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SPIN-$\frac{1}{2}$ ISOTROPIC XY CHAIN WITH DZYALOSHYNSKII-MORIYA INTERACTION IN RANDOM LORENTZIAN TRANSVERSE FIELD

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Спін-$\frac{1}{2}$ ізотропний XY ланцюжок з взаємодією Дзялошинського-Морія у випадковому лоренцовому поперечному полі

О.В.Держко, Т.М.Верхоляк

Анотація. Отримано точні результати для термодинамічних властивостей одновимірної спін-$\frac{1}{2}$ ізотропної XY моделі з взаємодією Дзялошинського-Морія у випадковому лоренцовому поперечному полі. Це дозволяє обговорити деякі наближені методи теорії невпорядкованих спінових систем. Запропоновано наближену схему дослідження термодинаміки одновимірної спін-$\frac{1}{2}$ XXZ моделі Гайзенберга з взаємодією Дзялошинського-Морія у випадковому лоренцовому полі.

Spin-$\frac{1}{2}$ isotropic XY chain with Dzyaloshinskii-Moriya interaction in random lorentzian transverse field

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Abstract. The exact results for thermodynamical properties of one-dimensional spin-$\frac{1}{2}$ isotropic XY model with Dzyaloshinskii-Moriya interaction in random lorentzian transverse field are obtained. This permits to discuss some approximate methods of disordered spin systems theory. The approximate scheme of examining the thermodynamics of one-dimensional spin-$\frac{1}{2}$ XXZ Heisenberg model with Dzyaloshinskii-Moriya interaction in random lorentzian field is suggested.

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Shortly after famous paper by E.Lieb, T.Schultz and D.Mattis [1], who making use of Jordan-Wigner transformation reformulated the Hamiltonian of spin $s=\frac{1}{2}$ XY chain in terms of non-interacting fermions and obtained a series of rigorous results, the random versions of such models attract much attention. Some exact results were derived by H.Nishimori [2] for isotropic XY chain in random lorentzian transverse field. It became possible because of the fact that after fermionization of such model one comes to a system of electrons on lattice that may transfer from site to site with the random (lorentzian) energy at sites. The average one-particle Green's function for such model was obtained first by P.Lloyd [3] (see also [4–7]).

What follows is based on the notation that similarly to [2] one can consider the case of more complicate interspin interaction including to it the so-called Dzyaloshinskii-Moriya interaction. It was introduced phenomenologically by I.E.Dzyaloshinskii [8] and then derived by T.Moriya [9] and it is widely used as one of microscopical mechanism (together with ANNNI model) of appearance of incommensurate phase in crystals [10]. V.M.Kontorovich and V.M.Tsukernik noted [11] that taking into account of such interaction in $s=\frac{1}{2}$ XY chain does not destroy the consideration proposed in Ref. [1] since after Jordan-Wigner transformation, as in the previous case, one comes to a quadratic in Fermi operators form. In Ref. [11] the problem about the possibility of appearance of spiral spin structure in such a chain was examined; for this purpose the pair equal-time spin correlation functions were evaluated. Other papers dealing with statistical mecanics of $s=\frac{1}{2}$ XY chains with Dzyaloshinskii-Moriya interaction [12–19] like the Ref. [11] are devoted to perfect (non-random) versions of the model.

The present paper contains some exact results of statistical mechanics of $1D$ $s=\frac{1}{2}$ isotropic XY model with Dzyaloshinskii-Moriya interaction in random lorentzian field. The paper is organized as follows. Fermionization and Lloyd's problem are considered in Section 1. Here the average one-fermion Green's functions, the average spectral density and the average fermion correlation functions are derived. Section 2 contains calculations of thermodynamical properties of the model in question. A discussion of the estimation of static spin correlations is also given in it. The comparison of exact results with the ones obtained within different approximate approaches (Bose commutation rules approximation, Tyablikov-like approximation, coherent potential approximation) are performed in Section 3. The developed in Sections 1 and 2 scheme may be used for the approximate study of $1D$ $s=\frac{1}{2}$ XXZ Heisenberg model with Dzyaloshinskii-Moriya interaction in random lorentzian field.

This possibility is discussed in Section 4. Conclusions are given in Section 5. Briefly the results of the present paper were reported in [20–22].

1. Fermionization, Lloyd's problem, average one-fermion Green's functions, average spectral density, average fermion correlation functions

A chain of N spins $s=\frac{1}{2}$ with interaction between nearest neighbours, that are in transverse fields with random component distributed according to lorentzian law is considered. The Hamiltonian of the model has the form

$$H=\sum_{j=1}^{N}(\Omega_0+\Omega_j)s_j^z+J\sum_{j=1}^{N-1}(s_j^xs_{j+1}^x+s_j^ys_{j+1}^y)$$
$$+D\sum_{j=1}^{N-1}(s_j^xs_{j+1}^y-s_j^ys_{j+1}^x)$$
$$=\sum_{j=1}^{N}(\Omega_0+\Omega_j)\left(s_j^+s_j^--\frac{1}{2}\right)$$
$$+\sum_{j=1}^{N-1}\left(\frac{J+iD}{2}s_j^+s_{j+1}^-+\frac{J-iD}{2}s_j^-s_{j+1}^+\right). \qquad (1.1)$$

After Jordan-Wigner transformation

$$c_1=s_1^-,c_j=(-2s_1^z)(-2s_2^z)...(-2s_{j-1}^z)s_j^-,\ j=2,...,N,$$
$$c_1^+=s_1^+,c_j^+=(-2s_1^z)(-2s_2^z)...(-2s_{j-1}^z)s_j^+,\ j=2,...,N,$$
$$\{c_i^+,c_j\}=\delta_{ij},\ \{c_i^+,c_j^+\}=0,\ \{c_i,c_j\}=0 \qquad (1.2)$$

one comes to the following quadratic in Fermi operators Hamiltonian

$$H=\sum_{j=1}^{N}(\Omega_0+\Omega_j)\left(c_j^+c_j-\frac{1}{2}\right)$$
$$+\sum_{j=1}^{N-1}\left(\frac{J+iD}{2}c_j^+c_{j+1}-\frac{J-iD}{2}c_jc_{j+1}^+\right), \qquad (1.3)$$

that can be treated like in Ref. [3].

Let's introduce the following retarded and advanced temperature two-times Green's functions [23]

$$G^{\mp}_{nm}(t)\equiv\mp i\theta(\pm t)<\{c_n(t),c^{+}_{m}(0)\}>. \qquad (1.4)$$

The goal of further consideration is to find the average Green's functions $\overline{G^{\mp}_{nm}(t)}$, where the average means

$$\overline{(...)}\equiv\int_{-\infty}^{+\infty}d\Omega_1...\int_{-\infty}^{+\infty}d\Omega_N p(...,\Omega_j,...)(...),$$
$$p(...,\Omega_j,...)=\prod_{j=1}^{N}\frac{1}{\pi}\frac{\Gamma}{\Omega_j^2+\Gamma^2} \qquad (1.5)$$

(i.e. Ω_js are independently distributed according to the lorentzian probability distribution density centred at $\Omega_j=0$ with the width Γ).

It is easy to get the equation of motion for (1.4), namely

$$i\frac{d}{dt}G^{\mp}_{nm}(t)=\delta(t)\delta_{nm}+(\Omega_0+\Omega_n)G^{\mp}_{nm}(t)$$
$$+\frac{J+iD}{2}G^{\mp}_{n+1,m}(t)+\frac{J-iD}{2}G^{\mp}_{n-1,m}(t). \qquad (1.6)$$

Using spectral representation of Green's functions (1.4)

$$G^{\mp}_{nm}(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\omega\exp{(-i\omega t)}G^{\mp}_{nm}(\omega),$$
$$G^{\mp}_{nm}(\omega)=\int_{-\infty}^{+\infty}dt\exp{(i\omega t)}G^{\mp}_{nm}(t) \qquad (1.7)$$

the equations (1.6) can be rewritten in the form

$$\omega G^{\mp}_{nm}(\omega)=\delta_{nm}+(\Omega_0+\Omega_n)G^{\mp}_{nm}(\omega)$$
$$+\frac{J+iD}{2}G^{\mp}_{n+1,m}(\omega)+\frac{J-iD}{2}G^{\mp}_{n-1,m}(\omega), \qquad (1.8)$$

or in the form that is initial for locator expansion

$$G^{\mp}_{nm}(\omega)=\frac{\delta_{nm}}{\omega-(\Omega_0+\Omega_n)}+\frac{\frac{J+iD}{2}G^{\mp}_{n+1,m}(\omega)+\frac{J-iD}{2}G^{\mp}_{n-1,m}(\omega)}{\omega-(\Omega_0+\Omega_n)}. \qquad (1.9)$$

Formal solution as a series with respect to intersite interaction reads

$$G^{\mp}_{nm}(\omega)=\frac{\delta_{nm}}{\omega-(\Omega_0+\Omega_n)}$$
$$+\frac{J+iD}{2}\frac{1}{\omega-(\Omega_0+\Omega_n)}\frac{\delta_{n+1,m}}{\omega-(\Omega_0+\Omega_{n+1})}$$
$$+\frac{J-iD}{2}\frac{1}{\omega-(\Omega_0+\Omega_n)}\frac{\delta_{n-1,m}}{\omega-(\Omega_0+\Omega_{n-1})}+... . \qquad (1.10)$$

It is easy to average $G^{\mp}_{nm}(\omega\pm i\epsilon)$ presented as (1.10). Really, one should perform the averaging of $[\frac{1}{\omega\pm i\epsilon-(\Omega_0+\Omega_j)}]^{k_j}$, i.e. to calculate the integral

$$\frac{1}{\pi}\int_{-\infty}^{+\infty}d\Omega_j\frac{\Gamma}{\Omega_j^2+\Gamma^2}\left[\frac{1}{\omega\pm i\epsilon-(\Omega_0+\Omega_j)}\right]^{k_j}$$
$$=\frac{1}{\pi}\int_{-\infty}^{+\infty}d\Omega_j\frac{\Gamma}{(\Omega_j+i\Gamma)(\Omega_j-i\Gamma)[\omega\pm i\epsilon-(\Omega_0+\Omega_j)]^{k_j}}. \qquad (1.11)$$

For retarded (advanced) Green's function the integrand has one pole in the lower (upper) half of the complex plane Ω_j and two poles in the upper (lower) half of the complex plane Ω_j. Thus expanding the contour of integration in the lower (upper) half of the complex plane and using the residuum theory one ends up with

$$\overline{\left[\frac{1}{\omega\pm i\epsilon-(\Omega_0+\Omega_j)}\right]^{k_j}}=\left[\frac{1}{\omega\pm i\epsilon-(\Omega_0\mp i\Gamma)}\right]^{k_j} \qquad (1.12)$$

that is the famous property of lorentzian distribution. In result the average series (1.10) can be summed up with the result

$$\overline{G^{\mp}_{nm}(\omega\pm i\epsilon)}=\frac{\delta_{nm}}{\omega\pm i\epsilon-(\Omega_0\mp i\Gamma)}$$
$$+\frac{\frac{J+iD}{2}\overline{G^{\mp}_{n+1,m}(\omega\pm i\epsilon)}+\frac{J-iD}{2}\overline{G^{\mp}_{n-1,m}(\omega\pm i\epsilon)}}{\omega\pm i\epsilon-(\Omega_0\mp i\Gamma)}. \qquad (1.13)$$

