Low-temperature properties of frustrated classical spin chain near the ferromagnet-helimagnet transition point

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Last year much attention has been paid to the 1D $J_1 - J_2$ Heisenberg model that exhibit frustration. Its Hamiltonian has a form

$$H = J_1 \sum \mathbf{S}_n \mathbf{S}_{n+1} + J_2 \sum \mathbf{S}_n \ \mathbf{S}_{n+2} \,, \tag{1}$$

where $J_1 < 0$ is the ferromagnetic (NN) coupling while $J_2 > 0$ is the antiferromagnetic (NNN) interaction. This model is basic one for the description of magnetic properties of recently synthesized edge-shared cuprate chain compounds. The model is characterized by the frustration parameter $\alpha = J_2/|J_1|$. The ground state of the model is ferromagnetic for $\alpha < 1/4$. At $\alpha = \alpha_{\rm c} = 1/4$ the ground state phase transition to the singlet phase with helical spin correlations takes place. Remarkably, this transition point does not depend on a spin value, including the classical limit $s = \infty$. The interesting question related to this model is the influence of the frustration on the low-temperature thermodynamics near the transition point. We study this problem for the classical version of the model (1). The calculation of the partition and spin correlation functions is reduced to quantum mechanics problem of a particle in a potential well. It is shown that exactly in the transition point at $T \to 0$ the correlation length $l_{\rm c} \sim T^{-1/3}$ and zero field susceptibility $\chi \sim T^{-4/3}$ in contrast with the Heisenberg ferromagnet ($\alpha = 0$) where $l_{\rm c} \sim T^{-1}$ and $\chi \sim T^{-2}$. Corresponding numerical factors for $l_{\rm c}$ and χ are found. The behavior of the low-temperature susceptibility in the helical phase near the transition point is described by the universal dependence of the scaling variable $t = T/(\alpha - \alpha_{\rm c})^{3/2}$. In particular, $\chi(T)$ has a maximum at $T_{\rm m} \sim (\alpha - \alpha_{\rm c})^{3/2}$ and $\chi_{\rm m} \sim (\alpha - \alpha_{\rm c})^{-2}$. The obtained dependence $\chi(T)$ is in qualitative agreement with that observed in edge-shared cuprates with α close to $\alpha_{\rm c}$.