

**Foreword****Reminiscences of unforgettable times of my collaboration with Nikolai N. Bogolubov (Jr.) \***

I would like to begin my reminiscences with a quite unique episode in my student life which many years later had a great impact on my future scientific and everyday life. As far back as early seventieth of the past century, being a student at the Physics Department of Lviv State University, walking along its corridors, I occasionally entered the Big Physical Hall amphitheater and found there a lot of young people sitting and listening attentively to a youthful speaker standing at the rostrum of the Hall and energetically explaining some overhanding visual aids filled with mathematical formulae. I also took a vacant seat and tried to catch what he was speaking about. . . On the front wall of the Hall there hung a long advertisement in big bold letters: “The All-Union Statistical Physics Conference Workshop – 1973” under the auspices of academician Nikolai N. Bogolubov. I understood that it could be interesting for me too, as I had then already taken lecture courses on quantum mechanics and statistical physics. I listened to the speaker until he finished and became a witness of a very hot and emotional discussion following the report. As during this discussion some people referred to the speaker “Professor Nikolai Bogolubov”, I right away understood that the speaker, being then young enough, was in reality Professor Nikolai N. Bogolubov (Junior) contrary to the Workshop chair academician Nikolai N. Bogolubov (Senior)! It is appropriate to say here that I had been already familiar with the name Nikolai N. Bogolubov (Sr.) since my university mate Dmytro Petryna had already told me a few things of him with great enthusiasm and had shown me his three-volume “Collected oeuvres” recently published in Kyiv. Moreover, his uncle Professor Dmytro Ya. Petryna, living then in Kyiv, was a disciple of academician Ostap S. Parasyuk, who was actively working together with academician Nikolai N. Bogolubov (Sr.), and had tried to urge his nephew Dmytro Petryna to study some fundamental N.N. Bogolubov’s (Sr.) works on statistical physics and quantum field theory. Having been intrigued by my friend’s scientific passion and persistence I also started looking through Bogolubov’s monographs stored in the libraries of Lviv University and Academy of Sciences. This way I soon became aware of such N. Bogolubov’s books as “Introduction to the theory of quantized fields”, “Lectures on statistical mechanics” and “Axiomatic approach backgrounds to the quantum field theory” etc. and tried to understand something therein. The first lesson I was taught while reading these and other related books, consisted in clear understanding that my mathematical education was quite insufficient for these books to be read with profit. Keeping deeply in mind this feeling, I started my “advanced” self-education by studying functional analysis and operator theory, differential geometry and topology, complex analysis and algebraic geometry, advanced algebra, differential equations and variational analysis etc. Being fascinated with very interesting problems in the quantum field theory and electrodynamics I secretly decided to try to start my graduate Ph.D. studies in the near future at the Institute for Theoretical Physics of NAS<sup>1</sup> in Kyiv and continue my already imagined theoretical physics investigations of such puzzling physical phenomena as electron-positron annihilation, physical vacuum polarization and so on. Now I would like to turn back to the episode of my occasional “participation” in the Statistical Physics Workshop held in the Lviv State University: when the discussion, following the Nikolai N. Bogolubov’s (Jr.) report, finished I was eager to approach him and ask whether he was also interested in quantum electrodynamics and what he thought of the electron-positron annihilation phenomenon that had bothered me for the last two years. To my regret, I failed to realize this intention because of a

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\* Author devotes these recollections to his Teacher and Friend Prof. Nikolai N. Bogolubov (Jr.) in honor of his 70<sup>th</sup> birthday.

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lot of people crowding the very narrow passages – when I descended along the bench rows to the rostrum, Prof. Nikolai N. Bogolubov (Jr.) had already disappeared in the neighboring rooms. Nonetheless, I promised myself that I would do my best to meet Prof. Nikolai N. Bogolubov (Jr.) and in the future, upon the University graduation, start my postgraduate physics studies under his supervision.

Having graduated from the Lviv State University and trying to make my beloved electrodynamics and quantum fields theory research, I soon became a non-formal disciple of academician Ostap S. Parasyuk, who was the head of Mathematical Department at the Institute for Theoretical Physics of NAS in Kyiv. To a great extent this became possible owing to the help of my friends Petro I. Holod (a former Ph.D. student of academician Ostap S. Parasyuk) and Ivan M. Kopych (a former Ph.D. student of Prof. Volodymyr P. Hachok from the Institute for Theoretical Physics of NAS in Kyiv and my Physics Department student-mate from Lviv). The scientific work under Prof. Ostap S. Parasyuk's supervision and friendly cooperation with Petro I. Holod had run successfully enough and I decided to enter the Institute for Theoretical Physics of NAS in Kyiv having applied for Ph.D. studies under Prof. Ostap S. Parasyuk's supervision. But very soon I was asked by Prof. Ostap S. Parasyuk to withdraw my Ph.D. application to him and instead apply for Ph.D. studies at the Institute of Mathematics of NAS in Kyiv under Nikolai N. Bogolubov's (Jr.) supervision, who worked in Moscow for the V. A. Steklov Mathematical Institute of RAS and had already agreed to take the duty of my supervisor. Thus, having soon passed my entrance exams with merit, I became a Ph.D. student of Prof. N.N. Bogolubov (Jr.) just as I had dreamed some years before being a Physics Department student at Lviv State University. Soon enough, in five months, I was directed by the Institute of Mathematics authority to continue my studies in Moscow at the V.A. Steklov Mathematical Institute of RAS, where I carried out some interesting research devoted both to the study of exact solutions to classical two-dimensional Thirring type two-dimensional field theory models and to the old mathematical integrability by quadratures problem of ordinary differential Riccati equations, making use essentially of the algebraic-geometrical tools, devised recently by such mathematicians as S.P. Novikov, B.A. Dubrovin, V.B. Matveev, L.D. Faddeev, P. Lax, J. Moser, M. Adler and some others. It should be mentioned here that most of the problems I was then working at had been posed by Prof. Ostap S. Parasyuk and which appeared to be also of great interest to Prof. Nikolai N. Bogolubov (Jr.). Having soon defended these results as a Ph.D. thesis (in 1980), I began jointly with Prof. Nikolai N. Bogolubov (Jr.) to investigate the problems posed by academician N.N. Bogolubov (Sr.) in early fortieth of the past century that consisted in the description of algebraic-analytical properties of physical solutions to dynamical systems of quantum statistical and quantum field theory physics making use of functional operator, Lie-algebraic and  $C^*$ -algebra representation methods and techniques. Amongst the problems studied jointly with Prof. Nikolai N. Bogolubov (Jr.) it is necessary to mention the following:

