ISING LECTURES IN LVIV (1997 - 2017)

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National Academy of Sciences of Ukraine
Institute for Condensed Matter Physics

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(1997–2017)

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Lviv, 2017

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The book is dedicated to the 20th anniversary of the "Ising Lectures", an annual workshop held in Lviv, Ukraine. Initiated as a meeting on phase transitions and critical phenomena it has subsequently broadened its scope into a more general context of complex systems. Besides the abstracts of talks and biographic information on each lecturer during the period 1997–2017, the book also contains a review article on the Ising model and Ernst Ising. This book will be of interest to students and researchers involved in the field of statistical and condensed matter physics, complexity and the history of science.

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Contents

Preface

Dear Reader,

You are holding in your hands the summary of twenty years of the "Ising Lectures", an annual workshop which has occurred since 1997 in Lviv, Ukraine. Initially the workshop aimed to promote and deepen the study of phase transitions and critical phenomena as well as to exchange information between scholars working in this field. It has subsequently expanded and broadened its horizons to include the study of complex systems allowing for a more interdisciplinary approach to lectures. Since its origins it has experienced significant growth in international participation with now more than half of all lectures being delivered by leading scholars from all over the world. This book is indicative of how a single spark can ignite into flourishing movement. Moreover, it demonstrates the evolution of a range of problems in the field of statistical physics and its interdisciplinary connections.

The set-up of the book is as follows: after a Foreword and a short description of how the Ising lectures started follows an article devoted to Ernst Ising and the Ising model. In the next sections we present lecture abstracts and short biographies on the speakers for the entire period between 1997–2017. Section titled "Gallery" contains some photos taken at lectures, meetings, discussions and also during traditional Carpathians hiking organised in conjuction with the Ising lectures. A selection of these lectures has been published by the World Scientific Publishers (Singapore) as a review volume series. We list the tables of content of these volumes at the end of the book.

On this occasion, we would like to thank all the organisers of these workshops; the lecturers, for taking time to deliver stimulating talks;

the participants for coming and generating fruitful scientific discussions; Thomas Ising for his support and the materials he has provided from family archive. Finally, special gratitude has to be given to an outstanding scientist Ernst Ising, whose model has brought us all together and gave a birth to this long standing interdisciplinary collaboration.

We hope You will enjoy reading!

Mariana Krasnytska,
Robin de Regt,
Petro Sarkanych

Nancy, Coventry, Lviv, 1.06.17

Foreword

One more book has been published. Its appearance was stimulated by the 20th anniversary of the Ising Lectures. Obviously it is not an event on a planetary scale, so some additional explanation is required at least for those who are uninitiated.

The Ising Lectures were set up in 1997 with the main motivation to create, in Ukraine, an additional platform for scientific discussions in the fields of phase transitions theory and critical phenomena by facilitating the presentations of leading expert lectures for students, postgraduates and young scientists. Such a motivation seemed significant and important because of the very limited travel opportunities for scientists from Ukraine at that time.

What has changed since then and what are the main results of interest for the wider audience?

Ukraine has recently been granted the right to visa-free travel to countries of the European Community. Obviously, this is a significant step forward, in particular in the direction of deepening international scientific relations. At the same time, the financial incentives for such travel are not always present, although for the past twenty years researchers from Ukraine have actively used the opportunities of the European scientific programs FP6, FP7 as well as Horizon 2020 that have opened up new options for strengthening scientific contacts.

More than 70 lectures have been presented over the last 20 years of the Ising Lectures. Most of them (about 40) were delivered by scientists from abroad. The first lecture texts appeared as preprints of the Institute for Condensed Matter Physics. Since 2004 they have been published in the series "Order, Disorder and Criticality: The Advanced Problems of Phase

Transition Theory" in World Scientific, Singapore. The fifth volume of this series is in preparation. Thus, the Ising lectures are now available to the international scientific community.

On the other hand, the experience of the Ising Lectures has also revealed another component that emphasizes its importance and relevance. The visits of leading experts to the Ukraine are mutually beneficial in opening up new prospects for cooperation and the launching of new projects. They are also interesting for guests through the discovery of Ukraine and Lviv as the city of the location for the Ising Lectures.

Can this book be of interest to a wider readership?

