The phases in the S=1/2 random antiferromagnetic Heisenberg spin chain (RAHC) and the applicability of the strong-disorder renormalization group (SDRG) method are investigated. Besides the SDRG method also the high-accuracy density-matrix renormalization group method (DMRG) is utilized. For uniform randomness both of the approaches observe the well-known random-singlet phase with formally infinite dynamical exponent. For dimerized randomness, both of the approaches reveal that the dynamical exponent is robust against dimerization, it remains the same as in case of uniform randomness, however a somewhat unusual random-singlet phase appears. This phenomenon for dimerized RAHC, suggested by both of the procedures, is contrary to the initial expectation, the appearance of Griffiths phase with dimerization-dependent dynamical exponents. In higher-S RAHC or in other quantum spin chains, it is very likely that the here described scenario is similarly present also for weaker uniform randomness. It is essential to notice, the SDRG results were justified by DMRG calculations; the SDRG can produce correct results even for the weaker strength of randomness, however in this parameter region very large number of random configurations must be generated in order to calculate the average quantities with the necessary accuracy. This observation suggests that in principle there is no limitation for the use of SDRG method although at weak randomness the accuracy of one basic RG-step, based on the perturbation theory, can be very poor, still the method can properly describe the properties of the system.