On the problem of option pricing. Heston model

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A derivation sequence of formula of call option pricing in Heston model is shown. The method is based on the solution of the equation for the evolution operator kernel in Merton – Garman model for the partial case, which corresponds to Heston model. The equation for the kernel is the second order evolutionary differential equation in variables $x = \ln(S)$, $V > 0$ – option price, $V > 0$ – volatility. The use of Fourier transform over variable $x$ and Laplace transform over variable $V$ allowed us to receive for the kernel image the first order differential equation. Indicated method was applied in [1] to solve the Fokker–Planck equation for transition probability density in Heston model. However in our case the received equation is inhomogeneous, the right part of which contains an unknown initial value of the kernel which is typical for the Laplace transforms. To determine inhomogeneous term the solution of the governing equation was studied in the neighborhood of point $V \approx 0$. As a result we received a closed equation for the kernel transform and found it analytical solution. A formula for option pricing is expressed in terms of Fourier transform of the kernel. It was shown that in limiting case of constant volatility the formula for option pricing turns into Black–Scholes formula. The comparison of the given paper option pricing formula with the known Heston one shows that they produce an identical results. However the received formula has more compact form for pricing and is more suitable in application. These formulas are also more suitable for numerical calculations.