Since the average Green's functions are translationally invariant the equations (1.13) can be solved with the help of transformation

$$\overline{G^{\mp}_{nm}(\omega\pm i\epsilon)}=\frac{1}{N}\sum_{\kappa}\exp[i(n-m)\kappa]\overline{G^{\mp}_{\kappa}(\omega\pm i\epsilon)}, \qquad (1.14)$$

$\kappa=\frac{2\pi n}{N}, n=-\frac{N}{2},-\frac{N}{2}+1,...,\frac{N}{2}-1$ for even N or $n=-\frac{N-1}{2},-\frac{N-1}{2}+1,...,\frac{N-1}{2}$ for odd N; the solution of algebraic equation for $\overline{G^{\mp}_{\kappa}(\omega\pm i\epsilon)}$ reads

$$\overline{G^{\mp}_{\kappa}(\omega\pm i\epsilon)}=\frac{1}{\omega-[\Omega_0+\sqrt{J^2+D^2}\cos{(\kappa+\varphi)}]\pm i(\epsilon+\Gamma)},$$
$$\cos\varphi\equiv\frac{J}{\sqrt{J^2+D^2}},\ \sin\varphi\equiv\frac{D}{\sqrt{J^2+D^2}}. \qquad (1.15)$$

The average Green's functions in site representation can be found in result of summation over κ in (1.14) that in thermodynamical limit reduces to the following integration

$$\overline{G^{\mp}_{nm}(\omega \pm i\epsilon)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\kappa \exp\left[i(n-m)\kappa\right] \times$$
$$\frac{1}{\omega - (\Omega_0 + \frac{\sqrt{J^2+D^2}}{2}\{\exp\left[i(\kappa+\varphi)\right] + \exp\left[-i(\kappa+\varphi)\right]\} \pm i(\epsilon+\Gamma))}. \quad (1.16)$$

Setting $z = \exp\left[i(\kappa+\varphi)\right]$ one comes to the contour integral on unit circle in complex plane z

$$\overline{G^{\mp}_{nm}(\omega \pm i\epsilon)} = -\frac{\exp\left[i\varphi(n-m)\right]}{2\pi i}$$
$$\times \oint dz \frac{z^{n-m}}{\frac{\sqrt{J^2+D^2}}{2}z^2 - [\omega - \Omega_0 \pm i(\epsilon+\Gamma)]z + \frac{\sqrt{J^2+D^2}}{2}}. \quad (1.17)$$

The denominator in the integrand in (1.17) should be presented as a product $a(z-z_1)(z-z_2)$ where

$$a = \frac{\sqrt{J^2+D^2}}{2},$$
$$z_1 = \frac{\omega - \Omega_0 \pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}} + \sqrt{\left[\frac{\omega - \Omega_0 \pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}}\right]^2 - 1},$$
$$z_2 = \frac{\omega - \Omega_0 \pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}} - \sqrt{\left[\frac{\omega - \Omega_0 \pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}}\right]^2 - 1}, \quad (1.18)$$

and since $z_1 z_2 = 1$ $z_1(z_2)$ is over (in) the unit circle. For $n \geq m$ the integration yields

$$\frac{\exp\left[i\varphi(n-m)\right]}{\sqrt{J^2+D^2}} \frac{\{\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}} - \sqrt{[\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}}]^2-1}\}^{n-m}}{\sqrt{[\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}}]^2-1}}, \quad (1.19)$$

whereas for $n \leq m$

$$\frac{\exp\left[i\varphi(n-m)\right]}{\sqrt{J^2+D^2}} \frac{\{\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}} - \sqrt{[\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}}]^2-1}\}^{m-n}}{\sqrt{[\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}}]^2-1}}. \quad (1.20)$$

Combing (1.19), (1.20) one finally gets for (1.17)

$$\overline{G^{\mp}_{nm}(\omega \pm i\epsilon)} =$$
$$\frac{\exp\left[i\varphi(n-m)\right]}{\sqrt{J^2+D^2}} \frac{\{\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}} - \sqrt{[\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}}]^2-1}\}^{|n-m|}}{\sqrt{[\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}}]^2-1}}. \quad (1.21)$$

The obtained result (1.21) gives the average elementary excitation spectral density

$$\overline{\rho(E)} \equiv \frac{1}{N}\sum_j \overline{\delta(E-\Lambda_j)} = -\frac{1}{\pi}\frac{1}{N}\sum_\kappa \mathrm{Im}\overline{G^{-}_{\kappa}(E+i\epsilon)}$$
$$= -\frac{1}{\pi}\mathrm{Im}\overline{G^{-}_{nn}(E)} = -\frac{1}{\pi}\mathrm{Im}\frac{1}{\sqrt{(E-\Omega_0+i\Gamma)^2-(J^2+D^2)}}. \quad (1.22)$$

It will be used in the next Section for examination of thermodynamical properties of the model in question.

In order to estimate spin correlations one should calculate average fermion correlation function $\overline{< c^{+}_{m}(0)c_n(t) >}$ that can be found from the relation [23]

$$\overline{< c^{+}_{m}(0)c_n(t) >} = -\frac{1}{\pi}\mathrm{Im}\int_{-\infty}^{\infty} d\omega \exp\left(-i\omega t\right)\frac{\overline{G^{-}_{nm}(\omega+i\epsilon)}}{\exp\left(\beta\omega\right)+1}. \quad (1.23)$$

In particular, the static average fermion correlation function at low tempereture limit comes out from the following calculation

$$\overline{< c^{+}_{m}(0)c_{m+p}(0) >} \equiv \overline{< c^{+}_{m}c_{m+p} >} = \overline{< c^{+}_{n-p}(0)c_n(0) >}$$
$$= -\frac{1}{\pi}\mathrm{Im}\int_{-\infty}^{0} d\omega \overline{G^{-}_{n,n-p}(\omega+i\epsilon)}$$
$$= -\frac{1}{\pi}\mathrm{Im}\exp\left(i\varphi p\right)\int_{-\infty}^{-\Omega} dy \frac{[y+i\gamma-\sqrt{(y+i\gamma)^2-1}]^{|p|}}{\sqrt{(y+i\gamma)^2-1}}$$
$$= \frac{1}{\pi\mid p\mid}\mathrm{Im}\left\{\exp\left(i\varphi p\right)\left[y+i\gamma-\sqrt{(y+i\gamma)^2-1}\right]^{|p|}\Big|^{y=-\Omega}_{y=-\infty}\right\}$$
$$= \frac{1}{\pi\mid p\mid}\mathrm{Im}\left\{\exp\left(i\varphi p\right)\right.$$
$$\times\left[-\omega_0+\sqrt{\frac{\sqrt{(\omega_0^2-\gamma^2-1)^2+4\gamma^2\omega_0^2}+\omega_0^2-\gamma^2-1}{2}}\right.$$
$$\left.\left.+i\gamma-i\sqrt{\frac{\sqrt{(\omega_0^2-\gamma^2-1)^2+4\gamma^2\omega_0^2}-\omega_0^2+\gamma^2+1}{2}}\right]^{|p|}\right\}, \quad (1.24)$$

where $y \equiv \frac{\omega}{\sqrt{J^2+D^2}} - \omega_0$, $\omega_0 \equiv \frac{\Omega_0}{\sqrt{J^2+D^2}}$, $\gamma \equiv \frac{\epsilon+\Gamma}{\sqrt{J^2+D^2}}$. The derived results (1.15), (1.21), (1.22), (1.24) remind the corresponding expressions obtained in slightly different cases in Refs. [24,25].

2. Thermodynamical properties. The influence of Dzyaloshinskii-Moriya interaction

The obtained in the previous Section results are of great use in understanding the thermodynamical properties of the model in question. Really, consider a model with a certain realization of transverse fields at sites. After exploiting Jordan-Wigner transformation (1.2) for the Hamiltonian (1.1) one comes to a quadratic in Fermi operators form that can be diagonalized by a canonical transformation $\eta_k = \sum_{j=1}^{N}(g_{kj}c_j + h_{kj}c_j^+)$ [1] (see also [26,27]) with the result $H = \sum_{k=1}^{N}\Lambda_k(\eta_k^+\eta_k - \frac{1}{2})$. Elementary excitation spectrum Λ_k and the coefficients g_{kj} and h_{kj} are determined from $\Lambda_k g_{kn} = \sum_{i=1}^{N} g_{ki}A_{in}$, $-\Lambda_k h_{kn} = \sum_{i=1}^{N} h_{ki}A_{in}^*$, where $A_{ij} = (\Omega_0 + \Omega_i)\delta_{ij} + \frac{J+iD}{2}\delta_{j,i+1} + \frac{J-iD}{2}\delta_{j,i-1}$. The calculation of free energy per site for this realization is straightforward

$$f = \lim_{N\to\infty}\frac{1}{N}\left\{-\frac{1}{\beta}\ln\prod_k\left[\exp\left(-\frac{\beta\Lambda_k}{2}\right)+\exp\left(\frac{\beta\Lambda_k}{2}\right)\right]\right\}$$
$$= -\frac{1}{\beta}\int dE\rho(E)\ln\left(2\cosh\frac{\beta E}{2}\right),$$
$$\rho(E) \equiv \lim_{N\to\infty}\frac{1}{N}\sum_k\delta(E-\Lambda_k) \qquad (2.1)$$

and the result of its averaging over configuration is given by

$$\overline{f} = -\frac{1}{\beta}\int dE\overline{\rho(E)}\ln\left(2\cosh\frac{\beta E}{2}\right)$$
$$= -\frac{1}{\beta}\int dE\overline{\rho(E)}\ln\left[1+\exp\left(-\beta E\right)\right] - \frac{\Omega_0}{2}, \qquad (2.2)$$

where $\overline{\rho(E)}$ is the average spectral density that has been found in the Section 1 (formula (1.22)).

