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A consistent description of a kinetics and hydrodynamics of
quantum Bose systems. Time correlation functions and collective
excitation spectrum

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Узгоджений опис кінетики та гідродинаміки квантових бозе-систем. Часові кореляційні функції і спектр колективних збуджень

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Анотація. На основі методу нерівноважного статистичного оператора Д.М.Зубарева проводиться опис динамічних властивостей рідкого He^4 при температурах вище T_λ . Отримані системи рівнянь переносу та рівнянь для часових кореляційних функцій. Аналізуються різні можливості наближеного розрахунку ядер переносу для отримання замкнутої системи рівнянь. Задача розглядається в контексті взаємозв'язку з даними експерименту з розсіяння нейтронів.

A consistent description of a kinetics and hydrodynamics of quantum Bose systems. Time correlation functions and collective excitation spectrum

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Abstract. On the basis of nonequilibrium statistical operator method the description of dynamic properties of liquid He^4 above T_λ is being carried out. The systems of transport equations and the equations for time correlation functions are obtained. Various possibilities of the approximate evaluation of transport kernels for obtaining of the closed chain of equations are being analyzed. The problem is considered in the context of relation with neutron scattering experiment data.

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Introduction

The study of dynamical properties of quantum fluids is actual among experimental and analytical investigations of condensed systems. In particular, the study of collective excitations in a hydrodynamic limit for semiquantum helium [1,2] is of great interest. An important problem of the consistent account for the one-particle (kinetic) and collective (hydrodynamic) effects in excitation spectrum at intermediate values of a wave vector and frequency is still unresolvable. In this study the time correlation functions and collective excitation spectrum of quantum Bose system are investigated using transport equations of consistent description of kinetics and hydrodynamics [3]. The transport processes of particles kinetic and potential energies are taken into account separately. The expressions for the dynamic structure factor, time correlation functions of the impulse density operators, kinetic and potential parts of the particles enthalpy are obtained. The spectrum of collective excitations with separate contributions from the kinetic and potential energies streams into the heat mode is obtained, using Markov approximation for the memory generalized functions. At a fixed values of a wave vector in a spectrum of collective excitations there are kinetic viscosity-elastic modes with partitioning of the contributions of kinetic and potential energies. In a limit of small densities of a quantum Bose gas the collective excitations spectrum of the corresponding equation is obtained.

1. Kinetics and hydrodynamics of nonequilibrium state near equilibrium

For kinetics and hydrodynamics of nonequilibrium processes which are close to an equilibrium state, the generalized transport equations [3] in the linear approximation transform into a transport equation for $f_{\mathbf{k}}(\mathbf{p}; t) = \langle \hat{n}_{\mathbf{k}}(\mathbf{p}) \rangle^t$, $h_{\mathbf{k}}^{\text{int}}(t) = \langle \hat{h}_{\mathbf{k}}^{\text{int}} \rangle^t$

$$\begin{aligned} \frac{\partial}{\partial t} f_{\mathbf{k}}(\mathbf{p}; t) + \frac{\mathbf{i}\mathbf{k} \cdot \mathbf{p}}{m} f_{\mathbf{k}}(\mathbf{p}; t) = & \quad (1.1) \\ - \frac{\mathbf{i}\mathbf{k} \cdot \mathbf{p}}{m} n f_0(p) c_2(\mathbf{k}) \sum_{\mathbf{p}'} f_{\mathbf{k}}(\mathbf{p}'; t) + i\Omega_{nh}(\mathbf{k}, \mathbf{p}) h_{\mathbf{k}}^{\text{int}}(t) - \\ \sum_{\mathbf{p}'} \int_{-\infty}^t dt' e^{\varepsilon(t'-t)} \varphi_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}'; t, t') f_{\mathbf{k}}(\mathbf{p}'; t') - \end{aligned}$$

$$\int_{-\infty}^t dt' e^{\varepsilon(t'-t)} \varphi_{nh}(\mathbf{k}, \mathbf{p}; t, t') h_{\mathbf{k}}^{\text{int}}(t'),$$

$$\frac{\partial}{\partial t} h_{\mathbf{k}}^{\text{int}}(t) = \sum_{\mathbf{p}} i\Omega_{hn}(\mathbf{k}, \mathbf{p}) f_{\mathbf{k}}(\mathbf{p}; t) - \quad (1.2)$$

$$\begin{aligned} \sum_{\mathbf{p}} \int_{-\infty}^t dt' e^{\varepsilon(t'-t)} \varphi_{hn}(\mathbf{k}, \mathbf{p}; t, t') f_{\mathbf{k}}(\mathbf{p}; t') - \\ \int_{-\infty}^t dt' e^{\varepsilon(t'-t)} \varphi_{hh}(\mathbf{k}; t, t') h_{\mathbf{k}}^{\text{int}}(t'), \end{aligned}$$

where $i\Omega_{nh}(\mathbf{k}, \mathbf{p})$, $i\Omega_{hn}(\mathbf{k}, \mathbf{p})$ are normalized static correlation functions:

$$i\Omega_{nh}(\mathbf{k}, \mathbf{p}) = \langle \hat{n}_{\mathbf{k}}(\mathbf{p}) \hat{h}_{-\mathbf{k}}^{\text{int}} \rangle_0 \Phi_{hh}^{-1}(\mathbf{k}), \quad (1.3)$$

$$i\Omega_{hn}(\mathbf{k}, \mathbf{p}) = \sum_{\mathbf{p}'} \langle \hat{h}_{\mathbf{k}}^{\text{int}} \hat{n}_{-\mathbf{k}}(\mathbf{p}') \rangle_0 \Phi_{\mathbf{k}}^{-1}(\mathbf{p}', \mathbf{p}) \quad (1.4)$$

where

$$\begin{aligned} \hat{h}_{\mathbf{k}}^{\text{int}} = \hat{\mathcal{E}}_{\mathbf{k}} - \int d\mathbf{p} d\mathbf{p}' \langle \hat{\mathcal{E}}_{\mathbf{k}} \hat{n}_{-\mathbf{k}}(\mathbf{p}') \rangle_0 \Phi_{\mathbf{k}}^{-1}(\mathbf{p}', \mathbf{p}) \hat{n}_{\mathbf{k}}(\mathbf{p}) = \\ \hat{\mathcal{E}}_{\mathbf{k}}^{\text{int}} - \langle \hat{\mathcal{E}}_{\mathbf{k}}^{\text{int}} \hat{n}_{-\mathbf{k}} \rangle_0 S_2^{-1}(\mathbf{k}) \hat{n}_{\mathbf{k}}, \end{aligned} \quad (1.5)$$

$$\hat{\mathcal{E}}_{\mathbf{k}}^{\text{int}} = \frac{1}{2} \sum_{l \neq j=1}^N \Phi(|\mathbf{r}_{lj}|) e^{-\mathbf{i}\mathbf{k} \cdot \mathbf{r}_l}, \quad \hat{n}_{\mathbf{k}} = \sum_{l=1}^N e^{-\mathbf{i}\mathbf{k} \cdot \mathbf{r}_l} \quad (1.6)$$

are the Fourier-components of densities for the interaction energy and the number of particles, respectively. Further, it is more convenient, instead of the dynamical variable of energy $\hat{\mathcal{E}}_{\mathbf{k}}$, to use the variable $\hat{h}_{\mathbf{k}}^{\text{int}}$ (1.5) which is orthogonal to $\hat{n}_{\mathbf{k}}(\mathbf{p})$ by means of the equality:

$$\langle \hat{h}_{\mathbf{k}}^{\text{int}} \hat{n}_{-\mathbf{k}} \rangle_0 = 0. \quad (1.7)$$