Each book (especially in science) has to open new horizons of knowledge for readers. In this particular case, information about a scientist who has only one scientific article in his list of publications can be found. The search for his name on Google Scholar issues over 460 thousand results, and the model, named in his honor, is mentioned in more than 160 thousand works. What was the fate of this Scientist and his famous model? You can find the answers to these questions in this book.

Finally, I would like to mention that the main protagonist in the appearance of this book and organizer of the Ising Lectures, Prof. Yurij Holovatch, met his 60th birthday on the last day of the 20th Ising Lectures. For more than a third of his life he has cared for the Ising Lectures and is the editor of the series "Order, Disorder and Criticality: Advanced Problems of Phase Transition Theory". Therefore, I finish with wishes of success to Prof. Holovatch and the Ising Lectures for many years to come. I would also like to thank our young colleagues who have performed the main work in the preparation of this book.

Ihor Mryglod
Director of the Institute
for Condensed Matter Physics
of the National Academy of
Sciences of Ukraine

Lviv, 20.07.17

Ising lectures (how all started)

The idea to organize in Lviv the "Ising lectures" – a seminar on the Ising model, its developments and different applications - arose during the 22nd Conference of the Middle European Cooperation in Statistical Physics (MECO 22) organized by Józef Sznajd and his colleagues in 1997 in Szklarska Poręba in Poland. There, a group of then young physicists from Lviv, that included Oleh Derzhko, Slavko Ilnytskyi, Volodia Tkachuk and myself – met Sigismund Kobe. We learned, from him, about Ernst Ising life – adventurous, hard, sometimes tragic, and extremely interesting. For us, back then, Ernst Ising appeared to be a figure from the beginning of 20th century and to our astonishment we learned that he is our contemporary and lives with his wife Jane (Johanna) in Peoria (Illinois, USA). Moreover, Sigismund Kobe was personally acquainted with the Ising family, visited them in the US and wrote papers on Ernst Ising and his model.

For us in Lviv, the Ising model was more than just an example from textbooks. First, we all worked in the field of statistical physics and to different extents were using the model in our research. Second, and even more important, we had all been students of Ihor Yukhnovskii who in 1970s published a famous series of papers on the 3D Ising model. These papers were, for us, the first meeting point with scientific research. So getting in touch with Ising at a personal level was really a strong emotional event for us.

On the way back home from Szklarska Poręba it was decided to organize a workshop in Lviv, discussing there different issues related to the Ising model and making our colleagues aware about Ernst Ising's life. This first seminar took place on May 12 1997. It was devoted to Ernst Ising 97th birthday, which was two days before, on May 10, and we sent him our greetings. At that moment we did not plan to continue the seminar, but after success of the first one, the second was organized on April 28, 1998 and since then the tradition of annual workshops was developed, with a single interruption in 2008. Ernst Ising passed away on May 11, 1998 and we keep contacts with his son, Thomas.

Originally, the principal topics of the workshops were centered upon phase transitions and criticality in their different incarnations. With the span of time the scope has gradually turned to a more general context of complexity. We were happy to welcome our colleagues, talented physicists from all over the world who came to lecture for us. To some extent this small but important west-east flow is also a compensation for permanently increasing east-west flow. The first lecture texts appeared as ICMP preprints, soon we changed the format and had them published as a review volume series with “World Scientific”, Singapore. So that’s how it started and that’s how it goes on. Needless to say that it does go on due to the drive and initiative coming from the scientific community and due to the constant support and encouragement coming from our institutions: the host, ICMP and also the guests, since all lectures are supported by their home institutions.

As long as these two driving forces act together, we have hope and we have a future. Thanks to all, who keep the Ising lectures alive already for more than twenty years!

Yurij Holovatch
(ICMP, Lviv and $\mathbb{L}^4$ collaboration,
Leipzig-Lorraine-Lviv-Coventry)

Lviv, 23.05.17

1 The fate of Ernst Ising and the fate of his Model [1]

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[1]The chapter is also available as preprint (arXiv:1706.01764) and will be published in J. Phys. Stud. **21** (2017).

Contents

1.1 Introduction

In seeking to explain a particular phenomenon in physics, namely the onset of ferromagnetism, Wilhelm Lenz proposed a model that was solved in one dimension by his PhD student, Ernst Ising in 1924. This event marked the start of a process that, over nearly 100 years, delivered tremendous and multiple successes in explaining collective behaviour in a vast variety of systems, including many beyond the natural sciences.

