Low-frequency electromagnetic field in Wigner crystal

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Let's consider low-frequency long-wave electromagnetic field in three-dimension Wigner (electron) crystal. It satisfies Maxwell equations with hydrodynamic currents. Equation for electron subsystem in continuum approximation in isotropic case is
\[
\rho_e \dot{\mathbf{v}}_{e_\alpha} = (A + B) \nabla^2 \mathbf{u}_\alpha / \partial \mathbf{x}_\alpha \partial \mathbf{x}_\alpha + B \mathbf{u}_\alpha / \partial \mathbf{x}_\alpha \partial \mathbf{x}_\alpha - e \mathbf{n}_i E_{\alpha},
\]
here \( \mathbf{u}_\alpha \) is deformation vector. Ion subsystem has been considered in jelly approximation \( \rho_i \mathbf{v}_{i_\alpha} = Z e \mathbf{n}_i E_{\alpha} \). We have such linearized Fourier-transformed equation system for wave-vector projections like \( \mathbf{E}_k / k = E_1 \) in dimensionless variables
\[
(E_{\parallel}, u_{\parallel}, v_{e_\parallel}, v_{i_\parallel}) \leftrightarrow \left( \frac{\delta_{\parallel} E_i}{m_e c}, u_{\parallel} k, \frac{v_{e_\parallel}}{c}, \frac{v_{i_\parallel}}{c} \right) t \leftrightarrow t k s, \text{ where } s^2 = (A + 2B) / \rho_{e_0}:\n
\dot{E}_{\parallel} = \Omega_2 v_{e_\parallel} - \Omega_1 v_{i_\parallel}, \quad \dot{u}_{\parallel} = v_{e_\parallel}, \quad \dot{v}_{e_\parallel} = E_1, \quad \dot{v}_{i_\parallel} = -u_{\parallel} - E_1.\n\]
Here \( \Omega_e \) and \( \Omega_i \) are electron and ion plasma frequencies. Low-frequency oscillation branch is \( \lambda = \pm \frac{1}{2} \sqrt{\Omega_2^2 + \Omega_1^2 - 1 - \sqrt{(\Omega_2^2 + \Omega_1^2 + 1)^2 - 4 \Omega_1^2}} \) that gives solution like ordinary ion sound in limit \( \Omega_2^2 \rightarrow \infty: \lambda = \pm i \Omega_1 / \Omega_e \). In the case of infinity heavy ions the sound disappears. For transversal oscillations like \( (\delta_{ab} - k_a k_b / k^2) E_{\beta} = E_{\alpha} \) we denote \( B \rightarrow [k / k, B] = Z \) and have \( (E^i, Z, u^i, v_{e^i}, v_{i^i}) \leftrightarrow \left( \frac{\delta_{\parallel} E_i}{m_e c}, \frac{c Z}{m_e c}, u^i k, \frac{v_{e^i}}{c}, \frac{v_{i^i}}{c} \right) \), \( t \rightarrow t k s, \epsilon \leftrightarrow c / s \) where \( s^2 = B / \rho_{e_0} \). In this variables we have \( \dot{E}^i = i c Z + \Omega_2 v_{e^i} - \Omega_1 v_{i^i}, \quad \dot{Z} = i c E^i, \quad \dot{u}^i = v_{e^i}, \quad \dot{v}_{e^i} = E^i, \quad \dot{v}_{i^i} = -u^i - E^i \). Solution branch \( \lambda = \pm \frac{1}{2} \sqrt{\Omega_2^2 + \Omega_1^2 + 1 + c^2 - \sqrt{(\Omega_2^2 + \Omega_1^2 + 1 + c^2)^2 - 4 (\Omega_1^2 + c^2)}} \) gives low-frequency oscillations that gives transversal sound in limit \( \Omega_1^2 \rightarrow \infty \) and condition \( \Omega_i \gg k c: \lambda = \pm i \Omega_1 / \Omega_e \). But in the case of infinity heavy ions a new low-frequency quadratic dispersion law appears \( \lambda = \pm i \Omega_1 / \Omega_e \). So resiliency modules of electron subsystem that are determined by short-acting potential enter in long wavelength oscillation frequencies: longitudinal \( \omega^2 = k^2 (A + 2B) / \rho_{i_0} \) and transversal \( \omega^2 = k^2 B / \rho_{i_0} + k^2 c^2 B / \rho_{e_0} \Omega_2^2 \). This consideration can be applied to covalent bounded isotropic crystals also.