From the structure of the dynamical variable $\hat{h}_{\mathbf{k}}^{\text{int}}$ (1.3) it can be seen that it corresponds to a potential part of the Fourier-component of the generalized enthalpy $\hat{h}_{\mathbf{k}}$, which is introduced in molecular hydrodynamics [5,1,2]:

$$\hat{h}_{\mathbf{k}} = \hat{\mathcal{E}}_{\mathbf{k}} - \langle \hat{\mathcal{E}}_{\mathbf{k}} \hat{n}_{-\mathbf{k}} \rangle_0 \hat{n}_{\mathbf{k}} = \hat{h}_{\mathbf{k}}^{\text{kin}} + \hat{h}_{\mathbf{k}}^{\text{int}}, \quad (1.8)$$

where

$$\hat{h}_{\mathbf{k}}^{\text{kin}} = \hat{\mathcal{E}}_{\mathbf{k}}^{\text{kin}} - \langle \hat{\mathcal{E}}_{\mathbf{k}}^{\text{kin}} \hat{n}_{-\mathbf{k}} \rangle_0 S_2^{-1}(\mathbf{k}) \hat{n}_{\mathbf{k}} \quad (1.9)$$

is a kinetic part of the generalized enthalpy,

$$\hat{\mathcal{E}}_{\mathbf{k}}^{\text{kin}} = \sum_{l=1}^N \frac{p_l^2}{2m} e^{-\mathbf{k} \cdot \mathbf{r}_l}$$

is the Fourier-component of the kinetic energy density. $S_2 = \langle \hat{n}_{\mathbf{k}} \hat{n}_{-\mathbf{k}} \rangle_0$ is a static structure factor of the system.

$$\Phi_{\mathbf{k}}^{-1}(\mathbf{p}'', \mathbf{p}') = \frac{\delta(\mathbf{p}'' - \mathbf{p}')}{n f_0(p'')} - c_2(\mathbf{k}), \quad (1.10)$$

where $c_2(\mathbf{k})$ denotes a direct correlation function which is connected with the correlation function $h_2(\mathbf{k})$ as: $h_2(\mathbf{k}) = c_2(\mathbf{k})[1 - n c_2(\mathbf{k})]^{-1}$.

$$\varphi_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}'; t, t') = \sum_{\mathbf{p}''} \langle I_n(\mathbf{k}, \mathbf{p}) T_0(t, t') I_n(-\mathbf{k}, \mathbf{p}'') \rangle_0 \Phi_{\mathbf{k}}^{-1}(\mathbf{p}'', \mathbf{p}'), \quad (1.11)$$

$$\varphi_{hn}(\mathbf{k}, \mathbf{p}; t, t') = \sum_{\mathbf{p}'} \langle I_h^{\text{int}}(\mathbf{k}) T_0(t, t') I_n(-\mathbf{k}, \mathbf{p}') \rangle_0 \Phi_{\mathbf{k}}^{-1}(\mathbf{p}', \mathbf{p}), \quad (1.12)$$

$$\varphi_{nh}(\mathbf{k}, \mathbf{p}; t, t') = \langle I_n(\mathbf{k}, \mathbf{p}) T_0(t, t') I_h^{\text{int}}(-\mathbf{k}) \rangle_0 \Phi_{hh}^{-1}(\mathbf{k}), \quad (1.13)$$

$$\varphi_{hh}(\mathbf{k}; t, t') = \langle I_h^{\text{int}}(\mathbf{k}) T_0(t, t') I_h^{\text{int}}(-\mathbf{k}) \rangle_0 \Phi_{hh}^{-1}(\mathbf{k}). \quad (1.14)$$

are generalized transport kernels (memory functions) which describe kinetic and hydrodynamic processes.

$$I_n(\mathbf{k}, \mathbf{p}) = (1 - \mathcal{P}_0) \hat{n}_{\mathbf{k}}(\mathbf{p}), \quad (1.15)$$

$$I_h^{\text{int}}(\mathbf{k}) = (1 - \mathcal{P}_0) \hat{h}_{\mathbf{k}}^{\text{int}} \quad (1.16)$$

are generalized fluxes in the linear approximation, $\hat{n}_{\mathbf{k}}(\mathbf{p}) = iL_N \hat{n}_{\mathbf{k}}(\mathbf{p})$, $\hat{h}_{\mathbf{k}}^{\text{int}} = iL_N \hat{h}_{\mathbf{k}}^{\text{int}}$, $T_0(t, t') = e^{(t-t')(1-\mathcal{P}_0)iL_N}$ is a time evolution operator with the projection operator \mathcal{P}_0 acts on the dynamical variables $\hat{A}_{\mathbf{k}}$

$$\begin{aligned} \mathcal{P}_0 \hat{A}_{\mathbf{k}'} &= \sum_{\mathbf{k}} \langle \hat{A}_{\mathbf{k}'} \hat{h}_{-\mathbf{k}}^{\text{int}} \rangle_0 \Phi_{hh}^{-1}(\mathbf{k}) \hat{h}_{\mathbf{k}}^{\text{int}} + \\ &\sum_{\mathbf{k}} \sum_{\mathbf{p}\mathbf{p}'} \langle \hat{A}_{\mathbf{k}'} \hat{n}_{-\mathbf{k}}(\mathbf{p}') \rangle_0 \Phi_{\mathbf{k}}^{-1}(\mathbf{p}', \mathbf{p}) \hat{n}_{\mathbf{k}}(\mathbf{p}). \end{aligned} \quad (1.17)$$

The system of transport equations (1.1), (1.2) is closed. We are used a Laplace transform with respect to time, assuming that at $t > 0$ the quantities $f_{\mathbf{k}}(\mathbf{p}; t = 0)$, $h_{\mathbf{k}}^{\text{int}}(t = 0)$ are known

$$A(z) = i \int_0^{\infty} dt e^{izt} A(t), \quad z = \omega + i\varepsilon, \quad \varepsilon \rightarrow +0. \quad (1.18)$$

Then, equations (1.1) and (1.2) are presented in the form:

$$z f_{\mathbf{k}}(\mathbf{p}; z) + \frac{i\mathbf{k} \cdot \mathbf{p}}{m} f_{\mathbf{k}}(\mathbf{p}; z) = -\frac{i\mathbf{k} \cdot \mathbf{p}}{m} n f_0(p) c_2(\mathbf{k}) \times \quad (1.19)$$

$$\sum_{\mathbf{p}'} f_{\mathbf{k}}(\mathbf{p}'; z) + \Sigma_{nh}(\mathbf{k}, \mathbf{p}; z) h_{\mathbf{k}}^{\text{int}}(z) -$$

$$\sum_{\mathbf{p}'} \varphi_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}'; z) f_{\mathbf{k}}(\mathbf{p}'; z) + f_{\mathbf{k}}(\mathbf{p}; t = 0),$$

$$z h_{\mathbf{k}}^{\text{int}}(z) = \sum_{\mathbf{p}'} \Sigma_{hn}(\mathbf{k}, \mathbf{p}'; z) f_{\mathbf{k}}(\mathbf{p}'; z) - \quad (1.20)$$

$$\varphi_{hh}(\mathbf{k}; z) h_{\mathbf{k}}^{\text{int}}(z) + h_{\mathbf{k}}^{\text{int}}(t = 0),$$

$$\Sigma_{nh}(\mathbf{k}, \mathbf{p}; z) = i\Omega_{nh}(\mathbf{k}, \mathbf{p}) - \varphi_{nh}(\mathbf{k}, \mathbf{p}; z), \quad (1.21)$$

$$\Sigma_{hn}(\mathbf{k}, \mathbf{p}; z) = i\Omega_{hn}(\mathbf{k}, \mathbf{p}) - \varphi_{hn}(\mathbf{k}, \mathbf{p}; z). \quad (1.22)$$

In the next subsection, on the basis of a system of transport equations for Fourier components of the nonequilibrium one-particle distribution function and the potential part of enthalpy (1.21), (1.22), we shall obtain equations for time correlation functions. We shall also investigate the spectrum of collective excitations and the structure of generalized transport coefficients.