When Ernst Ising started his work on the phenomenon of ferromagnetism, the nature of the microscopic, inter-atomic interactions was not yet understood. Indeed, it was not at all clear how a macroscopic magnetization could be generated by the interactions between elementary magnets. It was already known that magnetism is a quantum phenomenon but quantum theory was at a stage where the classical Bohr-Sommerfeld model was in disagreement with experiments. Ernst Ising succeeded in answering the

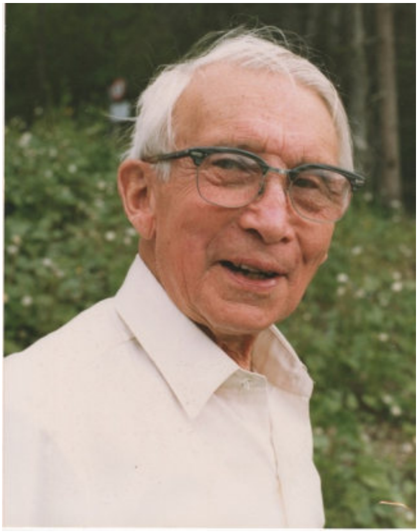


Figure 1.1: Ernst (Ernest) Ising (May 10, 1900 in Cologne, Germany - May 11, 1998 in Peoria, USA). Photo taken in 1987.

question in the specific context of Lenz's suggestion for a linear chain. At the same time, Wolfgang Pauli, who was also at Lenz's institute, contributed to quantum mechanics by suggesting that the electron possesses a two valued non-classical magnetic moment. While these were important steps towards answering the above questions, the full picture had to wait for further new ideas.

Nowadays literature based on the Ising model is abundant and there are several good reviews that report the history of the model [1–3] and its applications in different fields of science [4–6]. About 800 papers on the Ising model are published every year [3][(b)]. It has found applications in a range of different circumstances such as tumor modelling [7], seismic-hazard assessment [8] and sonification of science (instead of visualization) [9]. Quite recently a universal simulator modelling spin models has been found [10]. Such broad impact of the Ising model is also reflected in the contributions to the "Ising lectures" – an annual workshop in Lviv that in 2017 celebrates its 20th anniversary [11], the occasion for which this paper is written. The broadness of the appeal of the Ising model requires that we restrict our report to a small selection of topics. In particular, in what follows we discuss the history of the model and its formulation as we now know it (Section 1.3); exact solutions (Section 1.4); experimental realisations (Section 1.5); and its extension to other fields (Section 1.6). We start with the personal memoirs of Thomas Ising (Ernst's son) in the next section.

1.2 My Father – Ernest Ising

My father was a wonderful person who was in love with life. He thoroughly enjoyed teaching: "I got some of the students in the front row wet with my experiment." He often stated that no class was complete unless his students had laughed with him.

When I first got to know him, physics was something far away. He was only interested in keeping himself and our family alive in the middle of WWII. Starting with 1933, there were really only twelve very bad years for him.

He was born Ernst Ising, at the beginning of the 20th century on May 10th in Köln (or Cologne) near the cathedral. His mother, Thekla Loewe, came from a very successful Jewish merchant family in Duisburg. His father, Gustav, grew up in the small town of Rietberg in rural Westphalia, as son of the local blacksmith. We do not know how or when Thekla and Gustav met, but Kaufhaus Loewe had quite a few male and female employees. Thekla, my grandmother, spoke of the table usually being set for about forty people. Gustav and his brother (?) Bernard ran a very successful upscale women's clothing store in Bochum until the Hitler years. By the time his sister, Charlotte (Lotte), was born in 1904, his parents had a wonderful home in a wealthier part of Bochum. Their home became a stopping point for many artistes of the period. It included a two-story stained glass stairwell by Johan Thorn Prikker. This unfortunately did not survive the war.

My father liked acting and had a stage in the basement where he and his friend, Heinz Wildhagen put on plays. Heinz spent his life as an actor and theater owner. Later the actor Willie Busch became my father's dictation coach.

