Stochastic mechanism for improving selectivity of olfactory projection neurons

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5-th Conference on Statistical Physics: Modern Trends & Applications, July 3-6, 2019 Lviv, Ukraine

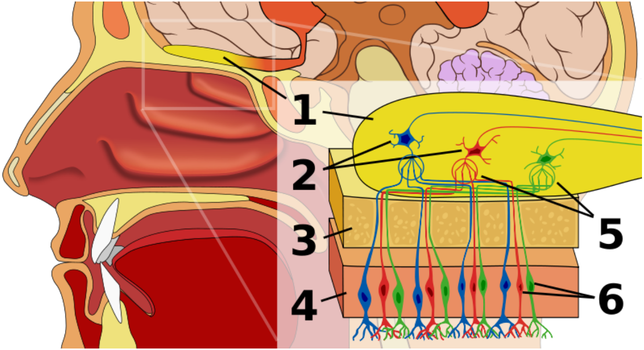
Outline

Quality of a sensory system

- Sensitivity
- Selectivity
- Speed of registration

Outline

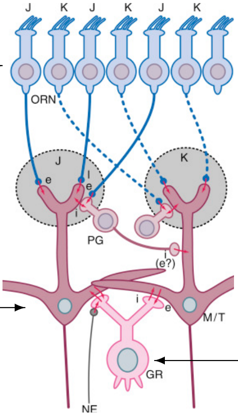
Structure of the olfactory system



From: Malnic,B., Hirono,J., Sato,T., Buck,L.B. Combinatorial receptor codes for odors. *Cell* **96**:713–723 (1999)

Outline

Lateral inhibition



receptor neurons

projection neurons

inhibitory neuron

From: Scott,K. Chapter 23 - Chemical Senses: Taste and Olfaction in: Squire,L.R., Berg,D., Bloom,F.E., du Lac,S., Ghosh,A., Spitzer,N.C.(ed.) Fundamental Neuroscience (Fourth Edition) pp.

Outline

No lateral inhibition at low c

From:

Duchamp,A.

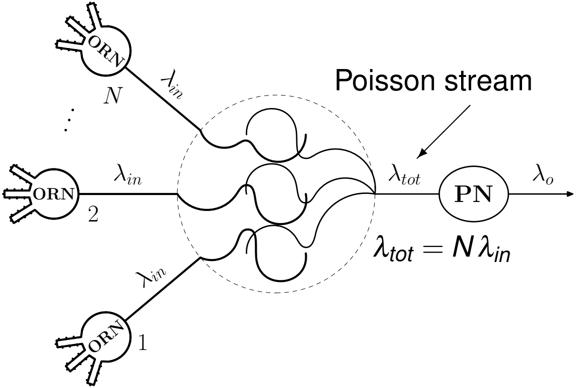
Electrophysiological responses of olfactory bulb neurons to odour stimuli in the frog. A comparison with receptor cells.

Chemical Senses **7**(2):191-210 (1982)

“The suppressive responses were therefore much more affected (about twice as much) than the excitatory ones by the decrease in stimulus concentration.”

Outline

Selectivity gain definition for odors O and $O^{'}$



$$\lambda'_{in} > \lambda_{in}, \quad \lambda'_{in} = \lambda_{in} + \Delta\lambda_{in}$$

$s = \frac{\Delta\lambda_{in}}{\lambda_{in}}$ – ORN's selectivity

$$\lambda'_o > \lambda_o, \quad \lambda'_o = \lambda_o + \Delta\lambda_o$$

$S = \frac{\Delta\lambda_o}{\lambda_o}$ – PN's selectivity

$$\lambda_o = \frac{1}{T_o}$$

<u>selectivity gain:</u> $g = \frac{S}{s}$, or $g = \frac{\lambda_{in}}{\lambda_o}\frac{d\lambda_o}{d\lambda_{in}}$, or $\boxed{g = -\frac{\lambda_{in}}{T_o}\frac{d\,T_o}{d\,\lambda_{in}}}$

Outline

Projection neuron, KKPT-model

perfect integrator

height of input impulse: h

states of depolarization: $0, h, 2h, 3h, \ldots$

numbers of states: $0, 1, 2, \quad 3, \ldots, N_0 - 1$

threshold depolarization: V_0

threshold depolarization: $N_0, \quad (N_0 - 1)h < V_0 \leq N_0 h$

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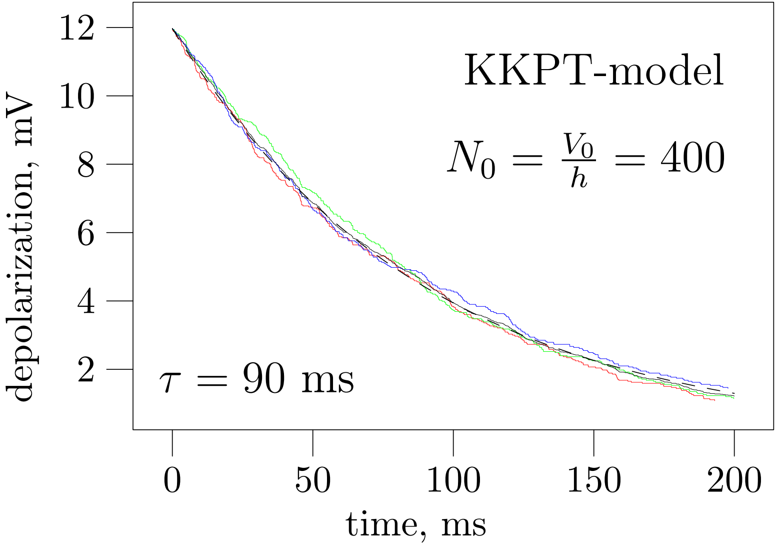
leakage