2. Time correlation functions, collective modes and generalized transport coefficients

With the help of combined equations (1.1), (1.2) one obtains a system for time correlation functions:

$$\Phi_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}'; t) = \sum_{\mathbf{p}''} \langle \hat{n}_{\mathbf{k}}(\mathbf{p}; t) \hat{n}_{-\mathbf{k}}(\mathbf{p}''; 0) \rangle_0 \Phi_{\mathbf{k}}^{-1}(\mathbf{p}'', \mathbf{p}'), \quad (2.1)$$

$$\Phi_{hn}(\mathbf{k}, \mathbf{p}; t) = \sum_{\mathbf{p}'} \langle \hat{h}_{\mathbf{k}}^{\text{int}}(t) \hat{n}_{-\mathbf{k}}(\mathbf{p}'; 0) \rangle_0 \Phi_{\mathbf{k}}^{-1}(\mathbf{p}', \mathbf{p}), \quad (2.2)$$

$$\Phi_{nh}(\mathbf{k}, \mathbf{p}; t) = \langle \hat{n}_{\mathbf{k}}(\mathbf{p}; t) \hat{h}_{-\mathbf{k}}^{\text{int}}(0) \rangle_0 \Phi_{hh}^{-1}(\mathbf{k}), \quad (2.3)$$

$$\Phi_{hh}(\mathbf{k}; t) = \langle \hat{h}_{\mathbf{k}}^{\text{int}}(t) \hat{h}_{-\mathbf{k}}^{\text{int}}(0) \rangle_0 \Phi_{hh}^{-1}(\mathbf{k}), \quad (2.4)$$

where $\hat{n}_{\mathbf{k}}(\mathbf{p}; t) = e^{-iL_N t} \hat{n}_{\mathbf{k}}(\mathbf{p}; 0)$, $\hat{h}_{\mathbf{k}}^{\text{int}}(t) = e^{-iL_N t} \hat{h}_{\mathbf{k}}^{\text{int}}(0)$.

One uses the Fourier transform with respect to time

$$\langle a \rangle_\omega = \int_{-\infty}^{\infty} dt e^{i\omega t} \langle a \rangle^t.$$

Then we write the system of equations (1.1), (1.2) in the form:

$$-i\omega \langle \hat{n}_{\mathbf{k}}(\mathbf{p}) \rangle_\omega = \sum_{\mathbf{p}'} \Sigma_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}'; \omega + i\varepsilon) \langle \hat{n}_{\mathbf{k}}(\mathbf{p}') \rangle_\omega + \Sigma_{nh}(\mathbf{k}, \mathbf{p}; \omega + i\varepsilon) \langle \hat{h}_{\mathbf{k}}^{\text{int}} \rangle_\omega, \quad (2.5)$$

$$-i\omega \langle \hat{h}_{\mathbf{k}}^{\text{int}} \rangle_\omega = \sum_{\mathbf{p}'} \Sigma_{hn}(\mathbf{k}, \mathbf{p}'; \omega + i\varepsilon) \langle \hat{n}_{\mathbf{k}}(\mathbf{p}') \rangle_\omega - \varphi_{hh}(\mathbf{k}; \omega + i\varepsilon) \langle \hat{h}_{\mathbf{k}}^{\text{int}} \rangle_\omega, \quad (2.6)$$

where

$$\Sigma_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}'; \omega + i\varepsilon) = i\Omega_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}') - \varphi_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}'; \omega + i\varepsilon), \quad (2.7)$$

$$\Sigma_{nh}(\mathbf{k}, \mathbf{p}; \omega + i\varepsilon) = i\Omega_{nh}(\mathbf{k}, \mathbf{p}) - \varphi_{nh}(\mathbf{k}, \mathbf{p}; \omega + i\varepsilon), \quad (2.8)$$

$$\Sigma_{hn}(\mathbf{k}, \mathbf{p}; \omega + i\varepsilon) = i\Omega_{hn}(\mathbf{k}, \mathbf{p}) - \varphi_{hn}(\mathbf{k}, \mathbf{p}; \omega + i\varepsilon). \quad (2.9)$$

It is more convenient to present the system of equations (2.5), (2.6) in a matrix form:

$$-i\omega \langle \tilde{a}_{\mathbf{k}} \rangle_\omega = \tilde{\Sigma}(\mathbf{k}; \omega + i\varepsilon) \langle \tilde{a}_{\mathbf{k}} \rangle_\omega, \quad (2.10)$$

where $\tilde{a}_{\mathbf{k}} = \text{col}(\hat{n}_{\mathbf{k}}(\mathbf{p}), \hat{h}_{\mathbf{k}}^{\text{int}})$ is a vector-column and

$$\tilde{\Sigma}(\mathbf{k}; \omega + i\varepsilon) = \begin{bmatrix} \sum_{\mathbf{p}'} \Sigma_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}'; \omega + i\varepsilon) & \Sigma_{nh}(\mathbf{k}, \mathbf{p}; \omega + i\varepsilon) \\ \sum_{\mathbf{p}'} \Sigma_{hn}(\mathbf{k}, \mathbf{p}'; \omega + i\varepsilon) & -\varphi_{hh}(\mathbf{k}; \omega + i\varepsilon) \end{bmatrix}, \quad (2.11)$$

$$\tilde{\Sigma}(\mathbf{k}; \omega + i\varepsilon) = \int_0^\infty dt e^{i(\omega + i\varepsilon)t} \tilde{\Sigma}(\mathbf{k}; t).$$

Now, one uses the solution to the Liouville equation in approximation [1,2] without introducing the projection operator $\mathcal{P}_q(t)$:

$$\varrho(x^N; t) = \varrho_q(x^N; t) - \int_{-\infty}^t dt' e^{\varepsilon(t'-t)} e^{iL_N(t'-t)} \left(\frac{\partial'}{\partial t'} + iL_N \right) \varrho_q(x^N; t').$$

Then, from the self-consistency conditions $\langle \tilde{a}_{\mathbf{k}} \rangle^t = \langle \tilde{a}_{\mathbf{k}} \rangle_q^t$ one obtains a system of equations which connects the average values $\langle \hat{n}_{\mathbf{k}}(\mathbf{p}) \rangle_\omega$ and $\langle \hat{h}_{\mathbf{k}}^{\text{int}} \rangle_\omega$ with spectral functions of time correlation functions:

$$i\omega \tilde{\Phi}(\mathbf{k}; \omega + i\varepsilon) \langle \tilde{a}_{\mathbf{k}} \rangle_\omega = \left[\tilde{\Phi}(\mathbf{k}) - i(\omega + i\varepsilon) \tilde{\Phi}(\mathbf{k}; \omega + i\varepsilon) \right] \langle \tilde{a}_{\mathbf{k}} \rangle_\omega, \quad (2.12)$$

where

$$\tilde{\Phi}(\mathbf{k}; \omega + i\varepsilon) = \begin{bmatrix} \sum_{\mathbf{p}'} \Phi_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}'; \omega + i\varepsilon) & \Phi_{nh}^{\text{int}}(\mathbf{k}, \mathbf{p}; \omega + i\varepsilon) \\ \sum_{\mathbf{p}'} \Phi_{hn}^{\text{int}}(\mathbf{k}, \mathbf{p}'; \omega + i\varepsilon) & -\Phi_{hh}^{\text{int}}(\mathbf{k}; \omega + i\varepsilon) \end{bmatrix}.$$

Let us multiply equation (2.10) by the matrix $\tilde{\Phi}(\mathbf{k}; \omega + i\varepsilon)$ and compare the result with equation (2.12). So we find

$$z \tilde{\Phi}(\mathbf{k}; z) = \tilde{\Sigma}(\mathbf{k}; z) \tilde{\Phi}(\mathbf{k}; z) - \tilde{\Phi}(\mathbf{k}), \quad z = \omega + i\varepsilon, \quad (2.13)$$

$$\tilde{\Phi}(\mathbf{k}; z) = i \int_0^\infty dt e^{izt} \tilde{\Phi}(\mathbf{k}; t), \quad \varepsilon \rightarrow 0$$

or in an explicit form:

$$z \Phi_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}'; z) = \sum_{\mathbf{p}''} \Sigma_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}''; z) \Phi_{nn}(\mathbf{k}, \mathbf{p}'', \mathbf{p}'; z) + \Sigma_{nh}(\mathbf{k}, \mathbf{p}; z) \Phi_{hn}^{\text{int}}(\mathbf{k}, \mathbf{p}'; z) - \Phi_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}'), \quad (2.14)$$

$$z \Phi_{nh}^{\text{int}}(\mathbf{k}, \mathbf{p}; z) = \sum_{\mathbf{p}''} \Sigma_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}''; z) \Phi_{nh}^{\text{int}}(\mathbf{k}, \mathbf{p}'', z) + \Sigma_{nh}(\mathbf{k}, \mathbf{p}; z) \Phi_{hh}^{\text{int}}(\mathbf{k}; z), \quad (2.15)$$

$$z \Phi_{hn}^{\text{int}}(\mathbf{k}, \mathbf{p}'; z) = \sum_{\mathbf{p}''} \Sigma_{hn}(\mathbf{k}, \mathbf{p}'', z) \Phi_{nn}^{\text{int}}(\mathbf{k}, \mathbf{p}'', \mathbf{p}'; z) - \varphi_{hh}(\mathbf{k}; z) \Phi_{hn}^{\text{int}}(\mathbf{k}, \mathbf{p}'; z), \quad (2.16)$$

$$z \Phi_{hh}^{\text{int}}(\mathbf{k}; z) = \sum_{\mathbf{p}''} \Sigma_{hn}(\mathbf{k}, \mathbf{p}'', z) \Phi_{nh}^{\text{int}}(\mathbf{k}, \mathbf{p}'', z) - \varphi_{hh}(\mathbf{k}; z) \Phi_{hh}^{\text{int}}(\mathbf{k}; z) - \Phi_{hh}(\mathbf{k}), \quad (2.17)$$

where the condition $\Phi_{hn}(\mathbf{k}, \mathbf{p}') = \Phi_{nh}(\mathbf{k}, \mathbf{p}) = 0$ is taken into account.