After completing Gymnasium in 1918 he spent a few compulsory months as a soldier near the end of WWI. On the day that the war ended he was on a ladder hanging a banner. He said he looked around and everyone was gone! The war was over and they had all left.

In 1919 he started at the university in Göttingen majoring in math and physics. Later he was at the University in Bonn.

In graduate school at Hamburg University in 1922 he came under the tutelage of Professor Wilhelm Lenz who suggested a doctoral thesis in ferromagnetism following up on his paper of 1920. The thesis was completed in 1924. One of his fellow students was Wolfgang Pauli. Also at this time his sister, Lotte, married Hermann Busch (Willie's brother) of the famous Busch family.

After receiving his PhD he went to work in the patent office of the AEG or the Allgemeine Elektrizitätsgesellschaft (General Electric) in Berlin. While he enjoyed the work, he knew that he preferred teaching. During this time he joined and hiked with members of the math and physics group where he met my mother. She had just received her Doctorate in Economics and was working for a professor at the university. In 1927 for a year Ernst worked as a teacher at the famous boarding school, Schule Schloss Salem, in Salem, near Lake Constance. He then went back to Berlin University in 1928 so he could begin studies on pedagogy and philosophy. Two years later, in 1930, he passed the state exams on higher education and they were married.

My parents moved to Strausberg where he had a teaching position and my mother could take the train to Berlin. This lasted for two wonderful years until April of 1933 when Jewish teachers were removed from their positions. This was the start of "12 years on a tightrope" as my mother described it.

There followed a year of searching, including a very temporary job at a school for emigrant children in Paris.

In 1934, he got a new job as a teacher for Jewish children at the Judishes Landschulheim in Caputh, a few miles from Potsdam (see Fig. 1.2). It was founded in 1931 by Gertrude Feiertag, who was a known progressive social educationalist. Next-door was the summerhouse of Albert Einstein. When Einstein permanently extended his USA visit in 1932, the school rented his house to be used as additional classrooms. This allowed the number of enrollees to increase due to the fact that Jewish children were being expelled from German public schools. Three years later my father took over the headmaster position. But as one survivor said, "the supposedly safe island threatened to go under the brown sea at any time, and the

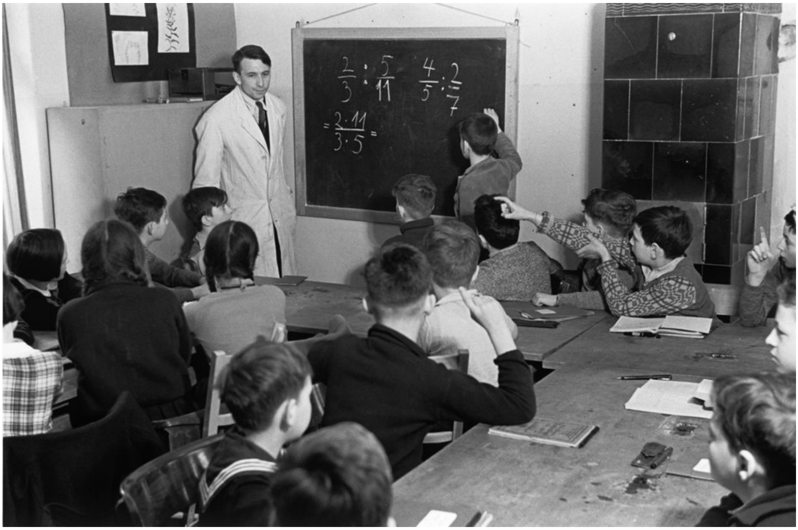


Figure 1.2: Ernst Ising teaching one of his classes at Landschulheim Caputh. ©Herbert Sonnenfeld, Judisches Museum Berlin.

children and teachers knew that too".

While they were able to live near the campus by the relaxing Havel River, it was possible to take a daily swim and take out their Klepper foldboat, although the Nazi threat was constant. Once when they thought my father was about to have a nervous breakdown, my mother persuaded him to take a camping trip down the Danube River in their foldboat (see Fig. 1.3). At the end my father said it was much better than a sanitarium.

On 10 November 1938, the school was destroyed, as part of Kristalnacht, a program to get rid of the Jewish people in Germany. The children had been prepared and were led in four groups through the woods to transportation, home or safety. As one survivor put it, "it was just like in the 'Sound of Music'".

