. Some examples are electrons and holes in semiconductor crystals, or particles in the early universe as well as in heavy-ion collisions [1]. In such systems, the fermion damping rate allows us to estimate the relaxation time in collisions as well as the quantum memory time [2] and radiation properties [3].

The damping rate of *massive* fermions interacting with *massless* bosons is an especially interesting problem in this context. From the phenomenological point of view this importance is due to the fact that all fundamental forces of nature involve massless gauge bosons. From the theoretical standpoint a similarly big interest lies in the infrared divergence associated with this particular problem.

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It arises from the fact, that fermions interacting with strictly massless bosons may emit an infinite number of such bosons with infinitely small energy. Such a concept is of course meaningless in the real world, and consequently the solution of the problem has to connect the abstract world of quantum physics with reality. At zero temperature such an infrared divergence occurs in QED as well as in QCD, and its removal (or regularization) has become a standard textbook content. Generally, the argument used is the finite energy resolution of every physical measuring apparatus.

At non-zero temperature, however, the question of the gauge-independent, causality preserving and therefore *physical* removal of the infrared divergence is far from settled. Most of the schemes discussed so far involve momentum scale cutoff factors or quasi-particles with "magnetic mass" [4]. As will be argued in the next section, such schemes violate several fundamental rules of quantum theory.

The present paper attempts a new solution to the infrared problem in hot systems, motivated by one of the few mathematically rigorous theorems of finite temperature quantum theory. This NRT theorem due to Narnhofer and Thirring states that particle-like excitations with infinite lifetime can exist in a hot system only if they do not interact [5].

The use of particle-like excitations in a perturbative scheme, which are asymptotically stable and only have a temperature dependent "mass" is therefore unjustified *a priori* and can only be obtained as a limiting case at the very end of all calculations.

Still deeper is a mathematical reason that the symmetry group of space and time in thermal states is not the Poincaré group as in the vacuum state, but rather a product of SO(3) and the four-dimensional translation group [6]. Its irreducible representations can be interpreted as having a continuous mass spectrum, i.e., the concept of a mass-shell (and thereby of stable asymptotic states) is not well-defined at non-zero temperature.

The solution of this problem has been pointed out by Landsman [7], using earlier work of Licht [8] and Wightman [9] and leading to a perturbative expansion in terms of *generalized free fields* without a mass-shell. To account for the proper temporal boundary conditions in a thermal system, these generalized free fields have to be embedded in a description with doubled Hilbert space, i.e., the single-particle propagators are 2×2 matrices [10]. Two flavours exist for such a formalism, the Schwinger-Keldysh (or closed time-path) method [11] and the method called thermo field dynamics (TFD) [12]. Within the latter this merger of different aspects of finite temperature field theory has been discussed in ref. [13], which also introduces the notation used in the following.

The mathematical framework used is a relativistic quantum field theory, however, as was pointed out above, the results apply to a wide range of physical systems also in the low energy sector.

The present paper is organized as follows: First, some general remarks are made about nonequilibrium propagators in terms of their spectral function. These remarks are then used to obtain well-defined approximations to the spectral functions of bosons and fermions in a hot medium.

In section 3 the problem of calculating one-loop diagrams with effective propa-

gators is discussed, and how they may be used for a self-consistent determination of the spectral function. Sections 4 and 5 are rather technical and are devoted to analytical approximations for the Fock self-energy function.

In section 6 these results are assembled, and consequently one obtains approximate solutions for the self-consistent Fock approximation, i.e., for the full fermion propagator calculated with a one-loop self-energy function at finite temperature. Finally these results, which are different from those obtained by other methods, are used to draw some conclusions.

2. Spectral functions

Appropriate for a description of dynamical phenomena in a thermal system is a formalism with 2×2 matrix valued Green functions. In thermo field dynamics (TFD) this "doubling" arises naturally because TFD contains two different (anti-) commuting representations of the canonical (anti-) commutation relations.

In the Schwinger-Keldysh method the doubling is introduced because a full description of statistical systems can be achieved only when using causal and anti-causal Green functions as well as Wigner functions. Consequently, the full propagator matrix is

$$S^{(ab)}(x,x') = -\mathrm{i} \begin{pmatrix} \left\langle \mathrm{T}\left[\psi_x \overline{\psi}_{x'}\right]\right\rangle & -\left\langle \overline{\psi}_{x'}\psi_x \right\rangle \\ \left\langle \psi_x \overline{\psi}_{x'}\right\rangle & \left\langle \widetilde{\mathrm{T}}\left[\psi_x \overline{\psi}_{x'}\right]\right\rangle \end{pmatrix}, \tag{1}$$

where $\langle \ldots \rangle$ denotes the statistical average, $\mathrm{T}[\ldots]$ the time ordered and $\widetilde{\mathrm{T}}[\ldots]$ the "anti-time ordered" product. The same matrix valued propagator is obtained in TFD (see [13,14] for detailed discussions of this equivalence).

By construction the propagator matrix obeys the linear relation $S^{11} + S^{22} - S^{12} - S^{21} = 0$. Furthermore, in an equilibrium state it has to fulfil the Kubo-Martin-Schwinger condition [15], i.e., an anti-periodic boundary condition in the imaginary time direction. In momentum space the KMS condition then reads [10,13,14]

$$\left(1 - n_{\mathrm{F}}(p_0)\right) S^{12}(p_0,\vec{p}) + n_{\mathrm{F}}(p_0) S^{21}(p_0,\vec{p}) = 0. \tag{2}$$

S^{12} and S^{21} are the Green functions without time ordering (Wigner functions) and $n_{\mathrm{F}}(E)$ is the fermion equilibrium distribution function at a given temperature, the Fermi-Dirac function

$$n_{\mathrm{F}}(E) = \frac{1}{\mathrm{e}^{\beta E} + 1}. \tag{3}$$

An almost identical KMS relation holds for bosons, but here the fields are periodic in the imaginary time direction. Consequently, occupation number factors are given as Bose-Einstein function

$$n_{\mathrm{B}}(E) = \frac{1}{\mathrm{e}^{\beta E} - 1}. \tag{4}$$

For simplicity only the case of zero chemical potential is discussed.

In interacting systems, the KMS boundary conditions lead to a *causal* propagator which has a cut along the real energy axis. Its analytical structure therefore is not easily understood, and especially when combining several of such propagators in a perturbative scheme one has to implement more or less complicated cutting rules to understand pieces of diagrams *physically* [16,17].

It is therefore generally a safe method to use only retarded and advanced propagators, and to condense the matrix structure of equation (1) into the vertices where such propagators join. This amounts to a *diagonalization scheme* for the matrix (1), which has been described in several publications [13,14].

2.1. Retarded and advanced propagators

The retarded and advanced propagators are, by definition, analytical functions of the energy parameter in the upper or lower complex energy half plane. Analytical functions obey the Kramers-Kronig relation, and this implies that the retarded propagator is known completely if only its imaginary part (or spectral function) $\mathcal{A}_{\rm F}$ is known along the real axis. Hence, for arbitrary complex E

$$S^{\rm R,A}(E,\vec{p}) = \int\limits_{-\infty}^{\infty} {\rm d}E'\ \mathcal{A}_{\rm F}(E',\vec{p})\ \frac{1}{E-E'\pm {\rm i}\epsilon}. \tag{5}$$

Trivially, the spectral function is recovered for real E,

$$\mathcal{A}_{\rm F}(E,\vec{p}) = \mp\frac{1}{\pi}\,{\rm Im}(S^{\rm R,A}(E,\vec{p})) = \frac{1}{2\pi{\rm i}}\left(S^{\rm A}(E,\vec{p}) - S^{\rm R}(E,\vec{p})\right). \tag{6}$$

The diagrammatic rules to combine these propagators in the calculation of physical quantities are well established. In terms of the spectral function $\mathcal{A}_{\rm F}(E,\vec{p})$, the matrix valued propagator of equation (1) can be expressed as [13,14]

$$\begin{aligned} S^{(ab)}(p_0,\vec{p}) = & \int\limits_{-\infty}^{\infty} {\rm d}E\,\mathcal{A}_{\rm F}(E,\vec{p}) \\ & \times \tau_3\left(\mathcal{B}(n_{\rm F}(E))\right)^{-1} \begin{pmatrix} \dfrac{1}{p_0-E+{\rm i}\epsilon} & \\ & \dfrac{1}{p_0-E-{\rm i}\epsilon} \end{pmatrix} \mathcal{B}(n_{\rm F}(E)). \end{aligned} \tag{7}$$

τ_3 is the diagonal Pauli matrix, and the transformation matrix $\mathcal{B}$ is

$$\mathcal{B}(n_{\rm F}(E)) = \begin{pmatrix} (1-n_{\rm F}(E)) & -n_{\rm F}(E) \\ 1 & 1 \end{pmatrix}. \tag{8}$$

It is a matter of a few lines to show that this matrix-valued propagator obeys relation (2) for each spectral function.

A similar relation holds for the boson case, see [13,14] for details of the corresponding $\mathcal{B}$. For both cases, the transformation matrices $\mathcal{B}$ have a special meaning in the TFD formalism, where they play the role of a thermal Bogoliubov transformation. However, the explicit form (5) of the propagator is the same in the Schwinger-Keldysh method.

2.2. Normalization and locality

The spectral function has got two features which are intimately related to fundamental requirements of quantum field theory. Here they are commented on only for the fermionic case, and refer to [18] for the case of bosons. Firstly, the quantization rules for fields,

$$\left\{\psi(t,\vec{x}),\psi^\dagger(t,\vec{y})\right\} = \delta^3(\vec{x}-\vec{y}\,) \tag{9}$$

require that the spectral function is *normalized*

$$\int_0^\infty \mathrm{d}E\,\mathrm{Tr}\left[\gamma^0\,\mathcal{A}_\mathrm{F}(E,\vec{p}\,)\right] = 2. \tag{10}$$

The second important feature of the spectral function is that its four dimensional Fourier transform into coordinate space must vanish for space-like arguments. This is equivalent to the Wightman axiom of *locality*, i.e., field operators must (anti-) commute for space-like separations in Minkowski space [19]:

$$\begin{aligned} C_\mathrm{F}(x,y) &= \left\langle\left\{\psi(x),\psi^\dagger(y)\right\}\right\rangle \\ &= \int\frac{\mathrm{d}E\,\mathrm{d}^3\vec{p}}{(2\pi)^3}\,\mathrm{e}^{-\mathrm{i}(E(x_0-y_0)-\vec{p}(\vec{x}-\vec{y}\,))}\,\mathcal{A}_\mathrm{F}(E,\vec{p}\,) \\ &= 0 \quad \text{if } x-y \text{ space-like.} \end{aligned} \tag{11}$$

In an interacting many-body system, one may very well expect non-locality in a causal sense: Wiggling the system at one side will certainly effect the other side after some time. The locality axiom ensures that this effect does not occur over space-like separations, i.e., faster than a physical signal can propagate. Thus, to distinguish between the *causal* non-locality and the violation of the locality axiom, the latter will henceforth be denoted as a violation of *causality*.