Further one finds the internal energy

$$\overline{e} = \overline{f} + \beta\frac{\partial\overline{f}}{\partial\beta} = \int dE\overline{\rho(E)}\frac{E}{1+\exp\left(\beta E\right)} - \frac{\Omega_0}{2}, \qquad (2.3)$$

the entropy

$$\overline{s} = \beta^2\frac{\partial\overline{f}}{\partial\beta} = \int dE\overline{\rho(E)}\left\{\ln\left[1+\exp\left(-\beta E\right)\right] + \frac{\beta E}{1+\exp\left(\beta E\right)}\right\}, \qquad (2.4)$$

and the specific heat

$$\overline{c} = -\beta\frac{\partial\overline{s}}{\partial\beta} = \beta^2\int dE\overline{\rho(E)}\frac{E^2}{(2\cosh\frac{\beta E}{2})^2}. \qquad (2.5)$$

Using magical property of $\overline{\rho(E)}$ (1.22)

$$\frac{\partial}{\partial\Omega_0}\overline{\rho(E)} = \frac{\partial}{\partial\Omega_0}\left[-\frac{1}{\pi}\mathrm{Im}\frac{1}{\sqrt{(E-\Omega_0+i\Gamma)^2-(J^2+D^2)}}\right]$$
$$= -\frac{\partial}{\partial E}\overline{\rho(E)} \qquad (2.6)$$

one finds the transverse magnetization

$$\overline{<\frac{1}{N}\sum_{j=1}^{N}s_j^z>} = \frac{\partial\overline{f}}{\partial\Omega_0} = \int dE\overline{\rho(E)}\frac{1}{1+\exp\left(\beta E\right)} - \frac{1}{2} \qquad (2.7)$$

and static transverse susceptibility

$$\overline{\chi_{zz}} = \frac{\partial\overline{<\frac{1}{N}\sum_{j=1}^{N}s_j^z>}}{\partial\Omega_0} = -\beta\int dE\overline{\rho(E)}\frac{1}{(2\cosh\frac{\beta E}{2})^2}. \qquad (2.8)$$

Note, that since at $T\to 0$ $\frac{1}{1+\exp(\beta E)}\to 0$ if $E>0$ and $\frac{1}{1+\exp(\beta E)}\to 1$ if $E<0$, the transverse magnetization and static transverse susceptibility at $\Omega_0 = 0$ in low tempereture limit are given by

$$\overline{<\frac{1}{N}\sum_{j=1}^{N}s_j^z>}$$
$$= \int_0^{-\Omega_0}dE'\left[-\frac{1}{\pi}\mathrm{Im}\frac{1}{\sqrt{(E'+i\Gamma)^2-(J^2+D^2)}}\right], \qquad (2.9)$$

$$\overline{\chi_{zz}} = -\frac{1}{\pi}\frac{1}{\sqrt{\Gamma^2+J^2+D^2}}. \qquad (2.10)$$

The results of numerical investigation of thermodynamical properties of the model (1.1) are depicted in Figs.1-5. The dependence on Dzyaloshinskii-Moriya interaction ($D=0,\ D=0.5J,\ D=J$) of the average spectral density $\overline{\rho(E)}$ ($\Omega_0=0$) for different values of the width of lorentzian distribution Γ ($\Gamma=0,\ \Gamma=0.25J,\ \Gamma=0.5J,\ \Gamma=J$) is shown in Fig.1. In Figs.2,3 it is shown the temperature behaviour of entropy and specific heat for $\Omega_0=0$ for different values of Dzyaloshinskii-Moriya interaction ($D=0,\ D=0.5J,\ D=J$) and different values of Γ ($\Gamma=0,\ \Gamma=J$). The changes in the dependence of transverse magnetization on the value of transverse field Ω_0 that are caused by Dzyaloshinskii-Moriya interaction ($D=0,\ D=0.5J,\ D=J$) for several values of Γ ($\Gamma=0,\ \Gamma=J$) at $T=0$ can be seen in Fig.4. In

Fig.5 the temperature behaviour of static transverse susceptibility in zero transverse field for few values of Dzyaloshinskii-Moriya interaction ($D = 0,\ D = 0.5J,\ D = J$) and Γ ($\Gamma = 0,\ \Gamma = J$) are presented.

As it can be easily seen from the formula for $\overline{\rho(E)}$ (1.22) and in Fig.1 the presence of Dzyaloshinskii-Moriya interaction formally causes

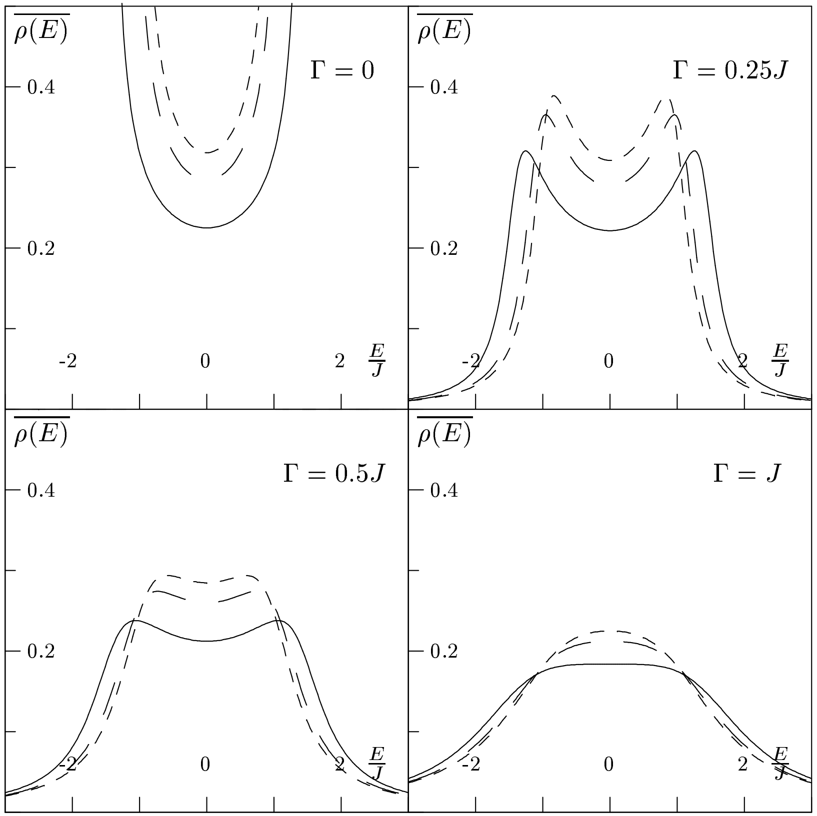

Figure 1: The average spectral density $\overline{\rho(E)}$ (1.22) vs. $\frac{E}{J}$ for different values of Γ; $\Omega_0 = 0$, $D = 0$ (dashed curves), $D = 0.5J$ (long dashed curves) and $D = J$ (solid curves).

the change of J^2 to $J^2 + D^2$. This leads to effective increase of interspin interaction and results in broadening of zones both with sharp edges (when $\Gamma = 0$) and with smooth ones because of randomness (when $\Gamma \neq 0$). Rather small quantitative changes in temperature behaviour of entropy and specific heat (Figs.2,3) reveal the tendency caused by

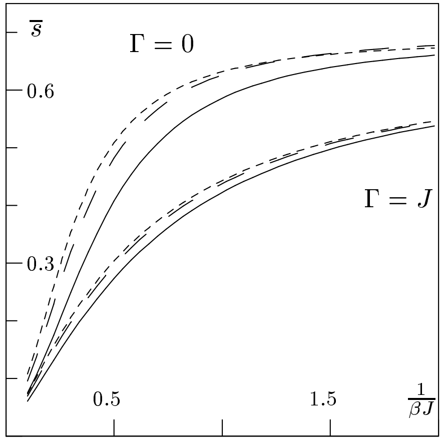

Figure 2: Temperature dependence of entropy $\overline{s}$ for $\Gamma = 0$ and $\Gamma = J$; $\Omega_0 = 0$, $D = 0$ (dashed curves), $D = 0.5J$ (long dashed curves) and $D = J$ (solid curves).

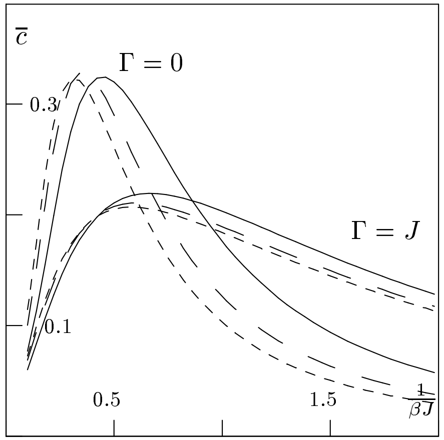

Figure 3: Temperature dependence of specific heat $\overline{c}$ for $\Gamma = 0$ and $\Gamma = J$; $\Omega_0 = 0$, $D = 0$ (dashed curves), $D = 0.5J$ (long dashed curves) and $D = J$ (solid curves).

Dzyaloshinskii-Moriya interaction. Dzyaloshinskii-Moriya interaction decreases the transverse magnetization for a given value of transverse field in both cases when $\Gamma = 0$ and when $\Gamma \neq 0$ (Fig.4) and decreases the value of static transverse susceptibility in zero transverse field in low temperature region. The obtained result are in agreement with the derived ones for non-random version of $1D$ $s = \frac{1}{2}$ isotropic XY model in transverse field [18,19].

Unfortunately, the obtained results do not permit to get any exact estimations for static spin correlation functions. Considering the simplest one $\overline{< s_j^z s_{j+n}^z >}$, using the relation $s_j^z = c_j^+ c_j - \frac{1}{2}$ and exploiting Wick-Bloch-de Dominicis theorem one finds

$$\begin{aligned}
&\overline{< s_j^z s_{j+n}^z >}\\
&= \overline{< c_j^+ c_j c_{j+n}^+ c_{j+n} >} - \frac{1}{2}\overline{< c_j^+ c_j >} - \frac{1}{2}\overline{< c_{j+n}^+ c_{j+n} >} + \frac{1}{4}\\
&= \overline{< c_j^+ c_j >< c_{j+n}^+ c_{j+n} >} - \overline{< c_j^+ c_{j+n}^+ >< c_j c_{j+n} >}\\
&+ \overline{< c_j^+ c_{j+n} >< c_j c_{j+n}^+ >} - \overline{< c_j^+ c_j >} + \frac{1}{4}.
\end{aligned} \tag{2.11}$$

Thus, knowing only $\overline{< c_m^+ c_n >}$ it is not possible to derive any rigorous results even for simplest (transverse) static spin correlation function. It is worthwhile to note that its calculation faces with the similar problems that were considered in [4]. Apparently, the problem of spin correlations can be solved within developed recently numerical approach [26–30].

3. Standard approximate approaches in the theory of disordered spin systems

Let's consider the results one faces with after adopting some standard approximate approaches in disordered spin systems theory while considering the thermodynamics of the model defined by (1.1).

3.1. Bose commutation rules approximation for spin operators s^+, s^-

Implying instead of Pauli commutation rules for spin operators s^+, s^- $[s_j^-, s_m^+] = \delta_{jm}(1 - 2s_j^+ s_j^-)$ Bose commutation rules, that is

$$[s_j^-, s_m^+] \approx \delta_{jm}, \tag{3.1}$$

one comes to a system of bosons on lattice that may transfer from site to site with random (lorentzian) energy at sites. Introducing the following

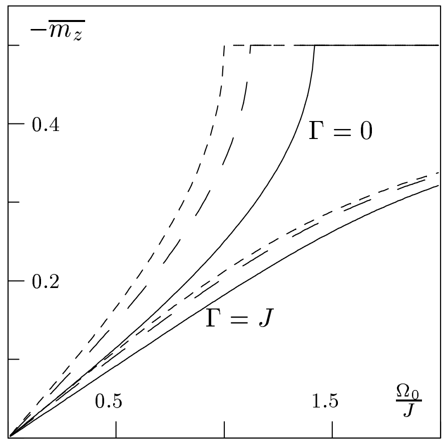

Figure 4: The dependence of transverse magnetization $\overline{m_z} \equiv \overline{< \frac{1}{N}\sum_{j=1}^N s_j^z >}$ on transverse field $\frac{\Omega_0}{J}$ for $\Gamma = 0$ and $\Gamma = J$; $1/\beta = 0$, $D = 0$ (dashed curves), $D = 0.5J$ (long dashed curves) and $D = J$ (solid curves).