On 27 January 1939, he was taken by the Gestapo and interrogated for four hours. He was only released after he promised that he and his wife would leave Germany. They gained entry to the closed borders of Luxembourg with the help of Dannie Heineman (of the Dannie Heineman physics prize) via his brother-in-law Hermann Busch and the Busch Quar-

Figure 1.3: Ernst Ising and his wife Jane (Johanna) Ehmer Ising during a camping trip down the Danube River in 1938.

tet. The quartet always gave two extra private performances in Belgium, one for the Queen and one for Mr. Heineman. Dannie Heineman had arranged for some 100 German-Jewish families to occupy vacant hotels and to pay their room and board. My parents had planned to emigrate to the United States, but at that time the quotas were full and they were forced to remain in Luxembourg where I was born. The Germans invaded on my father's 40th birthday. After the Germans arrived Mr. Heineman made arrangements for one last payment of a six-month allowance.

After that they survived in Luxembourg during the whole war by my father doing mostly menial farm jobs. In between, there were ten months of teaching Jewish children denied public schools in Luxemburg City. Later there were several months of caring for sick and old Jews who had not yet been deported from the Cinqfontaines Monastery in northern Luxembourg. The Nazis had confiscated the Monastery and were using it as a deportation center to send the Jews to the camps. Near the end of the war, he was forced with other mixed married Jewish men to help dismantle rails of the Maginot line to be sent to the eastern front. He was left relatively

Figure 1.4: Thekla Ising, Tom Ising, Ernst Ising and Johanna Ehmer Ising (from left to right) during their five-day hike through the Swiss Alps (1946).

unthreatened as he had both a non-Jewish wife and an “Aryan” son.

During this whole uncertain time they had acquired two used bicycles and we were able to escape on long bike rides through the countryside. Except for German control we were never in a war zone until the end. On September 10, 1944 allied soldiers arrived in Mersch and the horror was over. Later we escaped the Battle of the Bulge by only ten miles. Google indicates only 36 Jews survived in Luxemburg.

By 1946 we were able to take a month long vacation with my grandparents who had been lucky enough to survive the war safely in Basel, Switzerland. This included a five-day hike through the Alps with my 71-year-old grandmother (see Fig. 1.4).

It took over two years after the war ended for us to complete the paper work necessary to enter the US. In April 1947, we finally arrived in New York on the freighter “Lipscomb Lykes”. That spring my father went to a physics convention in Boston to get a job. There he was asked for the first time if he was the “Ising” of the Ising Model.

During that summer my parents found work at the Tapawingo Farm

Camp near Gouldsboro, PA. I was among other seven year olds who had Hanna Tillich as our housemother. Our English improved tremendously.

That fall my father started as a teacher at the State Teacher's College in Minot, North Dakota. He had to make a very radical change from teaching in a German high school nine years earlier to teaching in an American college in English. The next year he became a Physics Professor at Bradley University in Peoria, Illinois. His wife Jane (Americanized from Hanna) also became a teacher at the school. This was where they stayed. He retired in 1976. In 1953 we were granted our US citizenships. He officially became Ernest and my mother became Jane.

My parents soon made many lasting friendships. Every summer we, or they, went on a significant trip, even driving up to Alaska one year. They also took several trips to Europe and many other parts of the world. On a lonely beach in Oregon the summer after my graduation, they happened to meet one of my physics professors. He exclaimed that he was writing a chapter on the Ising model in his new book.

My father passed away at home one day after his 98th birthday after only five days in hospice.

1.3 From Lenz *Umklappmagnets* via Ising's Chain to Pauli's *Zweideutigkeit*

> *The most successful elaboration of technique in statistical mechanics exists in connection with the Ising model.* (G.H. Wannier 1966 [12])
>
> *Starting around 1925, a change occurred: With the work of Ising, statistical mechanics began to be used to describe the behavior of many particles at once.* (L.P. Kadanoff 2013)[2]

1.3.1 Lenz paper from 1920

Magnetism and especially ferromagnetism was a less understood phenomenon at the beginning of the 20th century. Pierre Curie discovered in 1895 that permanent magnets (ferromagnets) loose their magnetization if they

[2]In [13] Kadanoff cites together with Ising's 1925 paper Brush's review [1] who made a similar statement at the end of his paper.

are heated above a certain temperature T_C, now called Curie temperature [14]. Curie recognized that the behavior near the critical point in fluids and magnets seems to be the same and introduced a kind of universality[3] by pointing to the "analogy between the way in which the intensity of magnetization of a magnetic body increases under the influence of temperature and the intensity of the field, and the way in which the density of a fluid increases under the influence of temperature and of the pressure."