It is trivial to establish that normalization and locality axiom are satisfied for free fermions of mass M. Defining their on-shell energy as $\omega_\mathrm{p} = \sqrt{\vec{p}^{\,2}+M^2}$, the free spectral function is obtained as

$$\mathcal{A}^0_\mathrm{F}(E,\vec{p}\,) = \left(E\gamma^0 + \vec{p}\vec{\gamma} + M\right)\,\mathrm{sign}(E)\,\delta(E^2-\omega_\mathrm{p}^2). \tag{12}$$

Without loss of generality one may set $y=0$ in the free fermionic anti-commutator function, and obtain

$$C^0_\mathrm{F}(x,0) = (\mathrm{i}\partial_\mu\gamma^\mu + M)\,Z(x_0,\vec{x}), \tag{13}$$

where

$$Z(x_0,\vec{x}) = \frac{-\mathrm{i}}{4\pi}\left[\delta(x_0^2-\vec{x}^{\,2}) - \Theta(x_0^2-\vec{x}^{\,2})\frac{M}{2\sqrt{x_0^2-\vec{x}^{\,2}}}\,J_1\left(M\sqrt{x_0^2-\vec{x}^{\,2}}\right)\right], \tag{14}$$

and J_1 is a Bessel function of the first kind. Clearly this is zero for space-like arguments, i.e., for $|\vec{x}| > |x_0|$. However, from this prescription follows that for a general spectral function the locality axiom is not automatically guaranteed, a careful check is necessary in any application involving spectral functions.

2.3. Ghost poles

For an interacting system, the full fermion propagator and thus also the spectral function is defined in terms of a self energy function Σ:

$$\mathcal{A}_{\mathrm{F}}(p_0,\vec{p}) = \mp\frac{1}{\pi}\,\mathrm{Im}\left[\left(p_\mu\gamma^\mu - M - \Sigma^{\mathrm{R,A}}(p_0,\vec{p})\right)^{-1}\right]. \tag{15}$$

Note, that also the self-energy function is Dirac matrix valued, i.e., this equation contains the inversion of a 4×4 matrix.

However, the real part of the full propagator is *not* given by the real part of the quantity in square brackets of (15). The reason is, that according to Weinberg's theorem [20] the self-energy function in a relativistic theory behaves as

$$\Sigma(p_0,\vec{p}) \propto p_0\,\left(\log\left(\frac{p_0}{M}\right)\right)^n, \quad p_0\to\infty, \tag{16}$$

i.e., it is more than linearly divergent. This implies, in general, the presence of unwanted poles when the interaction is not asymptotically free. For fermions, these Landau ghost poles appear at four points in the complex energy plane [21, p. 636].

Since their appearance contradicts the definition of the retarded/advanced propagator as being free of poles at least in one half plane, they must be considered unphysical. For a sufficiently large coupling, they have a big effect on the propagator along the real axis, while for small coupling they are far away from the physical region. Hence, their consistent removal is mandatory for strongly interacting systems, and the only way to do so is by calculating the real part of the propagator by a dispersion integral according to (5).

It was pointed out in reference [22], that this indeed conforms to a *non-perturbative* correction to the propagator, in the sense that the correction term is not an analytical function of the coupling constant. In general, it will be of order $\exp(-1/g^2)$ instead. Hence, equation (5) is the only safe method to calculate a relativistic propagator, and the input needed for this calculation is a physically meaningful spectral function which obeys all the rules discussed here.

To demonstrate briefly *how* the removal of unphysical poles is achieved through a dispersion integral, consider a bosonic toy model spectral function of Lorentzian type which has got a peak along the real axis at some value ω_{p}

$$\mathcal{A}_0(E,\vec{p}) = \frac{1}{\pi}\,\frac{\gamma_{\mathrm{p}}}{\left(E-\omega_{\mathrm{p}}\right)^2+\gamma_{\mathrm{p}}^2}. \tag{17}$$

By definition, this spectral function may be used only for real energy arguments, and does not take on negative values. If this were violated, the probabilistic interpretation of the quantum (field) theory behind this formulation would be invalid.

The above function has a trivial expansion into simple poles in the complex plane,

$$\mathcal{A}_0(E,\vec{p}) = \frac{1}{2\pi\mathrm{i}}\left(\frac{1}{E-\omega_{\mathrm{p}}-\mathrm{i}\gamma_{\mathrm{p}}} - \frac{1}{E-\omega_{\mathrm{p}}+\mathrm{i}\gamma_{\mathrm{p}}}\right). \tag{18}$$

The real part of the corresponding propagator is obtained through

$$G_0^{\mathrm{R}}(p_0,\vec{p}) = \int\limits_{-\infty}^{\infty} \mathrm{d}E\, \mathcal{A}_0(E,\vec{p})\, \frac{1}{p_0 - E + \mathrm{i}\epsilon}. \tag{19}$$

The integrand is expanded into simple poles,

$$\frac{1}{E - \omega_{\mathrm{p}} \pm \mathrm{i}\gamma_{\mathrm{p}}}\, \frac{1}{p_0 - E + \mathrm{i}\epsilon} = \frac{1}{p_0 - (\omega_{\mathrm{p}} \mp \mathrm{i}\gamma_{\mathrm{p}})} \left(\frac{1}{E - \omega_{\mathrm{p}} \pm \mathrm{i}\gamma_{\mathrm{p}}} + \frac{1}{p_0 - E + \mathrm{i}\epsilon} \right). \tag{20}$$

The E-integration is convergent, and closing the integral in the upper complex energy half plane then picks up the residues of the corresponding poles. First consider the case $\mathrm{Im}(p_0) \geqslant 0$, where the equal sign is permitted because of the $\mathrm{i}\epsilon$ terms in the denominator of the dispersion integral. Clearly, the integrand then has three poles in the upper complex half plane, but two of the residues are cancelled.

In the second case $\mathrm{Im}(p_0) < 0$ the integrand has only one pole in the upper complex half plane. Assembled together the integral gives

$$G_0^{\mathrm{R}}(p_0,\vec{p}) = \begin{cases} \dfrac{1}{p_0 - (E_{\mathrm{p}} - \mathrm{i}\gamma_{\mathrm{p}})}\,, & \mathrm{Im}(p_0) \geqslant 0 \\ \dfrac{1}{p_0 - (E_{\mathrm{p}} + \mathrm{i}\gamma_{\mathrm{p}})}\,, & \mathrm{Im}(p_0) < 0 \end{cases}, \tag{21}$$

which is free of poles on both sides of the real axis. This may be cast into the statement, that retarded and advanced propagator have a common analytical continuation.

A similar effect occurs, when a self-energy function according to (16) is used, i.e., when ghost poles are present in a perturbative propagator: The dispersion integral always shifts complex (ghost) poles to the unphysical Riemann sheet.

As a matter of fact it has been shown very long ago in quantum field theory, that this is the only scenario compatible with causality: The retarded as well as the advanced propagator should be free of poles in the complex energy plane on both sides of the real energy axis [23].

2.4. Approximate spectral functions

The toy model spectral function of equation (17) cannot be used in a consistent calculation, since it propagates only the states of positive energy. More useful is a symmetric ansatz, which for bosons reads

$$\mathcal{A}_{\mathrm{B}}(E,\vec{k}) = \frac{1}{2\omega_k} \left(\mathcal{A}_0(E,\vec{k}) - \mathcal{A}_0(-E,\vec{k}) \right) = \frac{1}{\pi}\, \frac{2E\gamma_k}{\left(E^2 - \Omega_k^2\right)^2 + 4E^2\gamma_k^2}, \tag{22}$$

with $\Omega_k^2 = \omega_k^2 + \gamma_k^2$. This spectral function is normalized according to the canonical field commutation relations, i.e.,

$$\int\limits_0^{\infty} \mathrm{d}E\, E\, \mathcal{A}_{\mathrm{B}}(E,\vec{k}) = \frac{1}{2}\,. \tag{23}$$

Note, that in this ansatz the dependence of γ_k on the momentum of the relativistic boson is completely arbitrary, and one may also introduce a more general relationship between ω_k and momentum than in the free case.

The approximation of the full propagator by such an approximate spectral function amounts to the approximation of the denominator of a relativistic retarded boson propagator along the real axis as

$$k_\mu k^\mu - m^2 - \Pi^{\mathrm{R}}(k) = (k_0 - (\omega_k - \mathrm{i}\gamma_k))\,(k_0 + (\omega_k + \mathrm{i}\gamma_k)), \tag{24}$$

where Π is the boson self-energy function, and solving this equation for ω_k, γ_k. An example for such an approximation has been given in reference [13], and there the momentum dependence of ω_k as well as of γ_k was quite strong for the case of a pseudoscalar coupling between bosons and fermions.

Note at this point, that a *general* momentum dependence of the γ_k-parameter does not guarantee the causality of the model, i.e., it may violate the locality axiom as discussed above. For a physically correct model, it is mandatory that the Fourier transform of the spectral function vanishes for space-like coordinate arguments. Indeed one may show, that for the ansatz equation (22) this is achieved when considering an ω_k^2 which is quadratic in $\vec{k}$ and a constant γ_k.

This ansatz may be extended to the fermionic case. The spectral function then is slightly more complicated because of its Dirac matrix structure. In equation (10) the normalization was given for a fermion spectral function. The simplest ansatz compatible with this normalization, as well as with known properties of fermion systems, would be to multiply the spectral function from equation (22) by a factor $(E\gamma^0 + \omega_{\mathrm{p}})$.