In order to solve the system of equations (2.14)–(2.17) we also apply the projection procedure [4,6]. Let us introduce the dimensionless momentum $\xi = \frac{\mathbf{k}}{mv_0}$, $v_0^2 = (m\beta)^{-1}$. Then the system of equations (2.13)–(2.17) can be rewritten in the matrix form:

$$z \tilde{\Phi}(\mathbf{k}; \xi, \xi'; z) - \tilde{\Sigma}(\mathbf{k}; \xi, \xi''; z) \tilde{\Phi}(\mathbf{k}; \xi'', \xi'; z) = -\tilde{\Phi}(\mathbf{k}; \xi, \xi'), \quad (2.18)$$

where it is clear that the integration must be performed with respect to the repeating indices ξ'' . Further, let us introduce the scalar product of two functions, $\phi(\xi)$ and $\psi(\xi)$, as

$$\langle \phi | \psi \rangle = \sum_{\xi} \phi^*(\xi) f_0(\xi) \psi(\xi). \quad (2.19)$$

Then, the matrix element for some "operator" M can be determined as

$$\langle \phi | M | \psi \rangle = \sum_{\xi} d\xi' \phi^*(\xi) M(\xi, \xi') f_0(\xi') \psi(\xi'). \quad (2.20)$$

Let $\phi(\xi) = \{\phi_{\mu}(\xi)\}$ be the orthogonalized basis of functions with the weight $f_0(\xi)$, so that the following condition is satisfied:

$$\langle \phi_{\nu} | \phi_{\mu} \rangle = \delta_{\nu\mu}, \quad \sum_{\nu} |\phi_{\nu}\rangle \langle \phi_{\nu}| = 1, \quad (2.21)$$

where

$$\phi_{\mu}(\xi) = \phi_{lmn}(\xi) = (l!m!n!)^{-1/2} \bar{H}_l(\xi_x) \bar{H}_m(\xi_y) \bar{H}_n(\xi_z), \quad (2.22)$$

$\bar{H}_l(\xi) = 2^{-l/2} H_l(\xi/2)$, $H_l(\xi)$ is a Hermite polynomial. Then, each function in the system of equations (2.18), which depends on momentum variables ξ, ξ' , can be expanded over functions $\phi_{\mu}(\xi)$ in the series:

$$\tilde{\Phi}(\mathbf{k}; \xi, \xi'; z) = \sum_{\nu, \mu} \phi_{\nu}^*(\xi) \tilde{\Phi}_{\nu\mu}(\mathbf{k}; z) \phi_{\mu}(\xi') f_0(\xi'), \quad (2.23)$$

$$\tilde{\Sigma}(\mathbf{k}; \xi, \xi'; z) = \sum_{\nu, \mu} \phi_{\nu}^*(\xi) \tilde{\Sigma}_{\nu\mu}(\mathbf{k}; z) \phi_{\mu}(\xi') f_0(\xi'), \quad (2.24)$$

where

$$\tilde{\Phi}_{\nu\mu}(\mathbf{k}; z) = \langle \phi_{\nu} | \tilde{\Phi}(\mathbf{k}; \xi, \xi'; z) | \phi_{\mu} \rangle = \sum_{\xi'} \phi_{\nu}^*(\xi) f_0(\xi) \tilde{\Phi}(\mathbf{k}; \xi, \xi'; z) \phi_{\mu}(\xi'), \quad (2.25)$$

$$\tilde{\Sigma}_{\nu\mu}(\mathbf{k}; z) = \langle \phi_{\nu} | \tilde{\Sigma}(\mathbf{k}; \xi, \xi'; z) | \phi_{\mu} \rangle = \sum_{\xi'} \phi_{\nu}^*(\xi) f_0(\xi) \tilde{\Sigma}(\mathbf{k}; \xi, \xi'; z) \phi_{\mu}(\xi'). \quad (2.26)$$

Let us substitute expansions (2.23)–(2.26) into equation (2.18). As a result, one obtains:

$$z \tilde{\Phi}_{\nu\mu}(\mathbf{k}; z) - \sum_{\gamma} \tilde{\Sigma}_{\nu\gamma}(\mathbf{k}; z) \tilde{\Phi}_{\gamma\mu}(\mathbf{k}; z) = -\tilde{\Phi}_{\nu\mu}(\mathbf{k}). \quad (2.27)$$

In actual calculations, a finite number of functions from the set $\phi_{\nu}(\xi)$ is used. Taking into account this fact, let us introduce the projection

operator P which projects arbitrary functions $\psi(\xi)$ onto a finite set of functions $\phi_{\mu}(\xi)$:

$$P = \sum_{\nu=1}^n |\phi_{\nu}\rangle \langle \phi_{\nu}| = 1 - Q, \quad P|\psi\rangle = \sum_{\nu=1}^n \langle \psi | \phi_{\nu} \rangle \langle \phi_{\nu} |. \quad (2.28)$$

Here n denotes a finite number of functions. Then, from (2.27) we obtain a system of equations for a finite set of functions $\phi_{\mu}(\xi)$,

$$\sum_{\gamma=1}^n \left[z \tilde{\delta}_{\nu\gamma} - i \tilde{\Omega}_{\nu\gamma}(\mathbf{k}) + \tilde{D}_{\nu\gamma}(\mathbf{k}; z) \right] \tilde{\Phi}_{\gamma\mu}(\mathbf{k}; z) = -\tilde{\Phi}_{\nu\mu}(\mathbf{k}), \quad (2.29)$$

where

$$\tilde{D}_{\nu\mu}(\mathbf{k}; z) = \langle \phi_{\nu} | \tilde{\varphi}(\mathbf{k}; z) + \tilde{\Sigma}(\mathbf{k}; z) Q \left[z - Q \tilde{\Sigma}(\mathbf{k}; z) Q \right]^{-1} Q \tilde{\Sigma}(\mathbf{k}; z) | \phi_{\mu} \rangle \quad (2.30)$$

are generalized hydrodynamic transport kernels and

$$i \tilde{\Omega}_{\nu\mu}(\mathbf{k}) = \langle \phi_{\nu} | i \tilde{\Omega}(\mathbf{k}) | \phi_{\mu} \rangle \quad (2.31)$$

is a frequency matrix. Note that matrices $i \tilde{\Omega}(\mathbf{k})$ and $\tilde{\varphi}(\mathbf{k}; z)$ are defined according to (1.3), (1.4) and (1.11)–(1.14).