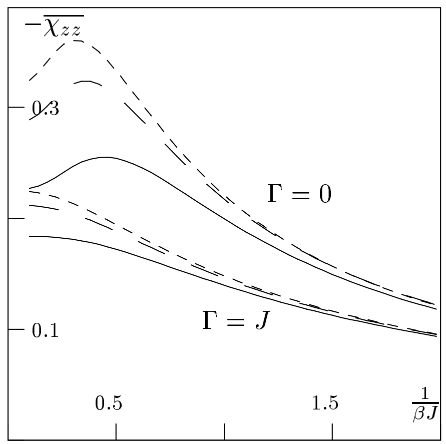

Figure 5: Temperature dependence of static transverse susceptibility $\overline{\chi_{zz}}$ for $\Gamma = 0$ and $\Gamma = J$; $\Omega_0 = 0$, $D = 0$ (dashed curves), $D = 0.5J$ (long dashed curves) and $D = J$ (solid curves).

Green's functions

$$D^{\mp}_{nm}(t) \equiv \mp i\theta(\pm t) < [s^-_n(t), s^+_m(0)] >, \quad (3.2)$$

and repeating the derivation of Section 1 one ends up with

$$\overline{D^{\mp}_{nm}(\omega\pm i\epsilon)} = \frac{\exp[i\varphi(n-m)]}{\sqrt{J^2+D^2}} \frac{\{\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}} - \sqrt{[\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}}]^2-1}\}^{|n-m|}}{\sqrt{[\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{\sqrt{J^2+D^2}}]^2-1}} \quad (3.3)$$

that gives for average spectral density formula (1.22). However, the average boson correlation function $\overline{< s^+_m(0)s^-_n(t) >} = -\frac{1}{\pi}\mathrm{Im}\int_{-\infty}^{\infty} d\omega \times \exp(-i\omega t)\frac{\overline{D^-_{nm}(\omega+i\epsilon)}}{\exp(\beta\omega)-1}$ contains the denominator that tends to 0 when $\omega\to 0$.

Let's discuss this problem in details. After assuming Bose commutation relation one has the quadratic in Bose operators s^+_j, s^-_j form (1.1) that with the help of linear canonical transformation $\gamma_k = \sum_{j=1}^N(f_{kj}s^-_j + d_{kj}s^+_j)$ can be diagonalized with the result $H = -\frac{1}{2}\sum_{j=1}^N(\Omega_0+\Omega_j) + \sum_{k=1}^N \mathcal{E}_k\gamma^+_k\gamma_k + const$. Since $\mathcal{E}_k$, f_{kj}, d_{kj} are determined from the equations $\mathcal{E}_k f_{kn} = \sum_{i=1}^N f_{ki}A_{in}$, $-\mathcal{E}_k d_{kn} = \sum_{i=1}^N d_{ki}A^*_{in}$, one immediately concludes that $\mathcal{E}_k = \Lambda_k$. The ground state energy of this Hamiltonian coincides with the exact value if $const = -\frac{1}{2}\sum_{k=1}^N\Lambda_k + \frac{1}{2}\sum_{j=1}^N(\Omega_0+\Omega_j)$ and thus finally $H = \sum_{k=1}^N\Lambda_k(\gamma^+_k\gamma_k - \frac{1}{2})$. Free energy per particle is given by

$$f = \lim_{N\to\infty}\frac{1}{N}\times \left\{-\frac{1}{\beta}\ln\prod_k \exp\left(\frac{\beta\Lambda_k}{2}\right)[1+\exp(-\beta\Lambda_k)+\exp(-2\beta\Lambda_k)+...]\right\}$$

$$\overset{\text{all }\Lambda_k>0}{=} \lim_{N\to\infty}\frac{1}{N}\left[-\frac{1}{\beta}\sum_k \ln\frac{\exp\left(\frac{\beta\Lambda_k}{2}\right)}{1-\exp(-\beta\Lambda_k)}\right]$$

$$= \frac{1}{\beta}\int dE\rho(E)\ln[1-\exp(-\beta E)] - \frac{1}{2}\int dE\rho(E)E. \quad (3.4)$$

Note, that the partition function for Bose system exists only when the condition all $\Lambda_\kappa > 0$ (or $\rho(E)=0$ if $E\le 0$) is valid. Really, otherwise the state without bosons is not the ground state, since one-boson state has smaller energy, the energy of two-bosons state is more smaller etc., the probability of their appearance increases respectively and thus the partition function tends to infinity. This difficulty arises because of approximate treating of elementary excitations as bosons (they are exactly fermionic objects in the case under consideration) and it is crushing if $\rho(E)\ne 0$ for $E\le 0$. Thus, one can consider the average thermodynamical quantities obtained within approximation (3.1), that is, free energy

$$\overline{f} = \frac{1}{\beta}\int dE\overline{\rho(E)}\ln[1-\exp(-\beta E)] - \frac{\Omega_0}{2}, \quad (3.5)$$

internal energy

$$\overline{e} = \int dE\overline{\rho(E)}\frac{E}{\exp(\beta E)-1} - \frac{\Omega_0}{2}, \quad (3.6)$$

entropy

$$\overline{s} = \int dE\overline{\rho(E)}\left\{-\ln[1-\exp(-\beta E)] + \frac{\beta E}{\exp(\beta E)-1}\right\}, \quad (3.7)$$

specific heat

$$\overline{c} = \beta^2\int dE\overline{\rho(E)}\frac{E^2}{(2\sinh\frac{\beta E}{2})^2}, \quad (3.8)$$

transverse magnetization

$$\overline{<\frac{1}{N}\sum_{j=1}^N s^z_j>} = \int dE\overline{\rho(E)}\frac{1}{\exp(\beta E)-1} - \frac{1}{2}, \quad (3.9)$$

and static transverse susceptibility

$$\overline{\chi_{zz}} = -\beta\int dE\overline{\rho(E)}\frac{1}{(2\sinh\frac{\beta E}{2})^2} \quad (3.10)$$

when the relation

$$\overline{\rho(E)} = 0 \text{ for } E\le 0 \quad (3.11)$$

holds true.

Considering at first the case $\Gamma = 0$, when

$$\rho(E) = \begin{cases} \frac{1}{\pi}\frac{1}{\sqrt{J^2+D^2-(E-\Omega_0)^2}}, & \Omega_0-\sqrt{J^2+D^2}<E<\Omega_0+\sqrt{J^2+D^2}, \\ 0, & \text{otherwise}, \end{cases} \quad (3.12)$$

one finds that (3.11) is valid only in the case of strong transverse fields

$$\Omega_0 > \sqrt{J^2+D^2}; \quad (3.13)$$

Dzyaloshinskii-Moriya interaction increases the value of the field behind which (3.11) becomes true. In the case $\Gamma \neq 0$ one immediately finds that (3.11) is never true, since there always will be elementary excitations with negative energy and therefore it is impossible to treat them as bosons. Thus, the consideration of the disordered version of the model demands the revision of the problem of validity of approximation that was suitable for study of non-random version of the model.

The results of numerical calculation of temperature behaviours of entropy and specific heat according to exact formulae (2.4), (2.5) (solid lines) and approximate ones (3.7), (3.8) (long dashed curves) in the case of validity of approximation (3.1) ($\Gamma = 0$, $\Omega_0 = (\sqrt{2} + 0.1)J$, $D = 0$, $D = 0.5J$, $D = J$) are presented in Figs.6,7. These results show that Bose commutation rules approximation for spin operators s^+, s^- gives suitable results only for low temperatures, and in the presence of Dzyaloshinskii-Moriya interaction only at very low temperatures.

3.2. Tyablikov-like approximation

Does not assuming Bose commutation rules for operators s^+, s^- (3.1), one faces with the equations of motion that contain more complicated Green's functions

$$\begin{aligned} i\frac{d}{dt}D^{\mp}_{nm}(t) &= \delta(t)\delta_{nm}(-2 < s^z_n >) + (\Omega_0 + \Omega_n)D^{\mp}_{nm}(t) \\ &+ \frac{J + iD}{2}(-2)(\mp i\theta(\pm t) < [s^z_n(t)s^-_{n+1}(t), s^+_m] >) \\ &+ \frac{J - iD}{2}(-2)(\mp i\theta(\pm t) < [s^-_{n-1}(t)s^z_n(t), s^+_m] >). \end{aligned} \tag{3.14}$$

Within Tyablikov-like approximation it is supposed that

$$\begin{aligned} &< s^z_n(t)s^-_{n+1}(t)s^+_m > \approx \overline{< s^z >} < s^-_{n+1}(t)s^+_m >, \\ &< s^+_m s^z_n(t)s^-_{n+1}(t) > \approx \overline{< s^z >} < s^+_m s^-_{n+1}(t) >, \\ &< s^-_{n-1}(t)s^z_n(t)s^+_m > \approx \overline{< s^z >} < s^-_{n-1}(t)s^+_m >, \\ &< s^+_m s^-_{n-1}(t)s^z_n(t) > \approx \overline{< s^z >} < s^+_m s^-_{n-1}(t) > . \end{aligned} \tag{3.15}$$

Then instead of (3.14) one has

$$\begin{aligned} i\frac{d}{dt}D^{\mp}_{nm}(t) &= -2\overline{< s^z >}\delta(t)\delta_{nm} + (\Omega_0 + \Omega_n)D^{\mp}_{nm}(t) \\ &+ (-2\overline{< s^z >})\frac{J + iD}{2}D^{\mp}_{n+1,m}(t) + (-2\overline{< s^z >})\frac{J - iD}{2}D^{\mp}_{n-1,m}(t). \end{aligned} \tag{3.16}$$

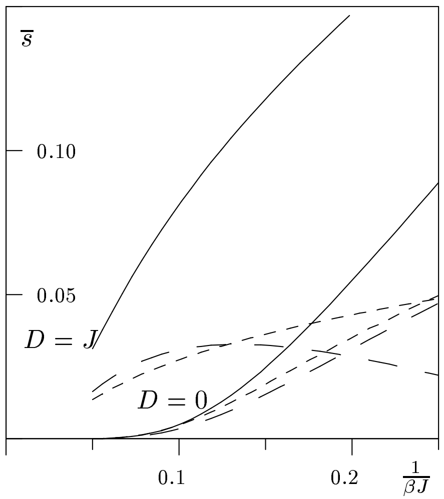

Figure 6: Entropy vs. temperature for $\Gamma = 0$, $\Omega_0 = (\sqrt{2} + 0.1)J$: exact results (solid curves), Bose commutation rules approximation (long dashed curves), Tyablikov-like approximation (dashed curves).