However, as one may show, such an ansatz violates the locality axiom, therefore it is not useful for a consistent calculation. Rather, one has to use a more complicated spectral function,

$$\begin{aligned}
\mathcal{A}_{\mathrm{F}}(E,\vec{p}) &= \frac{\gamma_{\mathrm{p}}}{\pi}\frac{\gamma^0\left(E^2+\omega_{\mathrm{p}}^2+\gamma_{\mathrm{p}}^2\right)+2E\vec{\gamma}\vec{p}+2EM}{\left(E^2-\omega_{\mathrm{p}}^2-\gamma_{\mathrm{p}}^2\right)^2+4E^2\gamma_{\mathrm{p}}^2} \\
&= \frac{1}{4\pi\mathrm{i}\omega_{\mathrm{p}}}\left(\frac{\omega_{\mathrm{p}}\gamma^0+\vec{p}\vec{\gamma}+M}{E-\omega_{\mathrm{p}}-\mathrm{i}\gamma_{\mathrm{p}}}-\frac{-\omega_{\mathrm{p}}\gamma^0+\vec{p}\vec{\gamma}+M}{E+\omega_{\mathrm{p}}-\mathrm{i}\gamma_{\mathrm{p}}}\right. \\
&\qquad \left.-\frac{\omega_{\mathrm{p}}\gamma^0+\vec{p}\vec{\gamma}+M}{E-\omega_{\mathrm{p}}+\mathrm{i}\gamma_{\mathrm{p}}}+\frac{-\omega_{\mathrm{p}}\gamma^0+\vec{p}\vec{\gamma}+M}{E+\omega_{\mathrm{p}}+\mathrm{i}\gamma_{\mathrm{p}}}\right),
\end{aligned} \tag{25}$$

where in principle γ_{p} may be momentum dependent, ${\Omega_{\mathrm{p}}}^2 = {\omega_{\mathrm{p}}}^2 + {\gamma_{\mathrm{p}}}^2$ and M is a constant which may be different from the physical fermion mass in the vacuum state.

As argued in the previous subsection, the Fourier transform of the spectral function has an important physical interpretation. Note, that in the special case where γ is independent of momentum the 4-dimensional Fourier transform of equation (25) is

$$C_{\mathrm{F}}(x,0) = \mathrm{e}^{-\gamma\,|x_0|}\left(\mathrm{i}\gamma^\mu\frac{\partial}{\partial x_\mu}+M\right)\,Z(x_0,\vec{x}), \tag{26}$$

with the function Z as defined in equation (13). Hence, for momentum independent γ and M, the locality axiom is fulfilled and interactions in the system are causal. One therefore has to limit the use of the spectral function (25) to this approximation, i.e.,

$$\begin{aligned} &\gamma_{\mathrm{p}} \equiv \gamma = \text{const}, \\ &\omega_{\mathrm{p}}^2 \equiv \omega^2 = \vec{p}^{\,2} + M^2, \\ &\quad \Rightarrow \left(\omega(\vec{p}+\vec{k})\right)^2 = \omega^2 + 2|\vec{p}|\,|\vec{k}|\eta + \vec{k}^2, \end{aligned} \tag{27}$$

where η is the cosine of the angle between the two momenta $\vec{p}$ and $\vec{k}$.

3. Fermion damping rate

The next step is to use spectral functions, together with the definition of the complete propagator by dispersion integral, in a "perturbative" expansion at finite temperature. One finds, that "perturbative" is certainly not the correct label for such a skeleton expansion, because in general due to the properties of the dispersion integral the results will *not* be expandable into power series in the coupling parameters (see section 2.3). Indeed up to correlation diagrams and vertex corrections, a perturbative expansion in terms of *full* propagators is exact already at the one-loop level [21, p. 476].

For simplicity, first consider a simple scalar coupling of a boson field having the spectral function $\mathcal{A}_{\mathrm{B}}$ to a fermion field having the spectral function $\mathcal{A}_{\mathrm{F}}$. In equations (39) and (40), some generalizations are investigated.

The "one-loop" diagram for the fermion self-energy, the Fock diagram, is given by the integral

$$\Sigma^{\mathrm{R}}(p_0,\vec{p}) = g^2 \int\frac{\mathrm{d}^3\vec{k}}{(2\pi)^3}\int_{-\infty}^{\infty}\mathrm{d}E\mathrm{d}E'\;\mathcal{A}_{\mathrm{F}}(E,\vec{p}+\vec{k})\,\mathcal{A}_{\mathrm{B}}(E',\vec{k})\;\left(\frac{n_{\mathrm{B}}(E')+n_{\mathrm{F}}(E)}{p_0+E'-E+\mathrm{i}\epsilon}\right). \tag{28}$$

This function is split into real and imaginary part as $\Sigma^{\mathrm{R}} = \mathrm{Re}\Sigma - \mathrm{i}\pi\Gamma$, and for the latter one obtains

$$\Gamma(p_0,\vec{p}) = g^2 \int\frac{\mathrm{d}^3\vec{k}}{(2\pi)^3}\int_{-\infty}^{\infty}\mathrm{d}E\;\mathcal{A}_{\mathrm{F}}(E+p_0,\vec{p}+\vec{k})\,\mathcal{A}_{\mathrm{B}}(E,\vec{k})\;\left(n_{\mathrm{B}}(E)+n_{\mathrm{F}}(E+p_0)\right). \tag{29}$$

To fulfil the Kubo-Martin-Schwinger boundary condition (2) for the fermion propagator including the above self-energy function, it is *absolutely essential* that both propagators in the above expression have the same equilibrium temperature. In other words, one may *not* disregard the fermion distribution function $n_{\mathrm{F}}(E+p_0)$ in the above expression in any case.

The corresponding "one-loop" result for the retarded scalar boson self-energy is, according to reference [13]

$$\Pi^{\mathrm{R}}(k_0,\vec{k}) = g^2 \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \int_{-\infty}^{\infty} \mathrm{d}E \mathrm{d}E' \, \mathrm{Tr}\left[\mathcal{A}_{\mathrm{F}}(E,\vec{p}+\vec{k})\mathcal{A}_{\mathrm{F}}(E',\vec{p})\right] \left(\frac{n_{\mathrm{F}}(E')-n_{\mathrm{F}}(E)}{k_0+E'-E+\mathrm{i}\epsilon}\right). \tag{30}$$

The imaginary part of $\Pi^{\mathrm{R}} = \mathrm{Re}\Pi - \mathrm{i}\pi\sigma$ is explicitly

$$\sigma(k_0,\vec{k}) = g^2 \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \int_{-\infty}^{\infty} \mathrm{d}E \, \mathrm{Tr}\left[\mathcal{A}_{\mathrm{F}}(E,\vec{p}+\vec{k})\mathcal{A}_{\mathrm{F}}(E-k_0,\vec{p})\right] (n_{\mathrm{F}}(E-k_0)-n_{\mathrm{F}}(E)). \tag{31}$$

The above imaginary part vanishes at $k_0 = 0$, more generally when the external energy parameter equals the boson chemical potential. This relation is *violated* if different propagators for the fermions are used on the two legs of the diagram, e.g., when inserting different "thermal masses" for the two legs [24]. This violation in turn leads to a violation of the Kubo-Martin-Schwinger (KMS) boundary conditions for the full boson propagator, i.e., it is *not* an equilibrium Green function.

In principle, one could now introduce a self-consistent calculation scheme: The expressions (28) and (30) are calculated with an approximate spectral function, then used as input to equation (15) and a similar equation for the full boson propagator in terms of its self-energy. This gives new spectral functions, which may be used again to determine self energies. Such a scheme has been used in refs. [25,26], and based on these papers one might hope that a few iterations are enough to determine the spectral function numerically quite well.

However, as pointed out, there is no way to guarantee that in such a scheme causality (in the representation of the locality axiom) is preserved. Moreover, in an entirely numerical scheme it would be impossible to point out the path to the solution of the infrared problem. Two strategies exist which may be used to circumvent this difficulty.

The first strategy is to enforce locality after each iterative step. This can be done by folding the resultant spectral functions with the Fourier transform of $\Theta(t^2-\vec{x}^{\,2})$, which makes this scheme numerically much more difficult than even the iterative procedure used in [25,26].

The second strategy is to use a simple causality-preserving parametrization of the spectral functions. The parameters are then determined self-consistently in a closed scheme. Apart from the possibility to achieve analytical approximations, this strategy also has the virtue that one can use it as an input to the first one. Thus, for obvious reasons this second course will be used henceforth.

3.1. Approximations to the boson spectral function

As was pointed out above, and is well known for many years, the boson propagator may have isolated poles only on the real energy axis, but not away from it on the physical Riemann sheet [23]. Furthermore it may exhibit cuts along the real

energy axis due to self-energy corrections with continuous non-zero imaginary part like in equation (31).

In the absence of condensation phenomena these self-energy corrections have got a zero imaginary part when the boson energy parameter is equal to the chemical potential. Consequently, the product of a Bose-Einstein distribution function (4) and a continuous part of the boson spectral function is always *infrared finite*.

The only possible source for an infrared divergence, therefore, are poles on the real axis, corresponding to particles with infinite lifetime. As argued above, such poles may not exist in a finite temperature system [5], and thus it is on the one hand clear that infrared divergences in finite temperature field theory are not present in the full theory but a mere artifact of perturbation theory.

On the other hand it is nevertheless instructive to study *how* these divergences are removed within a proper calculational scheme. Therefore, momentarily one may choose the worst case for a boson spectral function, corresponding to a kind of electromagnetic interaction without screening or damping:

$$\mathcal{A}^0_{\mathrm{B}} = \frac{1}{2k}\ (\delta(E-k) - \delta(E+k)) \tag{32}$$

with $k = |\vec{k}|$. It is obvious, that the product of this spectral function with the corresponding (particle plus antiparticle) Bose-Einstein distribution function is highly singular for $E \to 0$. In particular, the infrared dominant piece of the imaginary part (29) of the self-energy function is

$$\Gamma(p_0, \vec{p}\,) \approx g^2 \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3}\, \frac{T}{2k^2} \left(\mathcal{A}_{\mathrm{F}}(p_0 + k, \vec{p} + \vec{k}) + \mathcal{A}_{\mathrm{F}}(p_0 - k, \vec{p} + \vec{k}) \right) \mathrm{e}^{-k/T}, \tag{33}$$

i.e., the integral is *infrared finite* as long as the fermion spectral function is finite. That it is also ultraviolet finite is not obvious from the expansion made here, but follows from equation (29).