Let us find solutions to the system of equations (2.19) in the hydrodynamic region when a set of functions $\phi_{\mu}(\xi)$ present five moments of a one-particle distribution function:

$$\begin{aligned} \phi_1(\xi) &= 1, \quad \phi_2(\xi) = \xi_z, \quad \phi_3(\xi) = \frac{1}{\sqrt{6}}(\xi^2 - 3), \\ \phi_4(\xi) &= \xi_x, \quad \phi_5(\xi) = \xi_y. \end{aligned} \quad (2.32)$$

Then, the following relations are fulfilled:

$$\begin{aligned} \langle 1 | \hat{n}_{\mathbf{k}}(\xi) &= \sum_{\xi} \hat{n}_{\mathbf{k}}(\xi) = \hat{n}_{\mathbf{k}}, \\ \langle \xi_{\gamma} | \hat{n}_{\mathbf{k}}(\xi) &= \sum_{\xi} \hat{n}_{\mathbf{k}}(\xi) \xi_{\gamma} = \hat{p}_{\mathbf{k}}^{\gamma}, \\ \langle 6^{-1/2}(\xi^2 - 3) | \hat{n}_{\mathbf{k}}(\xi) &= \sum_{\xi} \hat{n}_{\mathbf{k}}(\xi) 6^{-1/2}(\xi^2 - 3) = \\ & \hat{\mathcal{E}}_{\mathbf{k}}^{\text{kin}} - 3 \hat{n}_{\mathbf{k}} \beta^{-1} = \hat{h}_{\mathbf{k}}^{\text{kin}}, \end{aligned} \quad (2.33)$$

for the Fourier components of densities for the number of particles, momentum and the kinetic part of generalized enthalpy. Besides that, the

microscopic conservation laws for densities of the number of particles and momentum can be written in the form:

$$\begin{aligned} \langle 1 | \hat{n}_{\mathbf{k}}(\xi) &= -ik_{\gamma} \hat{D}_{\mathbf{k}}^{\gamma} m^{-1}, \\ \langle \xi_{\alpha} | \hat{n}_{\mathbf{k}}(\xi) &= -ik_{\gamma} \hat{\mathbf{T}}_{\mathbf{k}}^{\gamma \alpha}, \end{aligned} \quad (2.34)$$

where $\hat{\mathbf{T}}_{\mathbf{k}}^{\gamma \alpha}$ is a Fourier component of the stress-tensor.

If we choose the direction of wavevector \mathbf{k} along oz -axis, then $\phi_{\nu}(\xi)$, $\nu = 1, 2, 3$ will correspond to longitudinal modes, whereas $\phi_{\nu}(\xi)$ at $\nu = 4, 5$ will be related to transverse modes.

From the system of equations (2.29), at $\nu = 4, 5$, $\phi_4(\xi) = \xi_x$, $\phi_5(\xi) = \xi_y$, one obtains an equation for the Fourier component of the time correlation function connected with the transverse component of the momentum density $\Phi_{44}(\mathbf{k}; z)$. From this equation one finds:

$$\Phi_{44}(\mathbf{k}; z) = \Phi_{pp}^{\perp}(\mathbf{k}; z) = -\frac{1}{z + D_{pp}^{\perp}(\mathbf{k}; z)}, \quad (2.35)$$

where

$$\Phi_{pp}^{\perp}(\mathbf{k}; z) = \langle \xi_x | \Phi_{nn}(\mathbf{k}, \xi, \xi'; z) | \xi'_x \rangle, \quad (2.36)$$

$$D_{pp}^{\perp}(\mathbf{k}; z) = D_{pp}^{\perp(\text{kin})}(\mathbf{k}; z) + D_{pp}^{\perp(\text{int})}(\mathbf{k}; z), \quad (2.37)$$

$$D_{pp}^{\perp(\text{kin})}(\mathbf{k}; z) = \langle \xi_x | \varphi_{nn}(\mathbf{k}, \xi, \xi'; z) | \xi'_x \rangle, \quad (2.38)$$

$$\begin{aligned} D_{pp}^{\perp(\text{int})}(\mathbf{k}; z) = \\ \langle \xi_x | \left[\tilde{\Sigma}(\mathbf{k}, \xi, \xi'; z) Q \left[z - Q \tilde{\Sigma}(\mathbf{k}, \xi, \xi'; z) Q \right]^{-1} Q \tilde{\Sigma}(\mathbf{k}, \xi, \xi'; z) \right]_{nn} | \xi'_x \rangle. \end{aligned} \quad (2.39)$$

$$D_{pp}^{\perp}(\mathbf{k}; z) = ik^2 \eta(\mathbf{k}; z) (mn)^{-1}, \quad (2.40)$$

where $\eta(\mathbf{k}; z)$ denotes a generalized coefficient of shear viscosity. Such a coefficient consists of two main contributions. The first one is $D_{pp}^{\perp(\text{kin})}(\mathbf{k}; z)$, whereas the second contribution $D_{pp}^{\perp(\text{int})}(\mathbf{k}; z)$ describes a relation of kinetic and hydrodynamic processes. If we put $\nu = 1, 2, 3$, $\phi_1(\xi) = 1$, $\phi_2(\xi) = \xi_z$, $\phi_3(\xi) = 6^{-1/2}(\xi^2 - 3)$ in the system of equation (2.29), then we obtain:

$$z \Phi_{na}(\mathbf{k}; z) - i\Omega_{np}(\mathbf{k}) \Phi_{pa}(\mathbf{k}; z) = -\Phi_{na}(\mathbf{k}), \quad (2.41)$$

$$\begin{aligned} z \Phi_{pa}(\mathbf{k}; z) - i\Omega_{pn}(\mathbf{k}) \Phi_{na}(\mathbf{k}; z) + D_{pp}^{\parallel}(\mathbf{k}; z) \Phi_{pa}(\mathbf{k}; z) - \\ \Sigma_{ph\text{kin}}(\mathbf{k}; z) \Phi_{h\text{kin}a}(\mathbf{k}; z) - \Sigma_{ph\text{int}}(\mathbf{k}; z) \Phi_{h\text{int}a}(\mathbf{k}; z) = -\Phi_{pa}(\mathbf{k}), \end{aligned} \quad (2.42)$$

$$\begin{aligned} z \Phi_{h\text{kin}a}(\mathbf{k}; z) - \Sigma_{h\text{kin}p}(\mathbf{k}; z) \Phi_{pa}(\mathbf{k}; z) + D_{h\text{kin}h\text{kin}}(\mathbf{k}; z) \Phi_{h\text{kin}a}(\mathbf{k}; z) + \\ D_{h\text{kin}h\text{int}}(\mathbf{k}; z) \Phi_{h\text{int}a}(\mathbf{k}; z) = -\Phi_{h\text{kin}a}(\mathbf{k}), \end{aligned} \quad (2.43)$$

$$\begin{aligned} z \Phi_{h\text{int}a}(\mathbf{k}; z) - \Sigma_{h\text{int}p}(\mathbf{k}; z) \Phi_{pa}(\mathbf{k}; z) + D_{h\text{int}h\text{kin}}(\mathbf{k}; z) \Phi_{h\text{kin}a}(\mathbf{k}; z) + \\ D_{h\text{int}h\text{int}}(\mathbf{k}; z) \Phi_{h\text{int}a}(\mathbf{k}; z) = -\Phi_{h\text{int}a}(\mathbf{k}), \end{aligned} \quad (2.44)$$

where $a = \{\hat{n}_{\mathbf{k}}, \hat{\mathbf{p}}_{\mathbf{k}}, \hat{h}_{\mathbf{k}}^{\text{kin}}, \hat{h}_{\mathbf{k}}^{\text{int}}\}$ and

$$\begin{aligned} \Sigma_{ph\text{kin}}(\mathbf{k}; z) &= i\Omega_{ph\text{kin}}(\mathbf{k}) - D_{ph\text{kin}}(\mathbf{k}; z), \\ \Sigma_{ph\text{int}}(\mathbf{k}; z) &= i\Omega_{ph\text{int}}(\mathbf{k}) - D_{ph\text{int}}(\mathbf{k}; z), \\ \Sigma_{h\text{kin}p}(\mathbf{k}; z) &= i\Omega_{h\text{kin}p}(\mathbf{k}) - D_{h\text{kin}p}(\mathbf{k}; z), \\ \Sigma_{h\text{int}p}(\mathbf{k}; z) &= i\Omega_{h\text{int}p}(\mathbf{k}) - D_{h\text{int}p}(\mathbf{k}; z), \end{aligned} \quad (2.45)$$