random decay of obtained impulses: $p = \mu\, dt$

on the average: $V(t+u) = V(t)\, e^{-\mu u}, \quad \mu = 1/\tau$

Korolyuk,V.S., Kostyuk,P.G., Pjatigorskii,B.Ya., Tkachenko,E.P. Mathematical model of spontaneous activity of some neurons in the CNS. *Biofizika* **12**(5):895-899 (1967)

Projection neuron model, check



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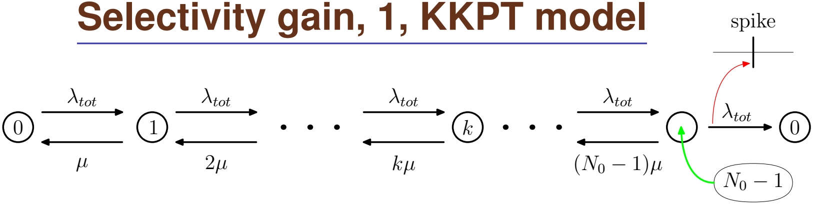
- Sensory systems
- The sense of smell (olfaction)
- Lateral Inhibition at high c
- No lateral inhibition at low c

Stochastic mechanism
- Definition of selectivity gain
- Projection neuron model
- Selectivity gain
- Numerical examples

Conclusions
- Prerequisites

Bibliography

Selectivity gain, 1, KKPT model



$$T_o = \sum_{0\le l\le N_0-1} \Lambda_l \sum_{0\le k\le l} \frac{1}{r_k^+ \Lambda_k},$$

$$\Lambda_0 = 1, \qquad \Lambda_n = \prod_{1\le k\le n} \frac{r_k^-}{r_k^+}, \qquad n \in \{1,\dots,N_0-2\},$$

$$\Lambda_{N_0-1} = \Lambda_{N_0-2} \frac{r_{N_0-1}^-}{r_{N_0-1}^+}. \qquad \boxed{r_k^+ = \lambda_{tot}, \quad r_k^- = k\mu}$$

A.K.Vidybida. *Stochastic models*. NAS of Ukraine, BITP, Kyiv; 2006.

Selectivity gain, 2, T_o

$$T_o = \frac{1}{\lambda_{tot}} \sum_{0 \le j \le N_0 - 1} \frac{1}{j+1} \left(\frac{\mu}{\lambda_{tot}} \right)^j \frac{N_0!}{(N_0 - 1 - j)!}, \quad \lambda_{tot} = N \lambda_{in}$$

$$\boxed{g = -\frac{\lambda_{in}}{T_o} \frac{d\,T_o}{d\,\lambda_{in}}}$$

Selectivity gain, 3, final expression

$$g = 1 + \frac{\displaystyle\sum_{j=0}^{N_0-1} \frac{j}{j+1} \left(\frac{\mu}{N\lambda_{in}}\right)^j \frac{1}{(N_0-j-1)!}}{\displaystyle\sum_{j=0}^{N_0-1} \frac{1}{j+1} \left(\frac{\mu}{N\lambda_{in}}\right)^j \frac{1}{(N_0-j-1)!}} .$$

Selectivity gain, 4, no leakage

$$g = 1 + \frac{\sum_{j=0}^{N_0-1} \frac{j}{j+1} \left(\frac{\mu}{N\lambda_{in}}\right)^j \frac{1}{(N_0-j-1)!}}{\sum_{j=0}^{N_0-1} \frac{1}{j+1} \left(\frac{\mu}{N\lambda_{in}}\right)^j \frac{1}{(N_0-j-1)!}}.$$

no leakage $\Rightarrow$ $\tau = \infty$ $\Rightarrow$ $\mu = 0$ $\Rightarrow$ $g = 1$ $\Rightarrow$ no gain

Selectivity gain, 5, high c

$$g = 1 + \frac{\sum_{j=0}^{N_0-1} \frac{j}{j+1} \left(\frac{\mu}{N\lambda_{in}}\right)^j \frac{1}{(N_0-j-1)!}}{\sum_{j=0}^{N_0-1} \frac{1}{j+1} \left(\frac{\mu}{N\lambda_{in}}\right)^j \frac{1}{(N_0-j-1)!}} .$$

high odor concentration:

high λ_{in} $\Rightarrow$ $\frac{\mu}{N\lambda_{in}} \approx 0$ $\Rightarrow$ $g \approx 1$ $\Rightarrow$ no gain

