Relation of entanglement of continuous variable graph states with graph properties

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Recently, continuous variable graph states attain considerable attention as a method of implementing a quantum computer. Unlike traditional quantum computation, which is based on discrete quantum variables and qubits, the continuous variable quantum computation is based on qumodes, quantum systems with an infinite-dimensional Hilbert space spanned by a continuum of orthogonal states.

In this report we are interested in the relation of quantum properties of continuous variable graph states with the properties of the underlying graph. To this end, we consider the graph states defined as

$$\psi(\mathbf{x}) = \psi(x_1, x_2, ..., x_N) = \sqrt{\frac{\alpha^N}{\pi^N}} \exp\left(-\sum_j \frac{\alpha}{2} x_j^2 + i \sum_{j \neq k} \frac{a_{jk}}{2} x_j x_k \right),$$

here α is constant, a_{ij} are elements of a constant symmetric matrix $\hat{a}$, indices $j, k = (1, ..., N)$. The state can be obtained as a result of action of the unitary operator $\hat{U}_{jk} = \exp(i a_{jk} x_j x_k)$ on the ground state of a system of N noninteracting harmonic oscillators. Each oscillator being represented by graph vertex, the last term in the exponent can be understood as a sum over graph edges. Subsequently, matrix $\hat{a}$ is the adjacency matrix of the graph.

We analyse the geometric measure of entanglement in the graph state and find that the entanglement of a harmonic oscillator with other ones is defined by the value of its vertex degree. We also study correlation of the Fubini–Study distance between the graph states with Hamming and Hilbert–Schmidt distances between the corresponding graphs.