How does spin chain’s quantum critical point manifest itself in elastic properties?

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In the present work we consider a number of exactly solvable one-dimensional spin-$\frac{1}{2}$ models (Ising model in a longitudinal/transverse field, XX model in a transverse field, Heisenberg-Ising model with a Dzyaloshinskii-Moriya interaction) assuming that an underlying harmonic lattice may rearrange itself according to a demand from the interacting quantum spins. We calculate the ground-state compressibility of the deformable quantum spin chains to discuss how a quantum critical point, inherent in the spin systems at hand, may manifest itself through the elastic properties of the underlying lattice. The obtained dependences for the compressibility on a dimensionless magnetic field or pressure (normalized with respect to exchange constant) indicate in a rather obvious manner the change of a ground-state phase of interacting spins. While the inverse compressibility of the XX chain in a transverse field exhibits a finite jump at a quantum critical point, the inverse compressibility of the transverse Ising chain or Heisenberg-Ising chain logarithmically diminishes at a quantum critical point. Magnetic-field dependence of the compressibility at low temperatures is amenable to experimental measurements (see, e.g., Ref. [1] reporting such measurements for a spin-$\frac{1}{2}$ Heisenberg spin-chain material, the coordination polymer CuCCP). Our rigorous analysis may be useful for understanding experimental results in general.