Selectivity gain, 6, low concentration

$$g = 1 + \frac{\sum_{j=0}^{N_0-1} \frac{j}{j+1} \left(\frac{\mu}{N\lambda_{in}}\right)^j \frac{1}{(N_0-j-1)!}}{\sum_{j=0}^{N_0-1} \frac{1}{j+1} \left(\frac{\mu}{N\lambda_{in}}\right)^j \frac{1}{(N_0-j-1)!}}\,.$$

low odor concentration:

$$\lambda_{in} \to 0 \quad \Rightarrow \quad g \approx N_0$$

HIGH SELECTIVITY GAIN

Selectivity gain, 7, decrease with increasing *c*

$$g = 1 + \frac{\sum_{j=0}^{N_0-1} \frac{j}{j+1} \left(\frac{\mu}{N\lambda_{in}}\right)^j \frac{1}{(N_0-j-1)!}}{\sum_{j=0}^{N_0-1} \frac{1}{j+1} \left(\frac{\mu}{N\lambda_{in}}\right)^j \frac{1}{(N_0-j-1)!}}.$$

$$\frac{dg}{d\lambda_{in}} < 0 \quad \Rightarrow$$

g decreases with increasing concentration

for moderate conditions: $1 < g < N_0$

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Bibliography

Experimental numerical data

PN threshold depolarization, V_0, mV	height of EPSP, h, μV	ORN spikes frequency, λ_{in}, 1/ms	PN membrane relaxation time, τ, ms
5 - 12, [1, 2]	30 - 665, the mean is 131, [5]	10^{-3}, [3]	90, [4]

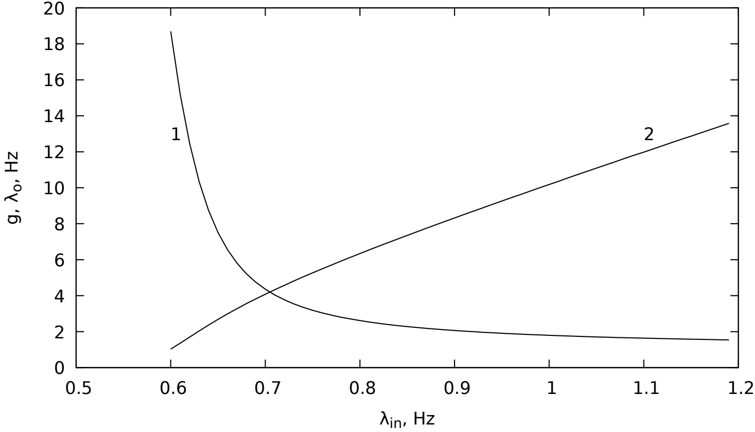
Experimental values for parameters, sources are indicated in brackets.

Calculated selectivity gain

threshold N_0	output frequency λ_o, 1/s	g
300	10.3	1.78
400	5.3	3.15
500	0.67	30.3

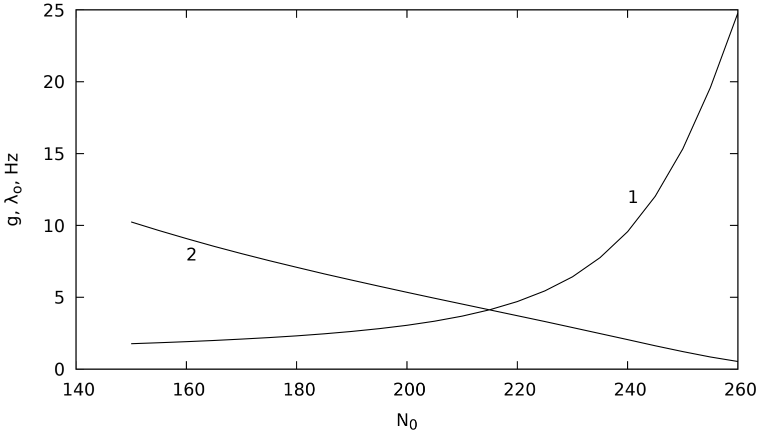
Results of numerical calculation.

Dependence on ORN's activity



Dependencies of g, 1 and λ_o, 2 on λ_{in} for threshold $N_0 = 300$, $N = 5000$, $\tau = 90$ ms. g is dimensionless.

Dependence on the firing threshold, N_0



Dependencies of g, 1 and λ_o, 2 on threshold N_0 for $\lambda_{in} = 0.5$ Hz.

Outline

Prerequisites

- Leakage in the projection neurons
- Stochastic nature of input to the PN
- Threshold-type response in the PN ($N_0 \gg 1$)

Accepted: *Neurophysiology* (Springer)
Preprint: **arXiv:1904.08767**

Missed reality

- The ORNs are not identical
- ORN's input is presynaptically inhibited
- ORN's axon arborizes: several inputs from a single ORN
- Dendritic preprocessing in the projection neuron
- Spontaneous activity in the ORNs

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  Electrophysiological analysis of mitral cells in the isolated turtle olfactory bulb.

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The Journal of Neuroscience. 1990;10(3):826.