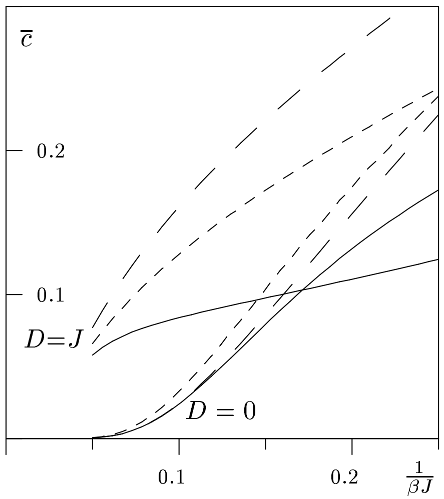

Figure 7: Specific heat vs. temperature for $\Gamma = 0$, $\Omega_0 = (\sqrt{2} + 0.1)J$: exact results (solid curves), Bose commutation rules approximation (long dashed curves), Tyablikov-like approximation (dashed curves).

Acting like in Section 1 one ends up with

$$\overline{D^{\mp}_{nm}(\omega\pm i\epsilon)} = \frac{\exp[i\varphi(n-m)]}{\sqrt{J^2+D^2}} \frac{\left\{\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{(-2\overline{<s^z>})\sqrt{J^2+D^2}} - \sqrt{[\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{(-2\overline{<s^z>})\sqrt{J^2+D^2}}]^2-1}\right\}^{|n-m|}}{\sqrt{[\frac{\omega-\Omega_0\pm i(\epsilon+\Gamma)}{(-2\overline{<s^z>})\sqrt{J^2+D^2}}]^2-1}} \quad (3.17)$$

that yields the following result for average spectral density

$$\overline{\rho(E)} = -\frac{1}{\pi}\mathrm{Im}\frac{1}{\sqrt{[\frac{E-\Omega_0+i\Gamma}{(-2\overline{<s^z>})}]^2-(J^2+D^2)}}. \quad (3.18)$$

The introduced average transverse magnetization at site is determined from the equation

$$\overline{<s^z>} = \overline{<s^+s^->} - \frac{1}{2} = -\frac{1}{\pi}\mathrm{Im}\int_{-\infty}^{\infty} d\omega \frac{\overline{D^-_{mm}(\omega+i\epsilon)}}{\exp(\beta\omega)-1} - \frac{1}{2}$$
$$= \int_{-\infty}^{\infty}\frac{d\omega}{\exp(\beta\omega)-1}\overline{\rho(\omega)} - \frac{1}{2}. \quad (3.19)$$

Equation (3.19) contains Bose factor $\frac{1}{\exp(\beta\omega)-1}$ and thus, apparently, Tyablikov-like approximation is possible if $\overline{\rho(\omega)} = 0$ for $\omega \leq 0$. The temperature behaviours of entropy and specific heat found within Tyablikov-like approximation (3.15) (dashed lines) in comparison with exact results (solid lines) and results obtained within approximation (3.1) (long dashed lines) are shown in Figs.6,7. Despite some improvment over the Bose commutation rules approximation that can be seen in Figs.6,7 the results obtained within Tyablikov-like approximation generally speaking are not closer to exact ones in comparison with the results derived within Bose commutation rules approximation.

3.3. Randomly disordered crystals theory methods

Let's consider $1D$ spin-$\frac{1}{2}$ isotropic XY model with Dzyaloshinskii-Moriya interaction in random (not necessary lorentzian) transverse field within usually used methods in the theory of disordered crystals [5,6,31]. The starting point is the equation for Green's functions that after introducing the notations

$$W_{pr} \equiv \Omega_p\delta_{pr}, \; V_{ps} \equiv \frac{J+iD}{2}\delta_{s,p+1} + \frac{J-iD}{2}\delta_{s,p-1} \quad (3.20)$$

can be written in the form

$$\omega G^{\mp}_{nm}(\omega) = \delta_{nm} + \Omega_0 G^{\mp}_{nm}(\omega) + W_{nr}G^{\mp}_{rm}(\omega) + V_{ns}G^{\mp}_{sm}(\omega) \quad (3.21)$$

(for $G^{\mp}_{nm}(\omega)$ defined by (1.4) or (3.2) within Bose commutation rules approximation) or

$$\omega D^{\mp}_{nm}(\omega) = -2<s^z_n>\delta_{nm} + \Omega_0 D^{\mp}_{nm}(\omega) + W_{nr}D^{\mp}_{rm}(\omega)$$
$$+(-2<s^z_n>)V_{ns}D^{\mp}_{sm}(\omega) \quad (3.22)$$

(for $D^{\mp}_{nm}(\omega)$ (3.2) within Tyablikov approximation, $<s^z_n>$ is only thermodynamically averaged value (without configurational averaging as in (3.15)) of transverse spin at site n); the summation over the repeating indices from 1 to N is implied. The different approaches of randomly disordered crystals theory are constructed from so called propagator and locator expansions.

Propagator expansion. Let's rewrite the Hamiltonian of the system in question (1.1) in the form

$$H = {}^aH + \sum_{j=1}^{N}\Omega_j s^z_j = {}^aH + \sum_{j=1}^{N}\Omega_j\left(s^+_j s^-_j - \frac{1}{2}\right) \quad (3.23)$$

and introduce Green's functions ${}^aG^{\mp}_{nm}(\omega)$ or ${}^aD^{\mp}_{nm}(\omega)$ for the system with Hamiltonian ${}^aH$ that, naturally satisfy the following equations

$$\omega\; {}^aG^{\mp}_{nm}(\omega) = \delta_{nm} + \Omega_0\; {}^aG^{\mp}_{nm}(\omega) + V_{ns}\; {}^aG^{\mp}_{sm}(\omega) \quad (3.24)$$

or

$$\omega\; {}^aD^{\mp}_{nm}(\omega) = -2<s^z_n>\delta_{nm} + \Omega_0\; {}^aD^{\mp}_{nm}(\omega)$$
$$+(-2<s^z_n>)V_{ns}\; {}^aD^{\mp}_{sm}(\omega). \quad (3.25)$$

Multiplying (3.21) (or (3.22)) by ${}^aG^{\mp}_{gn}(\omega)$ (${}^aD^{\mp}_{gn}(\omega)$) one comes to the following equations

$$G^{\mp}_{gm}(\omega) = {}^aG^{\mp}_{gm}(\omega) + {}^aG^{\mp}_{gn}(\omega)W_{nr}G^{\mp}_{rm}(\omega) \quad (3.26)$$

or

$$D^{\mp}_{gm}(\omega) = {}^aD^{\mp}_{gm}(\omega)\frac{<s^z_n>}{<s^z>} + {}^aD^{\mp}_{gn}(\omega)\tilde{W}_{nr}D^{\mp}_{rm}(\omega),$$
$$\tilde{W}_{nr} \equiv \frac{W_{nr}}{-2<s^z>} + V_{nr}\left(\frac{<s^z_n>}{<s^z>} - 1\right). \quad (3.27)$$

Note, that since generally speaking $<s^z_n> \neq <s^z>$ even in the case of diagonal disorder after Tyablikov-like approximation one faces with non-diagonal disorder problem. Expanding of (3.26) (or (3.27)) in degrees of $W_{nr}(\tilde{W}_{nr})$ leads to propagator expansion.

Locator expansion. Let's introduce locators

$$g^{\mp}_{nm}(\omega) \equiv g^{\mp}_{n}\delta_{nm} \equiv \frac{1}{\omega-(\Omega_0+\Omega_n)}\delta_{nm} \qquad (3.28)$$

or

$$d^{\mp}_{nm}(\omega) \equiv d^{\mp}_{n}\delta_{nm} \equiv \frac{-2 < s^z_n >}{\omega-(\Omega_0+\Omega_n)}\delta_{nm}. \qquad (3.29)$$

Then equations (3.21), (3.22) can be rewritten in the form

$$G^{\mp}_{nm}(\omega) = g^{\mp}_{nm}(\omega) + g^{\mp}_{np}(\omega)V_{ps}G^{\mp}_{sm}(\omega) \qquad (3.30)$$

or

$$D^{\mp}_{nm}(\omega) = d^{\mp}_{nm}(\omega) + d^{\mp}_{np}(\omega)V_{ps}D^{\mp}_{sm}(\omega). \qquad (3.31)$$

While expanding r.h.s. of (3.30), (3.31) in degrees of V_{ps} one comes to locator expansion.

Further analysis deals only with diagonal disorder when, for example, the propagator expansions have the form

$$G^{\mp}_{gm}(\omega) = {}^aG^{\mp}_{gm}(\omega) + {}^aG^{\mp}_{gn}(\omega)\Omega_n\, {}^aG^{\mp}_{nm}(\omega) + {}^aG^{\mp}_{gn}(\omega)\Omega_n\, {}^aG^{\mp}_{np}(\omega)\Omega_p\, {}^aG^{\mp}_{pm}(\omega) + \ldots \qquad (3.32)$$

or

$$D^{\mp}_{gm}(\omega) = {}^aD^{\mp}_{gm}(\omega) + {}^aD^{\mp}_{gn}(\omega)\tilde\Omega_n\, {}^aD^{\mp}_{nm}(\omega) + {}^aD^{\mp}_{gn}(\omega)\tilde\Omega_n\, {}^aD^{\mp}_{np}(\omega)\tilde\Omega_p\, {}^aD^{\mp}_{pm}(\omega) + \ldots,$$
$$\tilde\Omega_n = -\Omega_n/2\overline{< s^z >}. \qquad (3.33)$$

Extracting in (3.32) t-matrix

$$t_n \equiv \frac{\Omega_n}{1-\, {}^aG^{\mp}_{nn}(\omega)\Omega_n}, \qquad (3.34)$$

one can rewrite (3.32) as a series in degrees of t-matrix

$$\begin{aligned} G^{\mp}_{gm}(\omega) = {}&{}^aG^{\mp}_{gm}(\omega) + {}^aG^{\mp}_{gn}(\omega)\Omega_n\, {}^aG^{\mp}_{nm}(\omega) \\ &+ {}^aG^{\mp}_{gn}(\omega)\Omega_n\, {}^aG^{\mp}_{nn}(\omega)\Omega_n\, {}^aG^{\mp}_{nm}(\omega) \\ &+ {}^aG^{\mp}_{gn}(\omega)\Omega_n\, {}^aG^{\mp}_{np\ (n\neq p)}(\omega)\Omega_p\, {}^aG^{\mp}_{pm}(\omega) \\ &+ {}^aG^{\mp}_{gn}(\omega)\Omega_n\, {}^aG^{\mp}_{nn}(\omega)\Omega_n\, {}^aG^{\mp}_{nn}(\omega)\Omega_n\, {}^aG^{\mp}_{nm}(\omega) \\ &+ {}^aG^{\mp}_{gn}(\omega)\Omega_n\, {}^aG^{\mp}_{np\ (n\neq p)}(\omega)\Omega_p\, {}^aG^{\mp}_{pp}(\omega)\Omega_p\, {}^aG^{\mp}_{pm}(\omega) \\ &+ {}^aG^{\mp}_{gn}(\omega)\Omega_n\, {}^aG^{\mp}_{nn}(\omega)\Omega_n\, {}^aG^{\mp}_{nf\ (n\neq f)}(\omega)\Omega_f\, {}^aG^{\mp}_{fm}(\omega) \\ &+ {}^aG^{\mp}_{gn}(\omega)\Omega_n\, {}^aG^{\mp}_{np\ (n\neq p)}(\omega)\Omega_p\, {}^aG^{\mp}_{pf\ (p\neq f)}(\omega)\Omega_f\, {}^aG^{\mp}_{fm}(\omega) + \ldots \\ = {}&{}^aG^{\mp}_{gm}(\omega) + {}^aG^{\mp}_{gn}(\omega)t_n\, {}^aG^{\mp}_{nm}(\omega) \\ &+ {}^aG^{\mp}_{gn}(\omega)t_n\, {}^aG^{\mp}_{np\ (n\neq p)}(\omega)t_p\, {}^aG^{\mp}_{pm}(\omega) + \ldots\,. \end{aligned} \qquad (3.35)$$