Various methods exist to obtain approximate propagators or spectral functions. However, these methods either violate the NRT theorem, or causality. Introducing a modified (i.e., *less* singular) spectral function for the bosons, of course, modifies the result for Γ quantitatively, but only one direction for such a modification is possible since the calculation of Γ is dominated by the infrared sector. Any less singular spectral function than (32) therefore diminishes the value for Γ, because it leads to a "smearing" of the Bose pole with some distribution.

Consequently, one may consider the values, obtained in the following, an upper bound on the full calculation with screened massless bosons – as well as a quite good approximation to the case of "unscreened static magnetic fields".

3.2. Approximations to the fermion spectral function

Suppose, one really would approximate the full spectral function according to equation (15) by the simple parametrization of equation (25). This then would

amount to equate the denominators on the real energy axis as

$$\begin{aligned}\left(p_\mu - V_\mu(p) + \mathrm{i}\pi\Gamma^v_\mu(p)\right)^2 - \left(M + S(p) - \mathrm{i}\pi\Gamma^s(p)\right)^2 \\ = \left(p_0 - (\omega_\mathrm{p} - \mathrm{i}\gamma_\mathrm{p})\right)\left(p_0 + (\omega_\mathrm{p} + \mathrm{i}\gamma_\mathrm{p})\right). \end{aligned} \tag{34}$$

The functions of momentum appearing here are the components of the retarded self-energy function, split into real and imaginary part according to

$$\begin{aligned}\Sigma^\mathrm{R}(p) &= \mathrm{Re}\Sigma(p) - \mathrm{i}\pi\Gamma(p), \\ \mathrm{Re}\Sigma(p) &= S(p) + V_\mu(p)\gamma^\mu\ , \\ \Gamma(p) &= \Gamma^s(p) + \Gamma^v_\mu(p)\gamma^\mu. \end{aligned} \tag{35}$$

In the above expressions, the self-energy functions have been split into a Lorentz scalar and a Lorentz vector piece, henceforth simply abbreviated as scalar and vector part.

It was stated above, that in general this may give rise to a momentum dependent γ_p as well as to a non-quadratic dependence of ω_p on $\vec{p}$. Consequently, to check for the locality axiom, i.e., the preservation of causality in the interactions of the system, requires great numerical effort and one would lose all the virtues that come with a simple parametrization of $\mathcal{A}_\mathrm{F}$. It is therefore useful to study first, how the choice of spectral function (25) affects the imaginary part of the self-energy (29). Clearly it has got only two independent components, since its Lorentz scalar and vector parts are

$$\begin{aligned}\Gamma^v_0(p) &= \Gamma^\mathrm{I}(p_0, \vec{p}, \gamma), \\ \Gamma^v_i(p) &= \frac{\vec{p}_i}{M}\,\Gamma^\mathrm{II}(p_0, \vec{p}, \gamma), \\ \Gamma^s(p) &= \Gamma^\mathrm{II}(p_0, \vec{p}, \gamma). \end{aligned} \tag{36}$$

To study a slow *massive* fermion on its effective mass-shell, i.e., for $p_0 = \omega_\mathrm{p} = \sqrt{\vec{p}^2 + M^2}$, one may therefore make the approximation that the real part of the self energy function may be absorbed into the mass parameter M.

For the scalar coupling considered here, the self-consistency equation obtained from equation (34) reads

$$\gamma_\mathrm{S} = \pi\Gamma^\mathrm{I}(\omega, \vec{p}, \gamma_\mathrm{S}) + \pi\Gamma^\mathrm{II}(\omega, \vec{p}, \gamma_\mathrm{S})\,\left(1 - \frac{\vec{p}^2}{M^2}\right)\,\frac{M}{\omega}. \tag{37}$$

According to the assumption, the left side does not depend on the momentum $\vec{p}$. While one certainly cannot hope that the momentum dependence on the right side is cancelled completely, one may nevertheless expect that it possesses an expansion of the type

$$\gamma_\mathrm{S} = \nu_0 + \nu_1\frac{\vec{p}^2}{M^2} + \nu_2\frac{\vec{p}^4}{M^4} + \ldots. \tag{38}$$

Consequently, the approximations made here are reasonable if the coefficients in this expansion are at least of the same magnitude – and they should be considered

a failure when e.g. $|\nu_0| \ll |\nu_1|$. Checking this condition, therefore, will provide an a-posteriori test of the assumptions.

It is quite simple to generalize this scheme to the exchange of a massless vector boson, if the boson propagator is taken in Feynman gauge [21, p. 329]: The inclusion of Dirac matrices γ^μ, γ_μ at the vertices of the Fock diagram simply multiplies Γ^v by a factor –2 and Γ^s by a factor 4. The resulting *vector boson* self consistency relation is

$$\gamma_{\mathrm{V}} = -2\pi\Gamma^{\mathrm{I}}(\omega,\vec{p},\gamma_{\mathrm{V}}) + 4\pi\Gamma^{\mathrm{II}}(\omega,\vec{p},\gamma_{\mathrm{V}}) \left(1+\frac{\vec{p}^{\,2}}{2M^2}\right)\frac{M}{\omega}. \tag{39}$$

It is worthwhile to note, that the ansatz of a momentum independent imaginary part of the fermion self-energy ensures the gauge invariance of this self-consistent damping rate in the limit $\vec{p} \to 0$. Gauge invariance even of the first momentum dependent correction term requires to introduce a vertex correction into the equation (29) to satisfy the correct Ward identity [27].

Physically interesting is also the result for massless *pseudoscalar bosons*, where one has

$$\gamma_{\mathrm{P}} = \pi\Gamma^{\mathrm{I}}(\omega,\vec{p},\gamma_{\mathrm{V}}) - \pi\Gamma^{\mathrm{II}}(\omega,\vec{p},\gamma_{\mathrm{P}}) \left(1+\frac{\vec{p}^{\,2}}{M^2}\right)\frac{M}{\omega}. \tag{40}$$

3.3. Angular integrations

The integration over the angle between the two momenta $\vec{k}$ and $\vec{p}$ may be done analytically. Using the above spectral functions, one obtains as the γ^0-proportional imaginary part of the self-energy

$$\begin{aligned}\Gamma^{\mathrm{I}}(\omega,\vec{p},\gamma) = \\ \frac{g^2\,\gamma^0\,\gamma}{4\pi^3}\int_0^\infty \mathrm{d}k\,k\Big\{\Big[\big((k+\omega)^2 - k\omega + \gamma^2/2\big)\;I_1(k+\omega) + pk\,I_2(k+\omega)\Big] \\ \times\Big(n_{\mathrm{B}}(k) + n_{\mathrm{F}}(k+p_0)\Big) \\ -\Big[\big((k-\omega)^2 + k\omega + \gamma^2/2\big)\;I_1(k-\omega) + pk\,I_2(k-\omega)\Big] \\ \times\Big(n_{\mathrm{B}}(-k) + n_{\mathrm{F}}(-k+p_0)\Big)\Big\}.\end{aligned} \tag{41}$$

I_1 and I_2 are integrals over the angle between the momenta $\vec{p}$ and $\vec{k}$, and $k = |\vec{k}|$, $p = |\vec{p}|$ (see appendix A for technical details and the definition of the functions I_1 and I_2).

A similar expression can be found for the second (scalar) piece of the self-energy function,

$$\begin{aligned}\Gamma^{\mathrm{II}}(\omega,\vec{p},\gamma) &= \frac{g^2\,M\,\gamma}{4\pi^3}\int_0^\infty \mathrm{d}k\,k\Big\{(k+\omega)\,I_1(k+\omega)\Big(n_{\mathrm{B}}(k) + n_{\mathrm{F}}(k+p_0)\Big) \\ &\quad + (k-\omega)\,I_1(k-\omega)\Big(n_{\mathrm{B}}(-k) + n_{\mathrm{F}}(-k+p_0)\Big)\Big\}.\end{aligned} \tag{42}$$

The infrared problem addressed above finds its representation in the fact, that in the limit $\gamma \to 0$, I_1 and I_2 behave like a *negative* power $\leqslant -2$ of k. This becomes obvious when expanding the integrand around the value $k = 0$.

To check the conjecture of infrared finiteness made after equation (33), one may perform this expansion in the above expressions and obtains for the quantity in curly brackets of equation (41), at the point $p_0 = \omega$:

$$\lim_{k\to 0} \left\{ \dots \right\}_{p_0=\omega} = \gamma^0 \frac{T}{\gamma} \frac{\omega^2 + \gamma^2/2}{\omega^2 + \gamma^2/4} + \mathcal{O}(k^2). \tag{43}$$

Consequently, the infrared divergence of the fermion damping rate is removed by starting with a small non-zero γ.

4. "Vacuum" state

In this section, the temperature independent part of the Bose-Einstein and Fermi-Dirac distribution functions is used to calculate contributions to the self energy function of fermions. In this limit, $n^0_{\rm B}(E) = -\Theta(-E)$ and $n^0_{\rm F}(E) = \Theta(-E)$.

One may argue, that these are "vacuum" contributions, therefore not present in a properly renormalized theory. However, the propagators employed here are *not* the ordinary vacuum propagators for the fermion field, rather, they contain a non-zero damping rate as input. Only in the very end one may be able to find a self-consistent solution of zero damping rate in a "true" vacuum state.

Using the above step functions in the expression for the imaginary part of the self-energy leads to the simpler form

$$\Gamma^{\rm I}_{\rm vac}(\omega, \vec{p}, \gamma) = \frac{g^2\,\gamma}{4\pi^3} \int_0^\infty {\rm d}k\, k \left[\left((k-\omega)^2 + k\omega + \gamma^2/2\right)\, I_1(k-\omega) + pk\, I_2(k-\omega) \right] \Theta(\omega - k). \tag{44}$$

This part of the self-energy function has an asymptotic behaviour according to Weinberg's theorem [20]: The above integral is ultraviolet finite, but the corresponding real part is divergent and has to be renormalized. It is connected to the above imaginary part only through a *subtracted* dispersion relation. This asymptotic behaviour of the self-energy function for large values of p_0 implies, that the ghost problem discussed in section 2.3 becomes relevant.