$i\Omega_{ab}(\mathbf{k})$ and $D_{ab}(\mathbf{k}; z)$ are determined according to (2.31), (2.30). From the system of equations (2.41)–(2.44) one can define the Fourier components of the particle number density correlation functions

$$\Phi_{nn}(\mathbf{k}; z) = \Phi_{11}(\mathbf{k}; z) = \langle 1 | \Phi_{nn}(\mathbf{k}, \xi, \xi'; z) | 1 \rangle, \quad (2.46)$$

as well as of the longitudinal component of the momentum density

$$\Phi_{pp}^{\parallel}(\mathbf{k}; z) = \Phi_{22}(\mathbf{k}; z) = \langle \xi_z | \Phi_{nn}(\mathbf{k}, \xi, \xi'; z) | \xi'_z \rangle, \quad (2.47)$$

for the kinetic part of generalized enthalpy

$$\Phi_{h\text{kin}h\text{kin}}(\mathbf{k}; z) = \Phi_{33}(\mathbf{k}; z) = \langle 6^{-\frac{1}{2}}(\xi^2 - 3) | \Phi_{nn}(\mathbf{k}, \xi, \xi'; z) | 6^{-\frac{1}{2}}((\xi')^2 - 3) \rangle \quad (2.48)$$

as well as for the potential part of generalized enthalpy $\Phi_{h\text{int}h\text{int}}(\mathbf{k}; z)$ and cross correlation functions, especially $\Phi_{h\text{int}h\text{kin}}(\mathbf{k}; z)$, $\Phi_{nh\text{kin}}(\mathbf{k}; z)$, $\Phi_{nh\text{int}}(\mathbf{k}; z)$, $\Phi_{ph\text{kin}}(\mathbf{k}; z)$, $\Phi_{ph\text{int}}(\mathbf{k}; z)$. It is important to point out that the system of equations (2.41)–(2.44) corresponds to the system of equations for Fourier components of the average values of densities for the number of particles $\langle \hat{n}_{\mathbf{k}} \rangle_z$, longitudinal momentum $\langle \hat{\mathbf{p}}_{\mathbf{k}} \rangle_z$, kinetic $\langle \hat{h}_{\mathbf{k}}^{\text{kin}} \rangle_z$ and potential $\langle \hat{h}_{\mathbf{k}}^{\text{int}} \rangle_z$ parts of generalized enthalpy:

$$z \langle \hat{n}_{\mathbf{k}} \rangle_z - i\Omega_{np}(\mathbf{k}) \langle \hat{\mathbf{p}}_{\mathbf{k}} \rangle_z = -\langle \hat{n}_{\mathbf{k}}(t=0) \rangle, \quad (2.49)$$

$$z \langle \hat{\mathbf{p}}_{\mathbf{k}} \rangle_z - i\Omega_{pn}(\mathbf{k}) \langle \hat{n}_{\mathbf{k}} \rangle_z + D_{pp}^{\parallel}(\mathbf{k}; z) \langle \hat{\mathbf{p}}_{\mathbf{k}} \rangle_z = \quad (2.50)$$

$$\Sigma_{ph\text{kin}}(\mathbf{k}; z) \langle \hat{h}_{\mathbf{k}}^{\text{kin}} \rangle_z - \Sigma_{ph\text{int}}(\mathbf{k}; z) \langle \hat{h}_{\mathbf{k}}^{\text{int}} \rangle_z = -\langle \hat{\mathbf{p}}_{\mathbf{k}}(t=0) \rangle,$$

$$z \langle \hat{h}_{\mathbf{k}}^{\text{kin}} \rangle_z - \Sigma_{h\text{kin}p}(\mathbf{k}; z) \langle \hat{\mathbf{p}}_{\mathbf{k}} \rangle_z + D_{h\text{kin}h\text{kin}}(\mathbf{k}; z) \langle \hat{h}_{\mathbf{k}}^{\text{kin}} \rangle_z + \quad (2.51)$$

$$D_{h\text{kin}h\text{int}}(\mathbf{k}; z) \langle \hat{h}_{\mathbf{k}}^{\text{int}} \rangle_z = -\langle \hat{h}_{\mathbf{k}}^{\text{kin}}(t=0) \rangle,$$

$$z \langle \hat{h}_{\mathbf{k}}^{\text{int}} \rangle_z - \Sigma_{h\text{int}p}(\mathbf{k}; z) \langle \hat{\mathbf{p}}_{\mathbf{k}} \rangle_z + D_{h\text{int}h\text{kin}}(\mathbf{k}; z) \langle \hat{h}_{\mathbf{k}}^{\text{kin}} \rangle_z + \quad (2.52)$$

$$D_{h\text{int}h\text{int}}(\mathbf{k}; z) \langle \hat{h}_{\mathbf{k}}^{\text{int}} \rangle_z = -\langle \hat{h}_{\mathbf{k}}^{\text{int}}(t=0) \rangle.$$

This system of equations is similar in construction to the equations of molecular hydrodynamics [1]. The difference consists in the fact that instead of the equations for the Fourier component of the mean enthalpy density $\langle \hat{h}_{\mathbf{k}} \rangle^t$ which is introduced in molecular hydrodynamics, there are two connected equations for the Fourier components of mean values of the kinetic and potential parts of enthalpy density. Moreover, instead of the generalized thermal conductivity which appears in molecular hydrodynamics, the dissipation of energy flows is described in equations (2.49)–(2.52) by a set of generalized transport kernels $D_{h^{\text{kin}}h^{\text{kin}}}(\mathbf{k}; z)$, $D_{h^{\text{kin}}h^{\text{int}}}(\mathbf{k}; z)$, $D_{h^{\text{int}}h^{\text{kin}}}(\mathbf{k}; z)$, $D_{h^{\text{int}}h^{\text{int}}}(\mathbf{k}; z)$. Obviously, transport kernels give more detailed information on the dissipation of energy flows in the system because they describe the time evolution of dynamical correlations between the kinetic and potential flows of enthalpy density.

Solving the system of equation (2.41)–(2.44) at $a = n$, one obtains an expression for the correlation function “density-density” $\Phi_{nn}(\mathbf{k}; z)$

$$\Phi_{nn}(\mathbf{k}; z) = -S_2(\mathbf{k}) \left[z - \frac{i\Omega_{np}(\mathbf{k})i\Omega_{pn}(\mathbf{k})}{z + \bar{D}_{pp}^{\parallel}(\mathbf{k}; z)} \right]^{-1}, \quad (2.53)$$

where

$$\bar{D}_{pp}^{\parallel}(\mathbf{k}; z) = D_{pp}^{\parallel}(\mathbf{k}; z) - \quad (2.54)$$

$$\bar{\Sigma}_{ph^{\text{kin}}}(\mathbf{k}; z) [z + \bar{D}_{h^{\text{kin}}h^{\text{kin}}}(\mathbf{k}; z)]^{-1} \bar{\Sigma}_{h^{\text{kin}}p}(\mathbf{k}; z) - \Sigma_{ph^{\text{int}}}(\mathbf{k}; z) [z + D_{h^{\text{int}}h^{\text{int}}}(\mathbf{k}; z)]^{-1} \Sigma_{h^{\text{int}}p}(\mathbf{k}; z),$$

$$\bar{\Sigma}_{ph^{\text{kin}}}(\mathbf{k}; z) = \Sigma_{ph^{\text{kin}}}(\mathbf{k}; z) - \quad (2.55)$$

$$\Sigma_{ph^{\text{int}}}(\mathbf{k}; z) [z + D_{h^{\text{int}}h^{\text{int}}}(\mathbf{k}; z)]^{-1} D_{h^{\text{int}}h^{\text{kin}}}(\mathbf{k}; z),$$

$$\bar{\Sigma}_{h^{\text{kin}}p}(\mathbf{k}; z) = \Sigma_{h^{\text{kin}}p}(\mathbf{k}; z) - \quad (2.56)$$

$$D_{h^{\text{kin}}h^{\text{int}}}(\mathbf{k}; z) [z + D_{h^{\text{int}}h^{\text{int}}}(\mathbf{k}; z)]^{-1} \Sigma_{h^{\text{int}}p}(\mathbf{k}; z),$$

$$\bar{D}_{h^{\text{kin}}h^{\text{kin}}}(\mathbf{k}; z) = D_{h^{\text{kin}}h^{\text{kin}}}(\mathbf{k}; z) - \quad (2.57)$$

$$D_{h^{\text{kin}}h^{\text{int}}}(\mathbf{k}; z) [z + D_{h^{\text{int}}h^{\text{int}}}(\mathbf{k}; z)]^{-1} D_{h^{\text{int}}h^{\text{kin}}}(\mathbf{k}; z).$$

In expressions (2.54)–(2.57) we can observe an interesting renormalization of the functions Σ_{ab} and D_{ab} via the generalized transport kernels for fluctuations of flows of the potential part of enthalpy density. However, $\bar{D}_{pp}^{\parallel}(\mathbf{k}; z)$ is connected only with the generalized shear viscosity $\eta^{\parallel}(\mathbf{k}; z)$, since the densities of the number of particles $\hat{n}_{\mathbf{k}}$ and momentum $\hat{\mathbf{p}}_{\mathbf{k}}$ are included in the set of variables of an abbreviated description.