Within approximation of average t-matrix one assumes that

$$t_n \simeq \overline{t} \equiv \int d\Omega_1\ldots d\Omega_N p(\Omega_1,\ldots,\Omega_N)\frac{\Omega_n}{1-\, {}^aG^{\mp}_{nn}(\omega)\Omega_n}, \qquad (3.36)$$

where

$${}^aG^{\mp}_{nm}(\omega) = \frac{\exp\left[i\varphi(n-m)\right]}{\sqrt{J^2+D^2}}\frac{\left[\frac{\omega-\Omega_0\pm i\epsilon}{\sqrt{J^2+D^2}} - \sqrt{(\frac{\omega-\Omega_0\pm i\epsilon}{\sqrt{J^2+D^2}})^2-1}\right]^{|n-m|}}{\sqrt{(\frac{\omega-\Omega_0\pm i\epsilon}{\sqrt{J^2+D^2}})^2-1}}. \qquad (3.37)$$

In result one is able to sum the series for average Green's functions (3.35)

$$\overline{G^{\mp}_{gm}(\omega)} = \left(\frac{{}^aG^{\mp}(\omega)}{1-\, {}^aG^{\mp}(\omega)\overline{t}}\right)_{gm}. \qquad (3.38)$$

Within coherent potential approximation one should seek the Green's functions of the system with Hamiltonian

$$\check H = \sum_{j=1}^{N}(\Omega_0+\check\Omega)s^z_j + J\sum_{j=1}^{N-1}(s^x_js^x_{j+1}+s^y_js^y_{j+1}) + D\sum_{j=1}^{N-1}(s^x_js^y_{j+1}-s^y_js^x_{j+1}), \qquad (3.39)$$

where $\check\Omega$ is unknown coherent field. These Green's functions are given by

$$\check G^{\mp}_{nm}(\omega) = \frac{\exp\left[i\varphi(n-m)\right]}{\sqrt{J^2+D^2}}\frac{\left[\frac{\omega-\Omega_0-\check\Omega\pm i\epsilon}{\sqrt{J^2+D^2}} - \sqrt{(\frac{\omega-\Omega_0-\check\Omega\pm i\epsilon}{\sqrt{J^2+D^2}})^2-1}\right]^{|n-m|}}{\sqrt{(\frac{\omega-\Omega_0-\check\Omega\pm i\epsilon}{\sqrt{J^2+D^2}})^2-1}}. \qquad (3.40)$$

Since

$$H = \check H + \sum_{j=1}^{N}(\Omega_j-\check\Omega)\left(s^+_js^-_j-\frac{1}{2}\right), \qquad (3.41)$$

$$\omega\check G^{\mp}_{nm}(\omega) = \delta_{nm} + (\Omega_0+\check\Omega)\check G^{\mp}_{nm}(\omega) + V_{ns}\check G^{\mp}_{sm}(\omega), \qquad (3.42)$$

one can get acting like while deriving (3.26) the following equation

$$G^{\mp}_{gm}(\omega) = \check G^{\mp}_{gm}(\omega) + \check G^{\mp}_{gn}(\omega)\check W_{nr}G^{\mp}_{rm}(\omega),$$
$$\check W_{nr} = W_{nr} - \check\Omega\delta_{nr} = (\Omega_n-\check\Omega)\delta_{nr}, \qquad (3.43)$$

and hence the following propagator expansion

$$G^{\mp}_{gm}(\omega) = \check{G}^{\mp}_{gm}(\omega) + \check{G}^{\mp}_{gn}(\omega)(\Omega_n - \check{\Omega})\check{G}^{\mp}_{nm}(\omega)$$
$$+\check{G}^{\mp}_{gn}(\omega)(\Omega_n - \check{\Omega})\check{G}^{\mp}_{np}(\omega)(\Omega_p - \check{\Omega})\check{G}^{\mp}_{pm}(\omega) + \ldots . \tag{3.44}$$

This series can be rewritten as an expansion in degrees of $\check{t}$ -matrix

$$\check{t}_n \equiv \frac{\Omega_n - \check{\Omega}}{1 - \check{G}^{\mp}_{nn}(\omega)(\Omega_n - \check{\Omega})}; \tag{3.45}$$

namely,

$$G^{\mp}_{gm}(\omega) =$$
$$\check{G}^{\mp}_{gm}(\omega)+\check{G}^{\mp}_{gn}(\omega)\check{t}_n\check{G}^{\mp}_{nm}(\omega)+\check{G}^{\mp}_{gn}(\omega)\check{t}_n\check{G}^{\mp}_{np\ (n\neq p)}(\omega)\check{t}_p\check{G}^{\mp}_{pm}(\omega)+\ldots . \tag{3.46}$$

Determining the coherent field $\check{\Omega}$ from the condition

$$\overline{\check{t}_n} \equiv \int d\Omega_1 \ldots d\Omega_N p(\Omega_1, \ldots, \Omega_N) \frac{\Omega_n - \check{\Omega}}{1 - \check{G}^{\mp}_{nn}(\omega)(\Omega_n - \check{\Omega})} = 0, \tag{3.47}$$

where accorging to (3.40)

$$\check{G}^{\mp}_{nn}(\omega \pm i\epsilon) = \frac{1}{\sqrt{(\omega - \Omega_0 - \check{\Omega}\pm i\epsilon)^2 - (J^2 + D^2)}}, \tag{3.48}$$

one finds that

$$\overline{G^{\mp}_{gm}(\omega)} = \check{G}^{\mp}_{gm}(\omega) + \check{G}^{\mp}_{gn}(\omega)\overline{\check{t}_n}\check{G}^{\mp}_{nm}(\omega)$$
$$+\check{G}^{\mp}_{gn}(\omega)\overline{\check{t}_n\check{G}^{\mp}_{np\ (n\neq p)}(\omega)\check{t}_p}\check{G}^{\mp}_{pm}(\omega) + \ldots \simeq \check{G}^{\mp}_{gm}(\omega), \tag{3.49}$$

that is the desired result within coherent potential approximation.

In the case of lorentzian transverse field the equation for coherent field $\check{\Omega}$ (3.47) reads

$$\int_{-\infty}^{\infty} d\Omega_j \frac{1}{\pi}\frac{\Gamma}{(\Omega_j + i\Gamma)(\Omega_j - i\Gamma)}$$
$$\times\frac{(\Omega_j - \check{\Omega})\sqrt{(\omega - \Omega_0 - \check{\Omega}\pm i\epsilon)^2 - (J^2 + D^2)}}{\sqrt{(\omega - \Omega_0 - \check{\Omega}\pm i\epsilon)^2 - (J^2 + D^2)} - \Omega_j + \check{\Omega}} = 0. \tag{3.50}$$

Supposing that $\mathrm{Im}[\check{\Omega} + \sqrt{(\omega - \Omega_0 - \check{\Omega}\pm i\epsilon)^2 - (J^2 + D^2)}] \gtrless 0$ one can perform integration with the help of the residuum theory getting in result

instead of (3.50)

$$\frac{(\mp i\Gamma - \check{\Omega})\sqrt{(\omega - \Omega_0 - \check{\Omega}\pm i\epsilon)^2 - (J^2 + D^2)}}{\sqrt{(\omega - \Omega_0 - \check{\Omega}\pm i\epsilon)^2 - (J^2 + D^2)} + \check{\Omega}\pm i\Gamma} = 0. \tag{3.51}$$

Equation (3.51) has solutions $\check{\Omega} = \mp i\Gamma$ and after insertion them into (3.40) one gets exact result (1.21). It can be proved *post priory* the possibility of assumed displacement of poles in (3.50) at least for $\omega \to \infty$. Thus, in the case of lorentzian transverse field the coherent potetial approximation contains exact result for average Green's functions $\overline{G^{\mp}_{nm}(\omega)}$.

Consider now another version of random transverse field that is given by probability density

$$p(\Omega_1, \ldots, \Omega_N) = \prod_{j=1}^{N}[x\delta(\Omega_j) + (1 - x)\delta(\Omega_j - \mho)]. \tag{3.52}$$

Then the equation for coherent field $\check{\Omega}$ (3.47) reads

$$x\frac{-\check{\Omega}}{1 - \check{G}^{\mp}_{nn}(\omega)(-\check{\Omega})} + (1 - x)\frac{\mho - \check{\Omega}}{1 - \check{G}^{\mp}_{nn}(\omega)(\mho - \check{\Omega})} = 0 \tag{3.53}$$

and after some calculation reduces to 3th order algebraic equation for $\check{\Omega}$

$$\check{\Omega}^3 +$$
$$\frac{(x^2-2x)\mho^2-(J^2+D^2)+4(1-x)\mho(\omega-\Omega_0)+(\omega-\Omega_0)^2}{2(x\mho+\Omega_0-\omega)}\check{\Omega}^2 +$$
$$\frac{(1-x)\mho(J^2+D^2)-(1-x)^2\mho^2(\omega-\Omega_0)-(1-x)\mho(\omega-\Omega_0)^2}{x\mho+\Omega_0-\omega}\check{\Omega} +$$
$$\frac{-(1 - x)^2\mho^2(J^2 + D^2) + (1 - x)^2\mho^2(\omega - \Omega_0)^2}{2(x\mho + \Omega_0 - \omega)}$$
$$= \check{\Omega}^3 + a\check{\Omega}^2 + b\check{\Omega} + c = 0. \tag{3.54}$$

It is generally-known [32] that at first one should substitute $\check{\Omega} = y - \frac{a}{3}$ obtaining in result $y^3 + py + q = 0$ with $p = -\frac{a^2}{3} + b$, $q = 2\left(\frac{a}{3}\right)^3 - \frac{ab}{3} + c$. Then the real "noncomlete" cubic equation has

- one real and two conjugate complex roots if $Q > 0$;
- three real roots at least two of which coincide if $Q = 0$;
- three different real roots if $Q < 0$;

here $Q \equiv \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2$. The solution can be presented in the trigonometrical form

- if $Q \geq 0,\ p > 0$, then

$$y_1 = -2\sqrt{\frac{p}{3}}\cot 2\alpha,\ y_{2,3} = \sqrt{\frac{p}{3}}\left(\cot 2\alpha \pm i\sqrt{3}\csc 2\alpha\right),$$
$$\tan\alpha = \sqrt[3]{\tan\frac{\beta}{2}}\ \left(|\alpha| \leq \frac{\pi}{4}\right),\ \tan\beta = \frac{2}{q}\sqrt{\left(\frac{p}{3}\right)^3}\ \left(|\beta| \leq \frac{\pi}{2}\right); \quad (3.55)$$

- if $Q \geq 0,\ p < 0$, then

$$y_1 = -2\sqrt{-\frac{p}{3}}\csc 2\alpha,\ y_{2,3} = \sqrt{-\frac{p}{3}}\left(\csc 2\alpha \pm i\sqrt{3}\cot 2\alpha\right),$$
$$\tan\alpha = \sqrt[3]{\tan\frac{\beta}{2}}\ \left(|\alpha| \leq \frac{\pi}{4}\right),\ \sin\beta = \frac{2}{q}\sqrt{\left(-\frac{p}{3}\right)^3}\ \left(|\beta| \leq \frac{\pi}{2}\right); \quad (3.56)$$

- if $Q < 0$ (and hence $p < 0$), then

$$y_1 = 2\sqrt{-\frac{p}{3}}\cos\frac{\alpha}{3},\ y_{2,3} = -2\sqrt{-\frac{p}{3}}\cos\left(\frac{\alpha}{3} \pm \frac{\pi}{3}\right),$$
$$\cos\alpha = -\frac{q}{2\sqrt{\left(-\frac{p}{3}\right)^3}} \quad (3.57)$$

(all cubic roots are real).

