The $s = 1/2$ antiferromagnetic Heisenberg model on several bilayer lattices (square, honeycomb and triangular) with magnon states from the flat band with lowest energy in the presence of a strong magnetic field is considered. Due to the localized nature of the flat-band magnon states, these systems are mapped on the classical lattice gases of hard-core objects. Also, the standard strong-coupling perturbation theory is applied for constructing of the effective Hamiltonians. These effective models allowed to investigate the phase transitions related to the ordering of localized magnons. These phase transitions belong to the different classes of universality. For the antiferromagnetic model on a honeycomb bilayer lattice for a small deviations from the full frustration regime, a spin-flop transition, which occurs in a $XXZ$ model with an easy axis of magnetization, was found. On the basis of an effective model, constructed in the case of square geometry, a theory for a magnetic compound $\text{Ba}_2\text{CoSi}_2\text{O}_6\text{Cl}_2$ in an external magnetic field for the description of its low-temperature properties is developed. The results of experiments for this compound have been reproduced and new predictions have been made, which require new experimental studies to confirm them.

Also, the ground state of the quantum Heisenberg antiferromagnet on the bilayers (square and honeycomb) in the absence of magnetic field is investigated. A variational approach has been applied for that. By comparing the variational energies, the ground-state phase diagrams are constructed. The obtained results are compared with the ones obtained recently by more sophisticated methods. Qualitative consistency and good quantitative agreement for some critical points are observed.