In the light of our introductory remarks to this section it is instructive to study the integral in case of zero γ and momentum, where it becomes

$$\begin{aligned} \Gamma^{\rm I}_{\rm vac}(p_0, 0, 0) &= \frac{g^2}{4\pi^3} \frac{\pi}{8} p_0 \frac{(p_0^2 + 2Mp_0 - M^2)^2 (p_0^2 - M^2)}{p_0^4 (p_0^2 + M^2)} \Theta\left(|p_0| - M\right) \\ &\to \frac{g^2}{4\pi^3} \frac{\pi}{8} p_0, \quad p_0 \to \infty. \end{aligned} \tag{45}$$

This imaginary part vanishes at $p_0 = M$, more generally at $p_0 = \omega$ where the spectral width is calculated. We put it bluntly: Zero input γ gives zero output $\Gamma_{\rm vac}$ for "on-shell" particles.

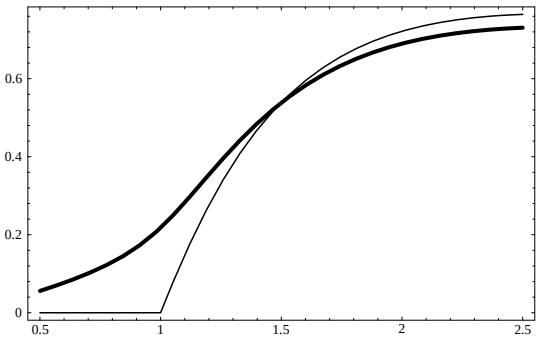

Figure 1. Imaginary part $\Gamma^{\mathrm{I}}_{\mathrm{vac}}$ of the self-energy function. Temperature $T = 0$, momentum $p = 0$, factor $g^2/(4\pi^3)$ set to one. Different constant values of γ: thin line: $\gamma = 0$, equation (45); thick line: $\gamma = 0.2M$

However, this holds no longer, once the integral in (44) is calculated with a non-zero positive γ as input. Instead, one obtains a non-zero imaginary part also at the point $p_0 = \omega$. In figure 1, $\Gamma^{\mathrm{I}}_{\mathrm{vac}}(p_0, 0, \gamma)$ is plotted for two different values of γ: At $p_0 = M$, the imaginary part of the Fock self-energy function is zero only in case $\gamma = 0$, otherwise it is positive.

One may therefore complete the statement above: Non-Zero input γ gives non-zero output Γ_{vac} even at $p_0 = \omega$.

One may now exploit the virtue of a simply parametrized spectral function for fermions, by expanding the continuous function Γ_{vac} around its value at $\gamma = 0$. The imaginary part of the self energy function with $p_0 = \omega$ is not an analytical function of γ: It has a cut in the complex γ-plane, starting at $\gamma = 0$. Thus, a Taylor expansion in γ is *not* possible, one has to perform an asymptotic expansion, explicitly taking into account the leading non-analyticity.

Various techniques exist for such an expansion, for the purpose of the present paper the leading terms were isolated by first substituting $k \to x\gamma$ and then expanding the integrand in powers of γ. It turns out, that for finite momentum p the scale for the expansion is $\omega^2 - p^2$ – which implies, that this asymptotic expansion is reliable only up to momenta $p \approx M$.

Within this limitation, the result for the vector imaginary part of the Fock self-energy function is

$$\begin{aligned}
\Gamma^{\mathrm{I}}_{\mathrm{vac}}(\omega, \vec{p}, \gamma) \;=\; & -\frac{g^2}{4\pi^3}\left\{\frac{\gamma}{4}\,\frac{\omega^2}{\omega^2-p^2}\,\left[\log\left(\frac{\gamma^2}{\omega^2-p^2}\right) - 1 + \frac{p}{\omega}\log\left(\frac{\omega+p}{\omega-p}\right)\right]\right. \\
& - \frac{\gamma^2\,\pi}{8}\,\frac{\omega(\omega^2+3p^2)}{(\omega^2-p^2)^2} \\
& + \frac{\gamma^3}{24}\frac{1}{(\omega^2-p^2)^3}\left[\frac{1}{6}\left(15\omega^4 + 122\omega^2p^2 - 9p^4\right)\right. \\
& \quad - \left(9\omega^4 + 20\omega^2p^2 + 3p^4\right)\,\log\left(\frac{\gamma^2}{\omega^2-p^2}\right) \\
& \quad \left.\left. - \frac{\omega}{p}\,\left(2\omega^4 + 18\omega^2p^2 + 12p^4\right)\,\log\left(\frac{\omega+p}{\omega-p}\right)\right]\right\} \\
& + \mathcal{O}(\gamma^4),
\end{aligned} \tag{46}$$

where $p = |\vec{p}|$ and $k = |\vec{k}|$. In figure 2, this expansion in γ is compared to the full numerical calculation of equation (44): With each additional order that is included, the quality of the approximation grows.

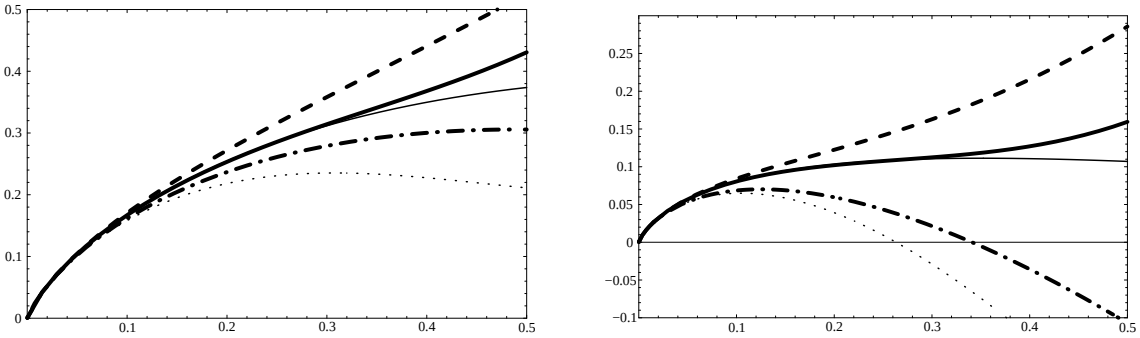

Figure 2. Imaginary part $\Gamma_{\rm vac}$: Numerical calculation vs. analytical approximation. Left panel: vector piece $\Gamma^{\rm I}_{\rm vac}$; right panel: scalar piece $\Gamma^{\rm II}_{\rm vac}$. Temperature $T = 0$, energy $p_0 = \omega$, momentum $p = 0.5M$, factor $g^2/4\pi^3$ set to one. Continuous thin line: full numerical calculation; dash-dotted thick line: expansion to order γ; dashed thick line: expansion to order γ^2; continuous thick line: expansion to order γ^3; dotted thin line: expansion to order γ^3, but *without* the term of order γ^2.

An important aspect of this expansion is the occurrence of the quadratic term: In a naive view of the integral involved, it is not present because $I_1(k-p_0)$ as well as $I_2(k-p_0)$ are odd functions of γ. However, as pointed out above, the expanded function is not analytical in γ – and this effect causes the appearance of the quadratic contribution. To demonstrate this, figure 2 also contains a curve where this quadratic term has been left out. Obviously in this case the third order contribution makes the approximation worse instead of improving it.

In view of the above results one has to conclude, that for not too large values of γ the asymptotic expansion to *second order* is well under control. Henceforth, to keep the results simple, the discussion is restricted to this next-to-leading log order.

A similar expansion is performed for the second piece of the imaginary part of the fermion self-energy. According to (42) one obtains in the vacuum

$$\Gamma^{\rm II}_{\rm vac}(\omega,\vec{p},\gamma) = -M\,\frac{g^2}{4\pi^3}\int\limits_0^\infty {\rm d}k\,k\,\gamma\,(k-\omega)\,I_1(k-\omega)\,\Theta(\omega-k), \tag{47}$$

leading consequently to

$$\begin{aligned}\Gamma^{\rm II}_{\rm vac}(\omega,\vec{p},\gamma) \;=\; & -\frac{g^2}{4\pi^3}\Bigg\{\frac{\gamma}{4}\,\frac{M\omega}{\omega^2-p^2}\,\left[\log\left(\frac{\gamma^2}{\omega^2-p^2}\right)+\frac{\omega}{p}\log\left(\frac{\omega+p}{\omega-p}\right)\right]\\ & -\frac{\gamma^2\,\pi}{8}\,\frac{M(3\omega^2+p^2)}{(\omega^2-p^2)^2}\\ & +\frac{\gamma^3}{8}\frac{M}{\omega(\omega^2-p^2)^3}\,\Bigg[\frac{1}{6}\left(21\omega^4+14\omega^2p^2-3p^4\right)\\ & -4\omega^2(\omega^2+p^2)\,\log\left(\frac{\gamma^2}{\omega^2-p^2}\right)\end{aligned}$$

$$- \frac{\omega}{p}\left(\omega^4 + 6\omega^2 p^2 + p^4\right) \log\left(\frac{\omega+p}{\omega-p}\right)\Bigg]\Bigg\} + \mathcal{O}(\gamma^4). \tag{48}$$

Note again, that as a sign of the non-analyticity in the parameter γ, a quadratic as well as a $\gamma\log(\gamma)$ contribution appear. In figure 2, the asymptotic expansion is compared to the fully numerical calculation of the above integral. Similar to the self-energy piece $\propto \gamma^0$, three aspects of this asymptotic expansion are noteworthy:

1. The approximation systematically improves from first to third order of γ, which proves that the non-analyticity was treated consistently.

2. The quadratic contribution (due to the non-analyticity of the integral) is necessary to provide a meaningful third order result.

3. For $\gamma/M \leqslant 0.1$, the use of a first order asymptotic expansion is sufficient.

5. Thermal state

In this section, the explicitly temperature dependent part of the distribution functions is used to perform a similar approximation as in the previous section. Contributions to the full imaginary part of the self-energy function according to (29) therefore contain an explicit temperature dependence.