Further one should act in a following way: 1) for a given x to calculate $a,\ b,\ c$ in (3.54), $p,\ q$ and Q; 2) comparing Q and p with the zero, to write down $y_{1,2,3}$; and hence $\check{\Omega}_{1,2,3}$; 3) to check whether $\check{\bar{t}}$ after inserting $\check{\Omega}_j$ is really equal to zero (since the algebraic transformation from (3.53) to (3.54) may lead to appearance of extra roots); 4) to substitute $\check{\Omega}_j$ into $\check{G}^-_{nn}(E + i\epsilon)$ (3.40); and 5) to calculate the average spectral density within coherent potential approximation (3.47)

$$\overline{\rho(E)} \simeq -\frac{1}{\pi}\mathrm{Im}\check{G}^-_{nn}(E) = -\frac{1}{\pi}\mathrm{Im}\frac{1}{\sqrt{(E - \Omega_0 - \check{\Omega}_j + i\epsilon)^2 - (J^2 + D^2)}}. \quad (3.58)$$

Further the case $J = 1,\ D = 0,\ \mho = 1$ is under consideration. Here the example of performing of this program is given:

$x = 0.01,$

$E = 0.02,$
$a = 47.015001,\ b = -97.000202,\ c = 48.985399,$
$p = -833.803646,\ q = 9267.099096,\ Q = 0.896875,$
$\check{\Omega}_1 = -49.014405 + i0.000000,$
$\check{\Omega}_2 = 0.999702 + i0.001967,$
$\check{\Omega}_3 = 0.999702 - i0.001967,$
$|\check{\bar{t}}|_{\check{\Omega}_1} = 1.012523\times10^{-13},\ |\check{\bar{t}}|_{\check{\Omega}_2} = 3.978199\times10^{-3},\ |\check{\bar{t}}|_{\check{\Omega}_3} = 3.064551\times10^{-12};$

$E = 0.03,$
$a = 22.505001,\ b = -47.985301,\ c = 24.480448,$
$p = -216.810318,\ q = 1228.762879,\ Q = 0.087101,$
$\check{\Omega}_1 = -24.504032 + i0.000000,$
$\check{\Omega}_2 = 0.999516 + i0.002358,$
$\check{\Omega}_3 = 0.999516 - i0.002358,$
$|\check{\bar{t}}|_{\check{\Omega}_1} = 1.187939\times10^{-14},\ |\check{\bar{t}}|_{\check{\Omega}_2} = 4.811497\times10^{-3},\ |\check{\bar{t}}|_{\check{\Omega}_3} = 2.295226\times10^{-12};$

$E = 0.04,$
$a = 14.331667,\ b = -31.640401,\ c = 16.308864,$
$p = -100.105961,\ q = 385.512229,\ Q = 0.023724,$
$\check{\Omega}_1 = -16.330344 + i0.000000,$
$\check{\Omega}_2 = 0.999339 + i0.002665,$
$\check{\Omega}_3 = 0.999339 - i0.002665,$
$|\check{\bar{t}}|_{\check{\Omega}_1} = 5.717649\times10^{-15},\ |\check{\bar{t}}|_{\check{\Omega}_2} = 5.488112\times10^{-3},\ |\check{\bar{t}}|_{\check{\Omega}_3} = 2.070775\times10^{-12};$

$E = 0.05,$
$a = 10.242501,\ b = -23.463002,\ c = 12.220623,$
$p = -58.432610,\ q = 171.921947,\ Q = 0.009691,$
$\check{\Omega}_1 = -12.240840 + i0.000000,$
$\check{\Omega}_2 = 0.999170 + i0.002918,$
$\check{\Omega}_3 = 0.999170 - i0.002918,$
$|\check{\bar{t}}|_{\check{\Omega}_1} = 1.362799\times10^{-14},\ |\check{\bar{t}}|_{\check{\Omega}_2} = 6.062763\times10^{-3},\ |\check{\bar{t}}|_{\check{\Omega}_3} = 3.098188\times10^{-12},$

etc. Since $\check{\bar{t}}$ with $\check{\Omega}_1$ for $x = 0$ and $x = 1$ is not equal to zero, this root has been rejected. Unfortunately, there are no exact results for a random model in question (3.52). Nevertheless, for arbitrary random spin-$\frac{1}{2}$ anisotropic XY model in transverse field it is possible to calculate numerically (see [26,27]) the quantity $R(E^2) = \frac{1}{N}\sum_{k=1}^{N}\delta(E^2 - \Lambda_k^2)$, the average value of which is connected with the average value of $\rho(E)$ by the relation

$$\overline{R(E^2)} = \frac{\overline{\rho(E)} + \overline{\rho(-E)}}{2\mid E\mid}. \tag{3.59}$$

In Fig.8 the results of calculations within coherent potential approximation (3.58), (3.59) (broken lines) are depicted together with the exact results (solid lines). The comparison shows just how little is the change in $\overline{R(E^2)}$ and thus in thermodynamical quantities. This seems to be conditioned by the fact that for the model in question (since it is described by Hamiltonian (1.3)) the thermodynamical averaging has been performed exactly.

4. One-dimensional spin-$\frac{1}{2}$ XXZ Heisenberg model with Dzyaloshinskii-Moriya interaction in lorentzian random external field

This Section is devoted to examining of somewhat more complicated than (1.1) case. Namely, now the intersite interaction contains the coupling of z components of neighbouring spins and the Hamiltonian reads

$$H=\sum_{j=1}^{N}(\Omega_0+\Omega_j)s_j^z+J\sum_{j=1}^{N-1}(s_j^x s_{j+1}^x+s_j^y s_{j+1}^y)+D\sum_{j=1}^{N-1}(s_j^x s_{j+1}^y-s_j^y s_{j+1}^x)$$
$$+J^z\sum_{j=1}^{N-1}s_j^z s_{j+1}^z=\sum_{j=1}^{N}(\Omega_0+\Omega_j)\left(s_j^+s_j^- -\frac{1}{2}\right)$$
$$+\sum_{j=1}^{N-1}\left(\frac{J+iD}{2}s_j^+s_{j+1}^- +\frac{J-iD}{2}s_j^-s_{j+1}^+\right)$$
$$+J^z\sum_{j=1}^{N-1}\left(s_j^+s_j^-s_{j+1}^+s_{j+1}^- -\frac{1}{2}s_j^+s_j^- -\frac{1}{2}s_{j+1}^+s_{j+1}^- +\frac{1}{4}\right). \tag{4.1}$$

After Jordan-Wigner transformation (1.2) one gets

$$H=-\frac{N\Omega_0}{2}+\frac{(N-1)J^z}{4}-\frac{1}{2}\sum_{j=1}^{N}\Omega_j+$$
$$\sum_{j=1}^{N}\left[\Omega_0+\Omega_j-\frac{J^z}{2}(2-\delta_{j,1}-\delta_{j,N})\right]c_j^+c_j+$$
$$\sum_{j=1}^{N-1}\left(\frac{J+iD}{2}c_j^+c_{j+1}-\frac{J-iD}{2}c_jc_{j+1}^+\right)+J^z\sum_{j=1}^{N-1}c_j^+c_jc_{j+1}^+c_{j+1}. \tag{4.2}$$

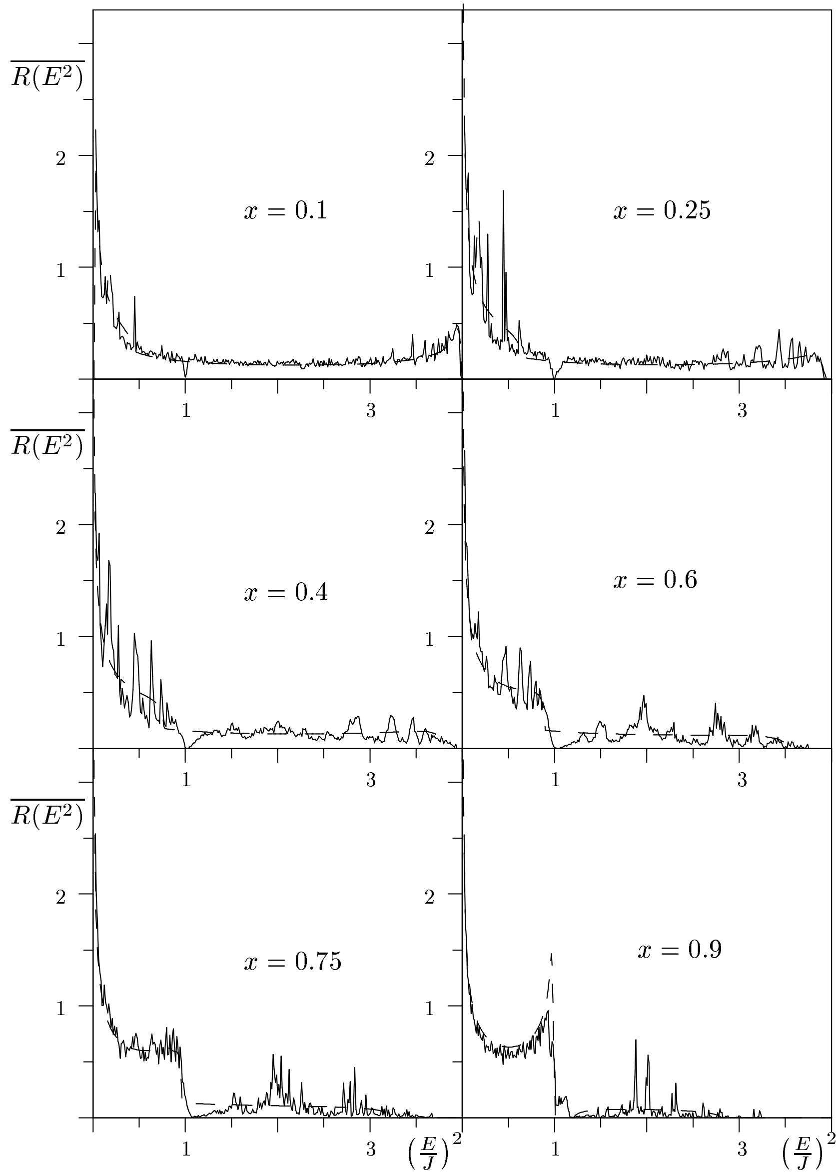

Figure 8: $\overline{R(E^2)}$ vs. E^2: exact results (solid curves) (the averaging is done only over few random realizations) and the results within coherent potential approximation (long dashed curves) for the model with disorder (3.52): $\Omega_0=0$, $J=1$, $D=0$, $\mho=1$.