- i) construction of a quantum analogue of the Bogolubov functional equations for many-particle distribution functions by means of the functional-operator methods and representation theory of the current algebras [1, 2];
- ii) the Lie-algebraic proof of the Hamiltonian structure for the classical Bogolubov functional equations [2];
- iii) the complete quantum integrability of new Schrodinger type nonlinear quantum dynamical systems with  $\delta$ - and  $\delta'$ -many-particle interaction [1];
- iv) devising an effective, so-called direct gradient-holonomic algorithm for studying the Lax type iso-spectral and parametric integrability of a wide class of nonlinear dynamical systems on functional (two-dimensional) topological jet-manifolds [1, 3–6];
- v) construction of the Delsart-Lions generalized transmutation operators by means of the de Rham-Hodge type theory to solve the multi-dimensional quantum inverse spectral transform problem [4, 7, 8];
- vi) application of Lie-algebraic and differential-geometric methods to construct quantum computer algorithms [9];

- vii) symplectic theory analysis of helicity and vortex type invariants of classical hydrodynamical and magneto-hydrodynamical systems [10–12].

It should be mentioned here that a wide cycle of investigations was also carried out by Nikolai N. Bogolubov (Jr.) jointly with his Moscow students on the well known approximating Hamiltonian method in quantum statistical mechanics. His works in this field have brought an outstanding contribution to the development of rigorous methods of statistical mechanics. Having been originally called for solving the modelling problems connected with fermion operators, many results by Bogolubov N.N. (Jr.) have subsequently found application in a wider range of problems. The known work by Ginibre [13] in its essential part follows the Bogolubov's N.N. (Jr.) remarkable work [14]. Recently the approximating Hamiltonian method has been also applied to rigorous problems related to Bose systems [15, 16]. In 1954 Bogolubov N.N. (Sr.) developed a new approach [17] for expressing physical observables as continual integrals. This approach was based on the representation of Green functions in terms of vacuum expectations of suitable chronological products. The averaging operation over the boson vacuum was interpreted as a functional integral. Later in 1981, Bogolubov N.N. and Bogolubov N.N. (Jr.) [18] developed this construction within the framework of quantum statistical mechanics. The measure that arises in this approach is a Gaussian measure in an appropriate space of continuous functions. The Gibbs equilibrium averages of the chronological products of operators are expressed as functional integrals with respect to this measure. Subsequently some mathematical problems of integration with respect to the Bogolubov's measure were considered [19]. It was found that the Bogolubov–Bogolubov (Jr.) approach is highly fruitful in quantum statistical mechanics side by side with the Feynman functional integration. Unlike the Feynman approach, the Bogolubov–Bogolubov (Jr.) approach is based on the well defined Gaussian measure. (As is well known, the natural analogue of Wiener measure with complex variance parameter is not a countably additive complex measure.)

Some of these studies were later defended as my doctor habilitation thesis in mathematics and physics sciences (in 1987) at the Laboratory of Theoretical Physics of the international Joint Institute for Nuclear Research in Dubna, Moscow region, of Russian Federation. During the next fifteen years I was much traveling abroad visiting research centers at the universities of Canada, USA and West Europe, and my contacts with Professor Nikolai N. Bogolubov (Jr.) had become more rare.

The situation has abruptly changed for the past ten years owing to our joint research collaboration grants obtained from the International Center for Theoretical Physics and the SISSA-International school for Advanced Studies in Trieste, Italy. This period was marked by turning back our research efforts to studying the old and new problems of classical and quantum electrodynamics and gravitation. Jointly with Prof. Nikolai N. Bogolubov (Jr.) and some of our co-authors, we have analyzed in detail, having based on the modern symplectic theory methods, the classical Dirac-Fock-Podolsky problem [20, 21] of embedding the well known Lorentz constraint into the canonical Hamiltonian formalism, suitable for quantization of the related Maxwell equations. Concerning the space-time and physical vacuum based description of the classical Maxwell equations we have recently developed jointly with Prof. Nikolai N. Bogolubov (Jr.) a new, so-called vacuum field theory approach [22–32] to the study of the Lorentz type particle charge interaction with external electromagnetic field. The obtained results have enabled us, in particular, to suggest a new physical interpretation of the inertial (dynamical) particle mass notion well fitting to be applied to the gravity theory, based on completely new no-geometry approach, and being free of well known singularity problems. Dropping the point finishing these recollections, I cordially express my sincere appreciation to my Teacher and Friend Prof. Nikolai N. Bogolubov (Jr.) for our many-year scientific relations and true friendship, owing to which I was able to do my beloved and God blessed Physics and Mathematics investigations of the beautiful Nature so freely, so happily and, I hope, fruitfully.

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