For calculational convenience they are split of the complete expressions according to

$$\begin{aligned}\Gamma(p_0,\vec{p},\gamma) &= \Gamma_{\text{vac}}(p_0,\vec{p},\gamma) + \Gamma_T(p_0\vec{p},\gamma)\\ &= \Gamma_{\text{vac}}(p_0,\vec{p},\gamma) + \Gamma_T^{\text{I}}(p_0,\vec{p},\gamma)\gamma^0 + \Gamma_T^{\text{II}}(p_0,\vec{p},\gamma)\left(1+\frac{\vec{p}\vec{\gamma}}{M}\right),\end{aligned} \tag{49}$$

similar to the decomposition for the vacuum part only in equation (36). For the moment it seems quite hopeless to obtain an *analytical* approximation to the full expression (41) valid for all temperatures. However, for not too high temperatures the Bose-Einstein and Fermi-Dirac distribution function may be expanded around their zero-temperature value,

$$\begin{aligned} n_{\text{B}}(k) + n_{\text{F}}(p_0+k) &\approx \mathrm{e}^{-k/T}\left[\frac{T}{k} + \frac{1}{2} + n_{\text{F}}(p_0) + \frac{k}{T}\left(\frac{1}{12} + n_{\text{F}}(p_0)^2\right)\right],\\ n_{\text{B}}(-k) + n_{\text{F}}(p_0-k) &\approx -\mathrm{e}^{-k/T}\left[\frac{T}{k} + \frac{1}{2} + \frac{k}{T}\frac{1}{12}\right]\\ &\quad - \mathrm{e}^{-k/T}\mathrm{e}^{p_0/T}\Theta(k-p_0)\left[n_{\text{F}}(p_0) + \frac{k}{T}n_{\text{F}}(p_0)\Big(1-n_{\text{F}}(p_0)\Big)\right]\\ &\quad + \underbrace{\Theta(p_0-k)\Big[-1}\ + n_{\text{F}}(p_0) + \frac{k}{T}n_{\text{F}}(p_0)\Big(1-n_{\text{F}}(p_0)\Big)\Big]. \end{aligned} \tag{50}$$

The term underlined with the curly brace constitutes the "vacuum" part as obtained in the previous section. The other terms are ordered according to their dominance, i.e., the contribution proportional to T/k is the largest due to the strongly peaked nature of the function $n_{\mathrm{B}}(k)$. The terms including the fermionic distribution functions are negligible for $p_0 \gg T$. Note however, that the negative energy states of the fermionic distribution function are properly taken care of since $n_{\mathrm{F}}(-p_0) = 1 - n_{\mathrm{F}}(p_0) \approx 1$.

It was stated in equations (33) and (43), that the introduction of a small non-zero γ removes the infrared divergence from the integral. Thus, similar to the vacuum case, one obtains a result for Γ_T which cannot be expanded in a Taylor series around the point $\gamma = 0$.

It was described above how to obtain an asymptotic expansion around this value. Here this is again achieved by rescaling $k \to x\gamma$ in the integrand, and then expanding everything but the exponential factor $\exp(-x\gamma/T)$ in powers of γ.

To improve the calculation, yet another trick is introduced: First, the momentum integral is calculated using only the leading term $T/k = 1/(\beta k)$ of the above expansion. The result is then improved by acting on it with a differential operator

$$\Gamma_T^{\mathrm{I}\,\prime}(\omega,\vec{p},\gamma) = \Gamma_T^{\mathrm{I}}(\omega,\vec{p},\gamma) - \left(\frac{1}{2}\frac{\partial}{\partial\beta} - \frac{1}{12T}\frac{\partial^2}{\partial\beta^2}\right)\frac{\Gamma_T^{\mathrm{I}}(\omega,\vec{p},\gamma)}{T}. \tag{51}$$

For both pieces of the self-energy function this is a quite laborious task, for our calculation it washily relied on symbolic computation using Mathematica$^{\mathrm{tm}}$. For the first piece one obtains

$$\begin{aligned}
\Gamma_T^{\mathrm{I}\,\prime}(\omega,\vec{p},\gamma) = \frac{g^2}{4\pi^3}\Bigg\{ & \frac{\pi T\omega}{4p}\,\log\left(\frac{\omega+p}{\omega-p}\right) \\
& + \frac{\gamma}{4}\,\frac{\omega^2}{\omega^2-p^2}\left[\log\left(\frac{\gamma^2\omega^2}{T^2(\omega^2-p^2)}\right) + \frac{\omega}{p}\log\left(\frac{\omega+p}{\omega-p}\right) + 2C_\Gamma - \frac{17}{3} + \frac{20T^2}{3\omega^2}\right] \\
& - \frac{\gamma^2\pi}{24T}\,\frac{\omega^4+18\omega^2T^2+18T^2}{(\omega^2-p^2)^2} \\
& + \frac{\gamma^3}{(\omega^2-p^2)^2}\Bigg[\frac{(\omega-p)^5}{8p}\left(\log\left(\frac{\gamma\omega}{T(\omega+p)}\right) + C_\Gamma - \frac{19}{12}\right) \\
& \quad - \frac{(\omega+p)^5}{8p}\left(\log\left(\frac{\gamma\omega}{T(\omega-p)}\right) + C_\Gamma - \frac{19}{12}\right) + \left(3\omega^2+p^2\right)\left(\frac{\omega^4}{216T^2} - \frac{5}{9}T^2\right)\Bigg]\Bigg\} \\
& + \mathcal{O}(\gamma^4),
\end{aligned} \tag{52}$$

where $C_\Gamma = 0.57721...$ is Euler's constant. A comparison of this approximation to different orders in γ with the full numerical calculation of the temperature dependent contribution is presented in figure 3. As before, it turns out that the results are already quite reliable in second order of γ, provided the temperature is not too high.

In case of the vacuum parts of the self-energy function it was possible to gather all terms of a given order in γ in this asymptotic expansion. For the present temperature dependent pieces this is *not* possible, because of the above expansion: It

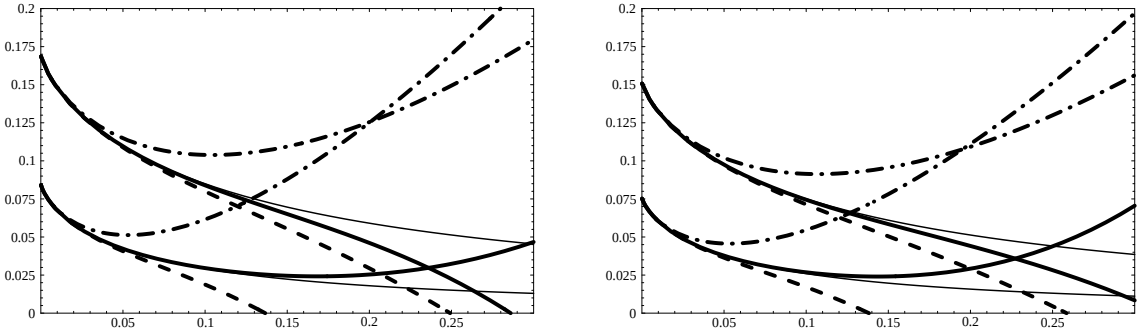

Figure 3. Imaginary part Γ'_T: Numerical calculation vs. analytical approximation. Left panel: vector piece ${\Gamma^{\rm I}_T}'$; right panel: scalar piece $\Gamma^{\rm II\prime}_T$. Energy $p_0 = \omega$, momentum $p = 0.5M$, factor $g^2/(4\pi^3)$ set to one. Temperatures are $T = 0.05M$ (bottom lines) and $T = 0.1M$ (top lines). Thin continuous lines: full numerical calculation; dash-dotted thick lines: asymptotic expansion to order γ; dashed thick lines: asymptotic expansion to order γ^2; thick continuous lines: expansion to order γ^3.

automatically counts T and γ to be of the same order, hence it is a simultaneous expansion in these parameters. As a small reminder of this fact appears a term γT^2 in the above expansion, and the $\mathcal{O}(\gamma^4)$ should be replaced by a $\mathcal{O}(\gamma^4, \gamma T^3)$

The net result is that in second order γ the asymptotic expansion is reliable up to $\gamma \approx T/2$, whereas it is good up to the temperature in third order. In anticipation of the following section, note that the quality of the approximation grows tremendously for very small temperatures $T \ll 0.1M$

With similar quality one may also obtain the second piece, asymptotically given by

$$\begin{aligned}
\Gamma^{\rm II\prime}_T(\omega,\vec{p},\gamma) = \frac{g^2}{4\pi^3}\Bigg\{ & \frac{\pi T M}{4p}\,\log\left(\frac{\omega+p}{\omega-p}\right) \\
& \frac{\gamma}{4}\,\frac{M\omega}{\omega^2-p^2}\,\left[\log\left(\frac{\gamma^2\omega^2}{T^2(\omega^2-p^2)}\right)+\frac{\omega}{p}\log\left(\frac{\omega+p}{\omega-p}\right)+2C_\Gamma-\frac{17}{3}\right] \\
& -\frac{\gamma^2\pi M\omega}{24T}\,\frac{\omega^2+12T^2}{(\omega^2-p^2)^2} \\
& +\frac{\gamma^3 M}{(\omega^2-p^2)^3}\Bigg[\frac{(\omega-p)^4}{8p}\,\left(\log\left(\frac{\gamma\omega}{T(\omega+p)}\right)+C_\Gamma-\frac{19}{12}\right) \\
& -\frac{(\omega-p)^4}{8p}\,\left(\log\left(\frac{\gamma\omega}{T(\omega-p)}\right)+C_\Gamma-\frac{19}{12}\right) \\
& +\frac{\omega^3}{216T^2}\left(3\omega^2+p^2\right)-\frac{1}{16\omega}\left(\omega^4+6\omega^2p^2+p^4\right)\Bigg]\Bigg\} \\
& +\mathcal{O}(\gamma^4,\gamma T^3).
\end{aligned} \tag{53}$$

To this expression one may apply the same considerations as before: Although there is no piece $\propto \gamma T^2$, one may trust this expansion in second order γ only up to $\gamma \approx T/2$.

The claims laid on the accuracy of our asymptotic expansions are supported by figure 3, where they are compared with a completely independent numerical calculation. The accuracy of the results was checked with various values for momentum and temperature, and found to persist up to momenta $p \approx M$.

To be completely sure of the findings, the corrections of orders γ^3 will be dropped henceforth: Unusual as it may seem for quantum field theory, one therefore has an approximate method with controlled accuracy in the next-to-leading-log order.

6. Fermion damping rate II

In this section, the results are assembled to obtain a consistent solution for the fermion damping rate in hot systems. This applies to the explicitly temperature dependent part as well as the "vacuum" part of the self-energy. Combining (46) with (52) and (48) with (53), *all the terms of order* $\gamma\,\log(\gamma)$ *cancel!*

The cancellation occurs for scalar and vector part of the self-energy function independently, hence it holds for all types of massless bosons that were considered in the equations (37)–(40). This fact is commented further in the following.

