Self-averaging in the two-dimensional random-bond Ising model

Vic. Dotsenko[a,b], Yu. Holovatch[c,d], M. Dudka[c,d], and M. Weigel[d,e]

[a] *LPTMC, Sorbonne University, 75252 Paris, France,*

[b] *L.D. Landau Institute for Theoretical Physics, 119334 Moscow, Russia,*

[c] *Institute for Condensed Matter Physics, National Acad. Sci. of Ukraine, 79011 Lviv, Ukraine*

[d] $\mathbb{L}^4$ *Collaboration & Doctoral College for the Statistical Physics of Complex Systems, Leipzig-Lorraine-Lviv-Coventry, D-04009 Leipzig, Germany*

[e] *Applied Mathematics Research Centre, Coventry University, Coventry, CV1 5FB, United Kingdom*

Effects of disorder on the properties of condensed systems are of great interest as a varying degree of impurities is present in every material studied in the laboratory. In some cases, disorder might lead to a loss of self-averaging, i.e., the behavior of a large sample with a specific realization of impurities will not be well described by the ensemble average normally calculated in an analytical or numerical approach. The question of (non)self-averaging is connected to the relevance of disorder answered by the Harris criterion (for pure systems with continuous phase transitions weak disorder is relevant only if the specific heat is divergent, i.e., the critical exponent $\alpha{>}0$). It was already shown that for pure systems with $\alpha{<}0$ the relative variance of thermodynamic observables of disordered counterparts weakly decreases as a power of system size L, indicating weak self-averaging, while for the case with $\alpha{>}0$ this ratio approaches a non-zero constant as $L{\to}\infty$, indicating a lack of self-averaging. The most intriguing case is given by the two-dimensional Ising model, where $\alpha{=}0$. We study sample-to-sample fluctuations in a critical two-dimensional Ising model with quenched random bonds. Using replica calculations in the renormalization group framework we derive explicit expressions for the probability distribution function of the critical internal energy and for the specific heat fluctuations [1]. It is shown that the distribution of internal energies is Gaussian, the typical sample-to-sample fluctuations and the average value scale with L like $\sim L \ln\ln L$. In contrast, the specific heat is shown to be self-averaging with a distribution function that tends to a δ-peak in the thermodynamic limit $L{\to}\infty$.

[1] Vic. Dotsenko *et al.*, PRE, **95**, 032118 (2017)