Recent developments in the theory of electrodynamic homogenization of random particulate systems

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We present results [1-3], derived within the compact group approach (CGA) and effectively incorporating many-particle polarization and correlation effects, for the effective quasistatic dielectric constant/electrical conductivity of model dispersions of hard-core–penetrable-shell particles embedded in a continuous matrix. Both the cores and shells are characterized by radially symmetrical distributions of their electric parameters. The local properties of overlapping constituents are governed by the distance from a given point to the nearest particle.

Let $x$ be an electric parameter of a system $\mathcal{D}$ and $x_{\text{eff}}$ its effective value. The key points behind [1-3] are as follows:

(1) $\mathcal{D}$ is electrically equivalent to an auxiliary system $\mathcal{S}$ formed by embedding $\mathcal{D}$’s constituents into a uniform host with $x = x_f$. $\mathcal{S}$ is a set of macroscopic regions (compact groups) large enough to have the properties of the entire $\mathcal{S}$, but point-like relative to the probing field;

(2) $x_{\text{eff}}$ is found as the proportionality coefficient in the relevant constitutive relation between the averaged induction/current and field. These averages are expressed through the statistical moments $\langle (x-x_f)^n \rangle$;

(3) Combining the CGA with the Hashin-Shtrikman variational theorem or the boundary conditions for electric fields, $x_f$ is proven to equal $x_{\text{eff}}$. This result makes the homogenization procedure internally closed;

(4) Finally, $x_{\text{eff}}$, as a functional of $\mathcal{D}$’s constituents conductivities and volume concentrations, is shown to obey an integral relation, rigorous in the quasistatic limit.

We demonstrate the validity of our results by: (a) contrasting them with analytical and numerical results for dispersions of graded dielectric spheres with power-law permittivity profiles; (b) mapping them onto extensive random resistor network simulation data for composite polymer electrolytes; and (c) applying them to real composite electrolytes.