As a consequence of this cancelation, the self-consistent damping rate for the fermion moving slowly through a hot medium, as defined in the equations (37)–(40), is the solution of an *algebraic equation* of the form

$$f_0(\omega,\vec{p},T) - \left(f_1(\omega,\vec{p},T) + \frac{4\pi}{g^2}\right)\gamma + f_2(\omega,\vec{p},T)\,\gamma^2 = 0. \tag{54}$$

Note however, that the results do not exclude the possibility of corrections of order $\gamma^3\log(\gamma)$ to this equation – but these are safely and even in view of the asymptotic expansions carried out to be considered of lower importance.

Equation (54) always has two solutions for γ, but, in general, one of them is negative or very large. Hence it is obvious how to choose the physical solution, and in the following this is the only one discussed.

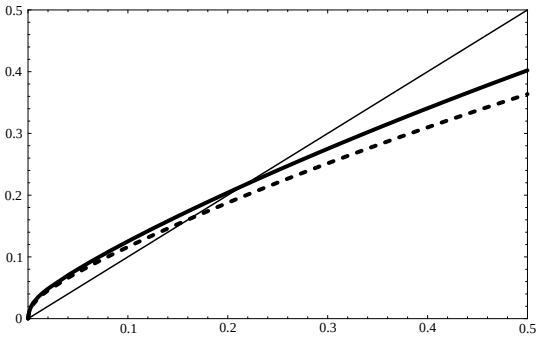

Figure 4. Self-consistent γ_S/M as function of T/M. Strong coupling $\alpha = 1$, compared to $\gamma = T$ (thin line). Two different momenta: $p = 0$ (thick continuous), $p = 0.5M$ (thick dashed).

The coupling constant appears only at one point, the three functions $f_0 - f_2$ do not depend on it. f_0 is proportional to the temperature T, f_1 contains terms which are not analytical around zero temperature, i.e., terms of order $T\,\log(T)$.

The solution γ of equation (54) therefore is a power series in the coupling constant – but still it includes a non-perturbative effect in the form of the non-analytical behaviour in the temperature, which is of the order $g^4\,T\,\log(T)$.

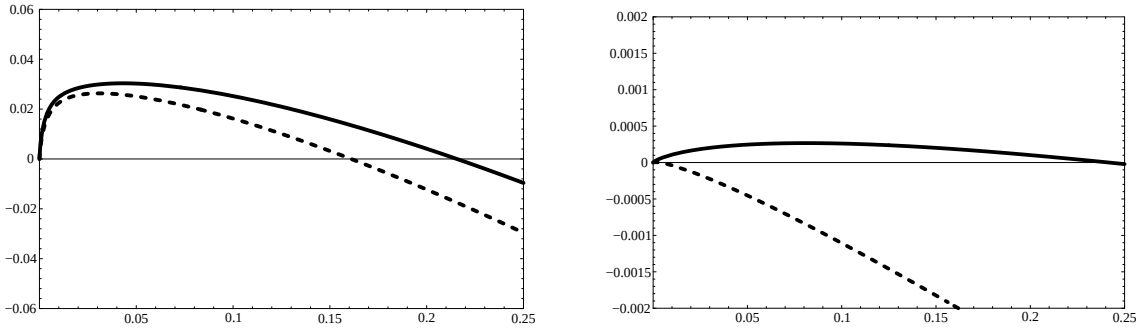

Figure 5. $(\gamma_S - \alpha T)/M$ as function of T/M. Left panel: strong coupling $\alpha = 1$; right panel: weak coupling $\alpha = 0.1$. Continuous lines: $p = 0$; dashed lines: $p = 0.5M$.

One might argue, that these non-analytical terms are not significant because in order g^4 the other two-loop diagrams that were neglected give a contribution as well. However, as may be expected from the results obtained here, their non-analytical contribution involving $\log(T)$ is suppressed with respect to their leading order contribution by a factor γ, which may be translated into a factor g^2T. This implies, that our results to order g^4 yield the *dominant* non-analyticity and are important for small temperatures (see figures).

Moreover, this chain of arguments also supports the conclusion that terms of order $g^2Tp^2/M^2 \log(1/g)$ cannot appear: Terms logarithmic in the coupling constant wear at least a coefficient g^6 or g^4T^2/M^2, even if vertex corrections are introduced in the calculation of the self-energy function.

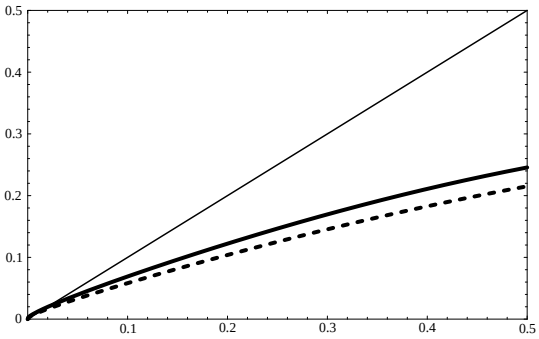

Figure 6. Self-consistent γ_V/M as function of T/M. Strong coupling $\alpha = 1$, compared to $\gamma = T$ (thin line). Two different momenta: $p = 0$ (thick continuous), $p = 0.5M$ (thick dashed).

Figures 4 and 5 depict the self consistent solution of equation (54) for scalar bosons coupled to the fermions. For small temperatures, the deviation from the linear temperature dependence is very striking: Due to the $T\log(T)$ terms in the self-energy functions, $\gamma_S(T)$ rises sharply at a very small temperature (see comment in the next section).

Before discussing this in detail, the results for the vector and pseudoscalar boson exchange are examined. The self-consistent γ_V due to massless vector boson exchange in Feynman gauge is plotted in figures 6 and 7. The curves basically employ the same features as for the scalar bosons: A pronounced non-analyticity of the function $\gamma_V(T)$ at small temperatures.

For the pseudoscalar boson the approach presented here leads to a result which is surprising at the first glance: The self-consistent solution for γ_P is identically zero at

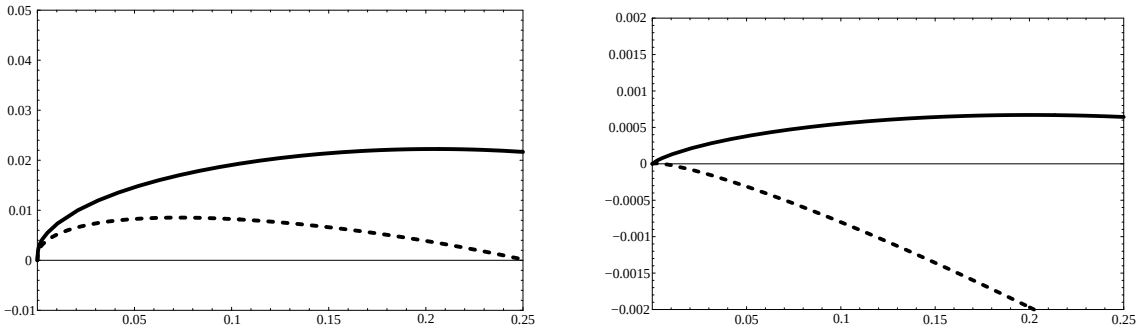

Figure 7. $(\gamma_{\rm V} - \alpha T)/M$ as function of T/M. Left panel: strong coupling $\alpha = 1$; right panel: weak coupling $\alpha = 0.1$. Continuous lines: $p = 0$; dashed lines: $p = 0.5M$.

all temperatures. This follows from the fact that the leading terms in the temperature dependence of equations (52) and (53) differ only by a factor of M/ω. Inspection of equation (40) then reveals that these leading terms are cancelled, consequently $f_0 \equiv 0$ in the above equation (54) and therefore $\gamma_{\rm P} \equiv 0$.

However, at a second glance this result is not surprising. The pseudoscalar coupling is a p-wave coupling, hence it vanishes for zero momentum transfer. This is reflected in the fact that the coefficients f_1 and f_2 start with power p^2 instead of possessing a constant piece. Therefore one expects a self-consistent $\gamma_{\rm P}$ which also starts with this power p^2 – clearly in contradiction to the ansatz spectral function.

Naturally it is possible to repeat the calculation with a corresponding ansatz for γ – but this would overstress the goals of the present paper. Therefore, this pseudoscalar result is considered merely as a consistency argument in favour of the approximations.

Also another feature of the results for $\gamma_{\rm S}$ and $\gamma_{\rm V}$ may be seen in the figures. Clearly, the momentum dependence of the self-consistent solution is small up to quite high temperatures. In view of equation (38) this means $|\nu_0| \gtrsim |\nu_1|$ for scalar and vector boson exchange. One may therefore conclude, that the ansatz of a momentum independent $\gamma_{\rm S}$ and $\gamma_{\rm V}$ is very well justified.

6.1. Analytical approximations for the damping rate

It is not necessary to repeat the analytical expressions for the self-energy pieces in order to give the three functions f_0 – f_2 in closed form. However, it is instructive to extract the dominant terms in the calculation of γ.

To this end, perform an expansion of the solution of (54) in powers of the coupling constant. Only even powers occur, hence the expansion parameter is

$$\alpha = g^2/4\pi. \tag{55}$$

The contributions of order T and of order $T\,\log(T)$ then are, to second order in α

$$\gamma_{\rm S} \approx \alpha T \left(1 - \frac{2}{3}\frac{p^2}{M^2} + \frac{8}{15}\frac{p^4}{M^4}\right)$$

$$- \alpha^2 \frac{T}{\pi} \left(\log \left[\frac{T}{M} \left(1 - \frac{11}{12} \frac{p^2}{M^2} + \frac{1061}{1440} \frac{p^4}{M^4} \right) \right] + \frac{25}{12} - C_\Gamma \right) \times \left(1 - \frac{2}{3} \frac{p^2}{M^2} + \frac{8}{15} \frac{p^4}{M^4} \right) + \mathcal{O}\left(\alpha \frac{p^6}{M^6}, \alpha^2 \frac{T^2}{M^2}, \alpha^3 \right), \tag{56}$$

where $p = |\vec{p}|$. Naturally this last expansion is only good in the weak coupling limit. Performing the same expansion for the vector boson exchange yields

$$\gamma_{\mathrm{V}} \approx \alpha T \left(1 - \frac{2}{3} \frac{p^2}{M^2} + \frac{8}{15} \frac{p^4}{M^4} \right) - \alpha^2 \frac{T}{\pi} \left(\log \left[\frac{T}{M} \left(1 - \frac{1}{3} \frac{p^2}{M^2} + \frac{31}{180} \frac{p^4}{M^4} \right) \right] + \frac{13}{3} - C_\Gamma \right) \times \left(1 - \frac{2}{3} \frac{p^2}{M^2} + \frac{8}{15} \frac{p^4}{M^4} \right) + \mathcal{O}\left(\alpha \frac{p^6}{M^6}, \alpha^2 \frac{T^2}{M^2}, \alpha^3 \right). \tag{57}$$

It is a crucial aspect of these results, that they are analytical functions of the coupling constant, i.e., in contrast to other calculations it is *not* proportional to $\log(1/g)$.