In our case $\bar{D}_{pp}^{\parallel}(\mathbf{k}; z)$ takes into account both thermal and viscous dynamical correlation processes. Excluding from (2.53) the imaginary part $\Phi_{nn}^{\parallel}(\mathbf{k}; \omega)$ of the correlation function $\Phi_{nn}(\mathbf{k}; z)$, one obtains an expression for the dynamical structure factor $S(\mathbf{k}; \omega)$ in which contributions of transport kernels corresponding to the kinetic and potential parts of the enthalpy density $\hat{h}_{\mathbf{k}}$ are separated. It is evident that the main contribution of the transport kernel $D_{h^{\text{int}}h^{\text{int}}}(\mathbf{k}; z)$ to the $S(\mathbf{k}; \omega)$ for liquids was in the hydrodynamical region (the region of small values of wavevector \mathbf{k} and frequency ω), whereas $D_{h^{\text{kin}}h^{\text{kin}}}(\mathbf{k}; z)$ will contribute to the kinetic region (orders of interatomic distances, small correlation times). In the region of intermediate values of wavevector \mathbf{k} and frequency ω , it is necessary to take into account all the transport kernels $\Sigma_{ph^{\text{kin}}}(\mathbf{k}; z)$, $\Sigma_{ph^{\text{int}}}(\mathbf{k}; z)$, $D_{h^{\text{kin}}h^{\text{kin}}}(\mathbf{k}; z)$, $D_{h^{\text{int}}h^{\text{kin}}}(\mathbf{k}; z)$, $D_{h^{\text{int}}h^{\text{int}}}(\mathbf{k}; z)$. Since it is impossible to perform exact calculations of the described above functions, it is necessary in each separate region to accept models corresponding to the physical processes. Obviously, it is necessary to provide modelling on the level of generalized transport kernels $\varphi_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}'; t, t')$, $\varphi_{nh}(\mathbf{k}, \mathbf{p}; t, t')$, $\varphi_{hn}(\mathbf{k}, \mathbf{p}'; t, t')$, $\varphi_{hh}(\mathbf{k}; t, t')$ (1.11)–(1.14). The modelling problems of transport kernels for intermediate values of \mathbf{k} and ω are reflected in the details of the description of spectra for collective excitations, as well as in the dynamical structure factor. In the limit $k \rightarrow 0$, $\omega \rightarrow 0$, the cross correlations between the kinetic and potential flows of energy and shear flows become not so important and the system of equations (2.53)–(2.57) gives a spectrum of the collective modes of molecular hydrodynamics [1,2]. For intermediate values of \mathbf{k} and ω , the spectrum of collective modes can be found from the condition

$$\begin{bmatrix} z & i\Omega_{np}(\mathbf{k}) & 0 & 0 \\ i\Omega_{pn}(\mathbf{k}) & z + D_{pp}^{\parallel}(\mathbf{k}; z) & \Sigma_{ph^{\text{kin}}}(\mathbf{k}; z) & \Sigma_{ph^{\text{int}}}(\mathbf{k}; z) \\ 0 & \Sigma_{h^{\text{kin}}p}(\mathbf{k}; z) & z + D_{h^{\text{kin}}h^{\text{kin}}}(\mathbf{k}; z) & D_{h^{\text{kin}}h^{\text{int}}}(\mathbf{k}; z) \\ 0 & \Sigma_{h^{\text{int}}p}(\mathbf{k}; z) & D_{h^{\text{int}}h^{\text{kin}}}(\mathbf{k}; z) & z + D_{h^{\text{int}}h^{\text{int}}}(\mathbf{k}; z) \end{bmatrix} = 0, \quad (2.58)$$

in which contributions of the kinetic and potential parts of generalized enthalpy are separated. This will be reflected in the structure of a heat mode at concrete model calculations of the wavevector- and frequency-dependent transport kernels $D_{h^{\text{kin}}h^{\text{kin}}}(\mathbf{k}; z)$, $D_{h^{\text{kin}}h^{\text{int}}}(\mathbf{k}; z)$, $D_{h^{\text{int}}h^{\text{kin}}}(\mathbf{k}; z)$, $D_{h^{\text{int}}h^{\text{int}}}(\mathbf{k}; z)$ depending on \mathbf{k} and ω .

The system of equations (2.41)–(2.44) for time correlation functions allows an extended description of collective modes in liquids, which includes both hydrodynamic and kinetic processes. Including on the basis

of functions $\phi_\nu(\xi)$ (2.32) some additional functions,

$$\psi_Q^l(\xi) = \frac{1}{5}(\xi^2 - 5)\xi_l, \quad \psi_\Pi^l(\xi) = \frac{\sqrt{2}}{2}(\xi_l\xi_j - \frac{1}{3}\xi^2\delta_{lj}), \quad (2.59)$$

which corresponds to a 13-moments approximation of Grad, one obtains a system of equations for time correlation functions of an extended set of hydrodynamic variables $\{\hat{n}_\mathbf{k}, \hat{\mathbf{p}}_\mathbf{k}, \hat{h}_\mathbf{k}^{\text{kin}}, \hat{\Pi}_\mathbf{k}, \hat{\mathbf{Q}}_\mathbf{k}, \hat{h}_\mathbf{k}^{\text{int}}\}$:

$$z\tilde{\Phi}^{\text{H}}(\mathbf{k}; z) - i\tilde{\Omega}^{\text{H}}(\mathbf{k})\tilde{\Phi}^{\text{H}}(\mathbf{k}; z) + \tilde{\varphi}^{\text{H}}(\mathbf{k}; z)\tilde{\Phi}^{\text{H}}(\mathbf{k}; z) = -\tilde{\Phi}^{\text{H}}(\mathbf{k}), \quad (2.60)$$

where

$$\tilde{\Phi}^{\text{H}}(\mathbf{k}; z) = \begin{bmatrix} \Phi_{nn} & \Phi_{np} & \Phi_{nh^{\text{kin}}} & \Phi_{n\Pi} & \Phi_{nQ} & \Phi_{nh^{\text{int}}} \\ \Phi_{pn} & \Phi_{pp} & \Phi_{ph^{\text{kin}}} & \Phi_{p\Pi} & \Phi_{pQ} & \Phi_{ph^{\text{int}}} \\ \Phi_{h^{\text{kin}}n} & \Phi_{h^{\text{kin}}p} & \Phi_{h^{\text{kin}}h^{\text{kin}}} & \Phi_{h^{\text{kin}}\Pi} & \Phi_{h^{\text{kin}}Q} & \Phi_{h^{\text{kin}}h^{\text{int}}} \\ \Phi_{\Pi n} & \Phi_{\Pi p} & \Phi_{\Pi h^{\text{kin}}} & \Phi_{\Pi\Pi} & \Phi_{\Pi Q} & \Phi_{\Pi h^{\text{int}}} \\ \Phi_{Qn} & \Phi_{Qp} & \Phi_{Qh^{\text{kin}}} & \Phi_{Q\Pi} & \Phi_{QQ} & \Phi_{Qh^{\text{int}}} \\ \Phi_{h^{\text{int}}n} & \Phi_{h^{\text{int}}p} & \Phi_{h^{\text{int}}h^{\text{kin}}} & \Phi_{h^{\text{int}}\Pi} & \Phi_{h^{\text{int}}Q} & \Phi_{h^{\text{int}}h^{\text{int}}} \end{bmatrix} \quad (2.61)$$

is a matrix of Laplace images of the time correlation functions, $\hat{\Pi}_\mathbf{k} = \sum_\xi \varphi_\Pi(\xi)\hat{n}_\mathbf{k}(\xi)$, $\hat{\mathbf{Q}}_\mathbf{k} = \sum_\xi \varphi_Q(\xi)\hat{n}_\mathbf{k}(\xi)$,