Note, that the key difficulty is that due to new intersite interaction one faces with the terms that are the products of four Fermi operators. It is easy to find the equations of motion for Green's functions (1.4)

$$\begin{aligned} & i\frac{d}{dt}G^{\mp}_{nm}(t) = \delta(t)\delta_{nm} + (\Omega_0+\Omega_n-J^z)G^{\mp}_{nm}(t) \\ & +\frac{J+iD}{2}G^{\mp}_{n+1,m}(t)+\frac{J-iD}{2}G^{\mp}_{n-1,m}(t) \\ & +J^z[\mp i\theta(\pm t) < \{(c_nc^+_{n+1}c_{n+1})(t), c^+_m(0)\} > \\ & \mp i\theta(\pm t) < \{(c^+_{n-1}c_{n-1}c_n)(t), c^+_m(0)\} >]. \end{aligned} \tag{4.3}$$

Equations (4.3) contain higher Green's functions and thus cannot be solved exactly. Nevertheless, making the approximation

$$\begin{aligned} & < \{(c^+_{n+1}c_{n+1}c_n)(t), c^+_m(0)\} > \approx \overline{< c^+_jc_j >} < \{c_n(t), c^+_m\} > \\ & -\overline{< c^+_{j+1}c_j >} < \{c_{n+1}(t), c^+_m\} >, \\ & < \{(c^+_{n-1}c_{n-1}c_n)(t), c^+_m(0)\} > \approx \overline{< c^+_jc_j >} < \{c_n(t), c^+_m\} > \\ & -\overline{< c^+_{j-1}c_j >} < \{c_{n-1}(t), c^+_m\} >, \end{aligned} \tag{4.4}$$

one gets instead of (4.3) the equations

$$\begin{aligned} & i\frac{d}{dt}G^{\mp}_{nm}(t) = \delta(t)\delta_{nm} + [\Omega_0+J^z(2\overline{< c^+_jc_j >}-1)+\Omega_n]G^{\mp}_{nm}(t) + \\ & \frac{J-2J^z\overline{< c^+_{j+1}c_j >}+iD}{2}G^{\mp}_{n+1,m}(t)+\frac{J-2J^z\overline{< c^+_{j-1}c_j >}-iD}{2}G^{\mp}_{n-1,m}(t) \end{aligned} \tag{4.5}$$

that can be treated like (1.6). Substituting $\Omega_0+J^z(2\overline{< c^+_jc_j >}-1)$ instead of Ω_0, $J-2J^z\overline{< c^+_{j\pm1}c_j >}\pm iD$ instead of $J\pm iD$ into (1.15) one finds that

$$\begin{aligned} & \overline{G^{\mp}_{\kappa}(\omega\pm i\epsilon)} = \frac{1}{\omega-\left[\overline{\overline{\Omega_0}}+J_+\exp{(i\kappa)}+J_-\exp{(-i\kappa)}\right]\pm i(\epsilon+\Gamma)}, \\ & \overline{\overline{\Omega_0}} \equiv \Omega_0+J^z(2\overline{< c^+_jc_j >}-1), \\ & J_\pm \equiv \frac{J-2J^z\overline{< c^+_{j\pm1}c_j >}\pm iD}{2}. \end{aligned} \tag{4.6}$$

Returning back to the site representation needs the calculation of the following integral (compare with (1.16)-(1.21))

$$\overline{G^{\mp}_{nm}(\omega\pm i\epsilon)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\kappa \exp{[i(n-m)\kappa]}$$

$$\times\frac{1}{\omega-\left[\overline{\overline{\Omega_0}}+J_+\exp{(i\kappa)}+J_-\exp{(-i\kappa)}\right]\pm i(\epsilon+\Gamma)}. \tag{4.7}$$

Putting $z=\exp{(\pm i\kappa)}$ (for $n\geq m$ and $n\leq m$, respectively) one comes to contour integral over unit circle centred at $z=0$ in complex plane z

$$\begin{aligned} & \overline{G^{\mp}_{nm}(\omega\pm i\epsilon)} = -\frac{1}{2\pi i}\oint dz\frac{z^{n-m}}{J_+z^2-[\omega-\overline{\overline{\Omega_0}}\pm i(\epsilon+\Gamma)]z+J_-} \\ & = -\frac{1}{2\pi i}\oint dz\frac{z^{n-m}}{J_+(z-z^+_1)(z-z^+_2)}, \\ & z^+_1 = \frac{\omega-\overline{\overline{\Omega_0}}\pm i(\epsilon+\Gamma)+\sqrt{[\omega-\overline{\overline{\Omega_0}}\pm i(\epsilon+\Gamma)]^2-4J_+J_-}}{2J_+}, \\ & z^+_2 = \frac{\omega-\overline{\overline{\Omega_0}}\pm i(\epsilon+\Gamma)-\sqrt{[\omega-\overline{\overline{\Omega_0}}\pm i(\epsilon+\Gamma)]^2-4J_+J_-}}{2J_+}, \\ & z^+_1z^+_2 = \frac{J_-}{J_+} = \frac{J-2J^z\overline{< c^+_{j-1}c_j >}-iD}{J-2J^z\overline{< c^+_{j+1}c_j >}+iD} \end{aligned} \tag{4.8}$$

for $n\geq m$,

$$\begin{aligned} & \overline{G^{\mp}_{nm}(\omega\pm i\epsilon)} = -\frac{1}{2\pi i}\oint dz\frac{z^{|n-m|}}{J_-z^2-[\omega-\overline{\overline{\Omega_0}}\pm i(\epsilon+\Gamma)]z+J_+} \\ & = -\frac{1}{2\pi i}\oint dz\frac{z^{|n-m|}}{J_-(z-z^-_1)(z-z^-_2)}, \\ & z^-_1 = \frac{\omega-\overline{\overline{\Omega_0}}\pm i(\epsilon+\Gamma)+\sqrt{[\omega-\overline{\overline{\Omega_0}}\pm i(\epsilon+\Gamma)]^2-4J_+J_-}}{2J_-}, \\ & z^-_2 = \frac{\omega-\overline{\overline{\Omega_0}}\pm i(\epsilon+\Gamma)-\sqrt{[\omega-\overline{\overline{\Omega_0}}\pm i(\epsilon+\Gamma)]^2-4J_+J_-}}{2J_-}, \\ & z^-_1z^-_2 = \frac{J_+}{J_-} = \frac{J-2J^z\overline{< c^+_{j+1}c_j >}+iD}{J-2J^z\overline{< c^+_{j-1}c_j >}-iD} \end{aligned} \tag{4.9}$$

for $n\leq m$. The result of integration can be easily obtained; it depends on the position of $z^{\pm}_{1,2}$ in complex plane z with respect to unit circle centred at the origin of complex plane. The introduced static average fermion correlation functions are given by

$$\overline{< c^+_jc_j >} = -\frac{1}{\pi}\mathrm{Im}\int_{-\infty}^{\infty} d\omega\frac{\overline{G^-_{jj}(\omega+i\epsilon)}}{\exp{(\beta\omega)}+1}, \tag{4.10}$$

$$\overline{< c_{j-1}^{+} c_j >} = \overline{< c_j^{+} c_{j+1} >} = -\frac{1}{\pi} \mathrm{Im} \int_{-\infty}^{\infty} d\omega \frac{\overline{G_{j,j-1}^{-}(\omega + i\epsilon)}}{\exp(\beta\omega) + 1}, \quad (4.11)$$

$$\overline{< c_{j+1}^{+} c_j >} = \overline{< c_j^{+} c_{j-1} >} = -\frac{1}{\pi} \mathrm{Im} \int_{-\infty}^{\infty} d\omega \frac{\overline{G_{j,j+1}^{-}(\omega + i\epsilon)}}{\exp(\beta\omega) + 1}. \quad (4.12)$$

The equations (4.8)-(4.12) permit to determine self-consistently correlation functions $\overline{< c_j^{+} c_j >}$, $\overline{< c_{j+1}^{+} c_j >}$, $\overline{< c_{j-1}^{+} c_j >}$ that were introduced in (4.4). Knowing them one can get the average spectral density that is given by

$$\overline{\rho(E)} = -\frac{1}{\pi} \mathrm{Im} \frac{1}{2\pi} \int_{-\pi}^{\pi} d\kappa \frac{1}{\omega - \left[\overline{\overline{\Omega_0}} + J_{+} \exp(i\kappa) + J_{-} \exp(-i\kappa)\right] + i(\epsilon + \Gamma)}$$

$$= \frac{1}{\pi} \mathrm{Im} \frac{1}{2\pi i} \oint \frac{dz}{J_{\pm}(z - z_1^{\pm})(z - z_2^{\pm})}. \quad (4.13)$$

Such approach can be viewed as a generalization of rather old ideas (see [33]) for the case of random lorentzian external field.

5. Conclusions

The results which have been obtained in the present paper are summarized as follows. Considering spin-$\frac{1}{2}$ isotropic XY chain with Dzyaloshinskii-Moriya interaction in random lorentzian transverse field (1.1) as a system of fermions (1.3) one is able to obtain exactly the average one-fermion Green's functions $\overline{G_{nm}^{\mp}(\omega \pm i\epsilon)}$ (1.21). This permits to get exact result for average spectral density (1.22) and therefore, for thermodynamical quantities (2.2)-(2.5), (2.7)-(2.10), to examine the influence of Dzyaloshinskii-Moriya interaction on them (Figs.1-5). Unfortunately, it appeared impossible to reach any exact results concerning spin correlations in model in question. The system that is considered does not represent accurately any physical system, but may serve as a testing ground for theory and numerical techniques and in this sence, has proved most valuable. The results of comparison of conclusions obtained within different approximate approaches with exact ones (Figs.6,7, Fig.8) represent some interest and may be useful in understanding the region of validity of various well-known methods of randomly disordered crystals theory. After introducing into the Hamiltonian of the interaction between z components of neighbouring spins one is unable to obtain exact results since these terms lead to appearance of products of four Fermi operators in (4.2). After adopting approximation (4.4) the problem reduces to already considered one, however, more complicated because of necessity to determine self-consistently three fermion correlation functions.

The performed exact calculations for the system with Hamiltonian (1.1) or (1.3) are anticipated to be useful in leading to an understanding of the properties of one-dimensional disordered models of condensed matter physics.

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