During the discussion of the self-consistency criterion (39) it was already stated, that the vector boson result is gauge independent in the limit $\vec{p} \to 0$. A vertex correction is needed to ensure gauge invariance for the momentum dependent parts (which in principle also require a completely different calculation scheme). However, this will at most give a regular modification of order $\alpha^2 p^2/M^2$, logarithmic terms may appear only in even higher order α.

The contribution of order $T \log(T)$, apart from being non-analytical around $T = 0$, has the effect of destroying the linear relationship between γ and T: Neither for high temperatures nor for low temperatures can one approximate the self-consistent γ by a linear function. This functional piece constitutes the leading non-analyticity observed numerically in figures (4)–(7).

6.2. Where does the log go ?

A striking feature of the above result is, that for a slow massive fermion there are no contributions to the damping rate of type $\log(1/g)$.

To understand this, consider an only partial assembly of the results neglecting the parts of the self energy function that are not explicitly temperature dependent. Taking the results of equations (52), (53) and (39) in the limit of zero momentum and neglecting the "vacuum part":

$$\overline{\gamma} = \frac{g^2}{4\pi} \left\{ T + \frac{\overline{\gamma}}{\pi} \left(\log \left(\frac{\overline{\gamma}}{T} \right) + C_\Gamma - \frac{11}{6} \right) \right\}, \tag{58}$$

which must be solved for $\overline{\gamma}$. Naively, one expects a leading order result $\overline{\gamma} \propto g^2 T$ with corrections of order $g^4 \log(1/g)$, but as will be shown below this naive expectation has to be taken with care.

To allow for comparison against other calculations one also may check the influence of a momentum space cutoff (seemingly taking into account only soft interactions) of the order gT. This would change the leading order behaviour in the

temperature dependent self energy part, equation (58), from $g^2(T + \gamma \log(\gamma))$ into $g^2(T + T \log(\gamma))$. In an approximate solution of the transcendental equation, this would shift the $\log(1/g)$ contribution from order $g^4 \log(1/g)$ into $g^2 \log(1/g)$ – but still and wrongly retain a logarithmic term in the final result.

Consequently to these two wrong results one may state, that the neglecting of the "vacuum" effects, i.e., the approximation of the Bose-Einstein function by a simple T/k, is an unnecessary oversimplification of calculations and should be avoided. Indeed, an improved treatment of the Bose-Einstein function is crucial in replacing the non-analyticity in the coupling constant by a non-analyticity in the temperature. Note also, that the momentum cutoff scheme furthermore violates causality as discussed in section 2.

7. Conclusion

This report presents a self-consistent calculation of the damping rate (=spectral width parameter) of a slow massive fermion in a gas of hot massless bosons. The result is obtained as a series in the coupling constant, the momentum of the fermion and the temperature. Instead of a non-analyticity in the coupling constant, which is sometimes quoted in the literature, the final results for the damping rate exhibit a non-analyticity in the temperature parameter arising from contributions of order $T \log(T)$.

To confirm and explain this discrepancy, several careful investigations were made. First of all, to control the quality of the approximations, every asymptotic expansion was made to higher order than assembled in the final result. On the other hand, all approximations were also compared to numerical calculations. Therefore, the quality of the series expansion is very well under control and considered *mathematically* sound.

Furthermore, there exists an independent *physical* support for this result, since it is in accordance with the symmetry of space and time: In the presence of temperature the Poincaré symmetry is broken to $\mathrm{SO}(3)\times\mathrm{T}_4$, and the symmetry restoration with temperature $T \to 0$ is expected to be singular. Consequently, on physical grounds one would *expect* a non-analyticity in the temperature parameter.

Another check to be made for *physical* reason is, whether the most dominant contribution to the fermion damping rate was taken into account. The "diagram" used in the present paper is the imaginary part of the self-consistent Fock diagram. Cutting this apart yields the proper scattering amplitudes hidden in the result – in the present case, these contain the coulomb scattering of the slow massive fermion by other fermions in the hot medium.

The only effect not included here is a polarization of the medium, i.e., a modification of the boson spectral function. However, such a modification could only weaken the boson propagator singularity, and consequently would give contributions which are *not* logarithmic. This has been discussed in more detail in section (3.1). Numerical calculations show, that using damped bosonic interactions in the self-consistent calculation is equivalent to inserting the sum of the fermionic γ and the bosonic

γ_B on the right side of equation (37), i.e., in the calculated imaginary parts of the self-energy function. Such a substitution does *not* affect the lowest order thermal contribution as obtained in section 5 of this work. Thus it must be concluded, that the use of a modified boson propagator does not change the leading order result for the fermionic spectral width, i.e., the non-analyticity in temperature.

By these careful investigations one is thus forced to conclude the correctness of the results at least to the accuracy given in the previous section: The damping rate of a slow massive fermion in a gas of moderately hot massless bosons is $\gamma \propto g^2 T + \mathcal{O}(g^4 T \log(T/M))$.

A second ingredient of the present work therefore was the investigation of how different mathematical or physical assumptions may produce damping rates $\gamma \propto g^2 T \log(1/g)$. It was shown, that generally the methods used to obtain such a result violate important axioms of quantum theory and therefore should be avoided.

This paper presents a straightforward application of the method of generalized free fields. It automatically accounts for the undeniable mathematical fact that at non-zero temperature a perturbative expansion in terms of quasi-particle states with infinite lifetime is ill-defined [5–7]. One may put this into the following form: In reality a fermion in a heat bath is always subject to some Brownian motion, hence it will never be in the same state for an infinite time.

The use of a proper finite temperature fermion propagator, which obeys this Narnhofer-Thirring theorem and therefore has no isolated poles anywhere in the complex energy plane, is completely sufficient to remove all infrared divergences from the calculations. In particular, no screening or damping of the bosonic interactions is needed to achieve this effect.

Continuous spectral functions appear (more or less) to be the central ingredient of the method. These functions are not a priori fixed in a theory involving generalized free fields [7], one may determine them from the experiment or use physically reasonable self-consistency schemes. Such a scheme was presented here: In assigning a constant spectral width to fermions, it goes one step beyond the introduction of temperature dependent ("thermal") masses. To our knowledge, this approximation is reasonable for systems which are dominated by the low-momentum sector of the interactions, i.e., those which involve massive fermions and massless gauge bosons.

For our approximate fermion spectral function, the summation of all nested Fock diagrams was carried out to infinite order. Apart from vertex corrections, such a summation constitutes the solution of the full one-body problem. As pointed out, such vertex corrections would lead to momentum dependent corrections to the damping rate - and therefore would not affect the momentum independent parts of our results.

One must stress the fact, that the calculation of the real part of the self-energy function by a dispersion integral was suppressed in the present paper. According to the arguments presented, the dispersion integral may still lead to non-analytical contributions in the coupling constant. However, these are expected to be of order $\exp(-1/g^2)$ due to the removal of the Landau ghost poles [22], and therefore they are varying slowly around the zero point of the coupling constant.

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A. Angular integration

In the following the explicit form of the angular integrals over the approximate fermion spectral function is given, which occur in the calculation of (41) and (42). In this appendix, η is the cosine of the angle between the two momenta $\vec{k}$ (internal) and $\vec{p}$ (external), and will henceforth be used $k = |\vec{k}|$, $p = |\vec{p}|$.

$$\begin{aligned} I_1(k \pm p_0) &= \int_{-1}^{1} \mathrm{d}\eta \, \frac{1}{(c_1 - 2pk\eta)^2 + c_2^2} \\ &= \frac{-1}{4pk(k \pm p_0)\gamma} \arctan\left(\frac{(k \pm p_0)^2 - t^2 - \gamma^2}{2(k \pm p_0)\gamma} \right) \Bigg|_{\omega_{p-k}}^{\omega_{p+k}} \end{aligned} \tag{59}$$

and

$$\begin{aligned} I_2(k \pm p_0) &= \int_{-1}^{1} \mathrm{d}\eta \, \frac{\eta}{(c_1 - 2pk\eta)^2 + c_2^2} \\ &= \frac{c_1}{2kp} I_1(k \pm p_0) \\ &\quad + \frac{1}{8p^2k^2} \log\left(\frac{\left(t^2 + \gamma^2 - (k \pm p_0)^2\right)^2 + 4\gamma^2(k \pm p_0)^2}{\gamma^2} \right) \Bigg|_{\omega_{p-k}}^{\omega_{p+k}} \end{aligned} \tag{60}$$

with

$$c_1 = \pm 2p_0k + p_0^2 - \omega^2 - \gamma^2, \qquad c_2 = 2(k \pm p_0)\gamma \tag{61}$$

and boundaries of the integration defined as

$$\omega_{p\pm k}^2 = \omega^2 \pm 2pk + k^2. \tag{62}$$

An important aspect of the above analytical results is to select the proper Riemann sheet for the arctan-functions: I_1 and I_2 are continuous functions in each variable.

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Константа затухання масивних ферміонів у гарячому середовищі

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У гарячому середовищі кожне збудження має скінченний час існування і проявляється ненульовою спектральною шириною. Звичайне затухання, так само як і квантові ефекти пам'яті, отримується з цієї нетривіальної спектральної функції. Дана робота представляє новий метод самоузгодженого розрахунку спектральної ширини ферміона, зв'язаного з безмасовим бозоном скалярного, векторного та псевдоскалярного типів. У відповідності до відомих з квантової електродинаміки процедур самоузгоджене сумування відповідних фоківських діаграм вилучає всі інфрачервоні розбіжності незважаючи на те, що бозони зовсім не екрануються. Розв'язок для константи ферміонного затухання є аналітичним за константою зв'язку g, але не є аналітичним за параметром температури, тобто $\gamma \propto g^2T + \mathcal{O}(g^4T\log(T/M))$.

Ключові слова: *квантова електродинаміка, теорія поля при скінченних температурах*

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