$$i\tilde{\Omega}^{\text{H}}(\mathbf{k}) = \begin{bmatrix} 0 & i\Omega_{np} & 0 & 0 & 0 & 0 \\ i\Omega_{pn} & 0 & i\Omega_{ph^{\text{kin}}} & i\Omega_{p\Pi} & 0 & i\Omega_{ph^{\text{int}}} \\ 0 & i\Omega_{h^{\text{kin}}p} & 0 & 0 & i\Omega_{h^{\text{kin}}Q} & 0 \\ 0 & i\Omega_{\Pi p} & 0 & 0 & i\Omega_{\Pi Q} & 0 \\ 0 & 0 & i\Omega_{Qh^{\text{kin}}} & i\Omega_{Q\Pi} & 0 & i\Omega_{Qh^{\text{int}}} \\ 0 & i\Omega_{h^{\text{int}}p} & 0 & 0 & i\Omega_{h^{\text{int}}Q} & 0 \end{bmatrix} \quad (2.62)$$

is an extended hydrodynamic frequency matrix,

$$\tilde{\varphi}^{\text{H}}(\mathbf{k}; z) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & D_{pp}^H & D_{ph^{\text{kin}}}^H & D_{p\Pi}^H & D_{pQ}^H & D_{ph^{\text{int}}}^H \\ 0 & D_{h^{\text{kin}}p} & D_{h^{\text{kin}}h^{\text{kin}}} & D_{h^{\text{kin}}\Pi} & D_{h^{\text{kin}}Q} & D_{h^{\text{kin}}h^{\text{int}}} \\ 0 & D_{\Pi p} & D_{\Pi h^{\text{kin}}} & D_{\Pi\Pi} & D_{\Pi Q} & D_{\Pi h^{\text{int}}} \\ 0 & D_{Qp} & D_{Qh^{\text{kin}}} & D_{Q\Pi} & D_{QQ} & D_{Qh^{\text{int}}} \\ 0 & D_{h^{\text{int}}p} & D_{h^{\text{int}}h^{\text{kin}}} & D_{h^{\text{int}}\Pi} & D_{h^{\text{int}}Q} & D_{h^{\text{int}}h^{\text{int}}} \end{bmatrix} \quad (2.63)$$

is a matrix of generalized memory functions, elements of which are transport kernels (2.30) projected on the basis of functions $\phi_\nu(\xi)$ (2.32), (2.59). For such a description, the spectrum of generalized collective modes of the system is determined for intermediate \mathbf{k} and ω by the relation

$\det[z\tilde{I} - i\tilde{\Omega}^{\text{H}}(\mathbf{k}) + \tilde{\varphi}^{\text{H}}(\mathbf{k})] = 0$ which takes into account kinetic and hydrodynamic processes. In the hydrodynamic limit $k \rightarrow 0$, $\omega \rightarrow 0$, when the contribution of cross dissipative correlations between the kinetic and hydrodynamic processes practically vanishes, the system of equations for the time correlation function (2.60), after some transformations, can be reduced to a system of equations for time correlation functions of densities for the number of particles $\hat{n}_\mathbf{k}$, momentum $\hat{\mathbf{p}}_\mathbf{k}$, total enthalpy $\hat{h}_\mathbf{k}$, the generalized stress tensor $\hat{\pi}_\mathbf{k} = (1 - \mathcal{P}_\text{H})iL_N\hat{\mathbf{p}}_\mathbf{k}$ and the generalized enthalpy flow $\hat{\mathbf{q}}_\mathbf{k} = (1 - \mathcal{P}_\text{H})iL_N\hat{h}_\mathbf{k}$, where \mathcal{P}_H is the Mori operator constructed on the dynamical variables $\{\hat{n}_\mathbf{k}, \hat{\mathbf{p}}_\mathbf{k}, \hat{h}_\mathbf{k}\}$. For such a system of equations, the spectrum of collective excitations is determined from [1,2]:

$$\begin{vmatrix} z & i\Omega_{np} & 0 & 0 & 0 \\ i\Omega_{pn} & z & i\Omega_{ph} & i\Omega_{p\pi} & 0 \\ 0 & i\Omega_{hp} & z & 0 & 0 \\ 0 & i\Omega_{\pi p} & 0 & z + \varphi_{\pi\pi} & i\Omega_{\pi Q} + \varphi_{\pi Q} \\ 0 & 0 & 0 & i\Omega_{Q\pi} + \varphi_{Q\pi} & \varphi_{QQ} \end{vmatrix}_{(\mathbf{k}; z)} = 0. \quad (2.64)$$

In the hydrodynamic limit this gives: the heat mode

$$z_\text{H}(k) = D_\text{T}k^2 + \mathcal{O}(k^4), \quad (2.65)$$

two complex conjugated sound modes

$$z_\pm(k) = \pm i\omega_\text{S}(k) + z_\text{S}(k), \quad (2.66)$$

where $\omega_\text{S}(k) = ck + o(k^3)$ is a frequency of sound propagation, $z_\text{S}(k) = \Gamma k^2 + \mathcal{O}(k^4)$ is a frequency of sound damping with the damping coefficient Γ ; two nonvanishing in the limit $k \rightarrow 0$ kinetic modes

$$\begin{aligned} z_\pi(k) &= \varphi_{\pi\pi}(0) + \mathcal{O}(k^2), \\ z_Q(k) &= \varphi_{QQ}(0) + \mathcal{O}(k^2). \end{aligned} \quad (2.67)$$

Here D_T denotes a thermal diffusion coefficient

$$D_\text{T} = \frac{v_{\text{T}Q}^2}{\gamma\varphi_{QQ}(0)} = \frac{\lambda}{mn c_p}, \quad v_{\text{T}Q}^2 = \frac{m\Phi_{QQ} - h^2}{nc_V}, \quad \gamma = c_p/c_V$$

c_p and c_V are, correspondingly, thermodynamic values of specific heats at the constant pressure and volume, λ is a thermal conductivity coefficient, h denotes a thermodynamic value of enthalpy, $c = \gamma/mnS(k=0)$ denotes an adiabatic sound velocity. In (2.66)

$$\Gamma = \frac{1}{2}(\gamma - 1)D_\text{T} + \frac{1}{2}\eta^\parallel \quad (2.68)$$

is a sound damping coefficient with

$$\eta_{\parallel} = \frac{v_{p\pi}^2}{\varphi_{\pi\pi}(0)} = \left(\frac{4}{3}\eta + \kappa \right) / nm, \quad v_{p\pi}^2 = \frac{mS(0)\Phi_{\pi\pi}(0) - \gamma}{mnS(0)},$$

where η and κ are shear and bulk viscosities. However, at fixed values of \mathbf{k} and ω , the transport kernels $\varphi_{\pi\pi}$, $\varphi_{\pi Q}$, $\varphi_{Q\pi}$, φ_{QQ} are expressed via the generalized transport kernels $D_{\nu\mu}(\mathbf{k}; z)$ of matrix (2.63), i.e. via $\varphi_{nn}(\mathbf{k}, \mathbf{p}, \mathbf{p}'; t, t')$, $\varphi_{nh}(\mathbf{k}, \mathbf{p}; t, t')$, $\varphi_{hn}(\mathbf{k}, \mathbf{p}'; t, t')$, $\varphi_{hh}(\mathbf{k}; t, t')$ (1.11)–(1.14), according to the definition $D_{\nu\mu}(\mathbf{k}; z)$ (2.30). Here, it is important to point out that passing from the system of transport equations of a self-consistent description of kinetics and hydrodynamics to the equations of generalized hydrodynamics, we can connect the generalized transport kernels (1.11)–(1.14) with the hydrodynamic transport kernels $D_{\nu\mu}(\mathbf{k}; z)$ in (2.58) or (2.63).

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УЗГОДЖЕНИЙ ОПИС КІНЕТИКИ ТА ГІДРОДИНАМІКИ КВАНТОВИХ
БОЗЕ-СИСТЕМ. ЧАСОВІ КОРЕЛЯЦІЙНІ ФУНКЦІЇ І СПЕКТР
КОЛЕКТИВНИХ ЗБУДЖЕНЬ

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