Phase transitions in the systems described by the Bose-Fermi-Hubbard model are investigated in the case of the weak boson-fermion interaction. Such type models can be applied for the description of atoms in optical lattices and intercalation of ions in crystals. Special attention is paid to the case of the lattice with two nonequivalent sublattices. Recently optical lattices of double wells were realized experimentally. We consider the case of hard-core bosons and use the pseudospin formalism. The phase diagrams in the plane of the chemical potential of bosons-bosonic hopping parameter were built and it was shown that the transition from the uniform to modulated phase can be of the first or the second order (in the regime of the fixed concentrations this leads to phase separation). The gap in the fermionic spectrum appears due to the presence of anisotropic hopping and this causes the narrowing of the parameter region of supersolid phase existence. The presence of the effective interaction between bosons via fermions leads to the first order phase transition between modulated phases at the presence of the double-well potential.