Already in 1911 Niels Bohr and independently Hendrika Johanna van Leeuwen discovered that magnetism is not a classical but a quantum mechanical phenomenon. They proved[4]: "At any finite temperature, and in all finite applied electrical or thermal fields, the net magnetization of a collection of electrons in thermal equilibrium vanishes identically" [16, 17]. Bohr postulated within his atomic model that the planetary-like electron orbiting around the atomic nucleus has a quantized angular momentum and induces a magnetic moment. This condition allowed one to introduce atomic (molecular) magnetic moments, which could respond to an external field and to set up models for para- and diamagnetism. In gases the magnets, according to the freedom of the atoms or molecules, could be oriented in every direction. On the basis of such assumptions Curie's law (the dependence of the susceptibility χ with temperature T as $\chi \sim c/T$) for paramagnets could be derived.

In 1920 Lenz questioned the assumption of free rotation of the elementary magnets in solids and suggested instead that they may change their direction just turning around by 180 degrees (Umklapp-Prozess) [18]. He then derived Curie's law. In the last paragraph of his short communication he suggested his two-state model also for ferromagnets in order to explain the appearance of a permanent magnetism at temperatures below T_C. He says[5]: "If one assumes that in ferromagnetic bodies the potential energy

[3] "Analogie entre la manière dont augmente l'intensité d'aimantation d'un corps magnétique sous l'influence de la température et l'intensité du champ, et la manière dont augmente la densité d'un fluide sous l'influence de la température et de la pression." [14]

[4] Bohr concluded in his thesis: *a piece of metal in electric and thermal equilibrium will not possess any magnetic properties whatever due to the presence of free electrons* (see [16], page 380).

[5] "Nimmt man an, daß im ferromagnetischen Körper die potentielle Energie eines Atoms (Elementarmagnets) gegenüber seinen Nachbarn in der Nullage eine andere ist als in der π Lage, so entsteht eine natürliche zum Kristallzustand gehörige Gerichtetheit der Atome und daher spontane Magnetisierung."

Figure 1.5: Ernst Ising and Wolfgang Pauli during the time in Hamburg about 1925.

of an atom (elementary magnet) with respect to its neighbors is different in the null position and in the π position, then there arises a natural directedness of the atom corresponding to the crystal state, and hence a spontaneous magnetization." (translation from [2]$^{(a)}$).

In Weiss's domain model for ferromagnetism [19] it is the reaction of already ordered domains to a magnetic field which leads to the Curie-Weiss susceptibility $\chi \sim c/(T - T_C)$, whereas in Lenz's suggestion it is an unknown kind of non-magnetic interaction between the two directions of the elementary magnets.

After Lenz took up the post of Chair of Theoretical Physics at the University of Hamburg he was able to lead a group of physicists and young students to work on the project he suggested in his short paper [20]. Einstein considered Lenz's papers on magnetism, although published incompletely, as "extremely important" ([20] p. 93). The first to become involved in this project was Ernst Ising, who was already a student in Hamburg when Lenz became full professor. In 1922 Lenz proposed the problem outlined in his 1920 paper for Ising's thesis, *Beitrag zur Theorie des Ferro- und Paramagnetismus* [21].[6]

In May of the same year (1922) Lenz managed to obtain Wolfgang Pauli

[6] *Contribution to the theory of the ferro- and paramagnetism.*

as assistant.[7] He was of the same age as Ising but already an internationally well-known physicist. He came from Göttingen where he worked with Max Born [34] on problems with the Bohr-Sommerfeld's atomic model and remained in Hamburg where he submitted his Habilitation on 17 January 1924. He stayed in Hamburg until March 1928 and moved to Zürich when he got a Chair at the Technical University. In 1923, during his stay at the institute of Lenz, he visited for almost one year Bohr's institute at Copenhagen. During this time Ising replaced Pauli until his return at the end of September 1923 (see Fig. 1.5).

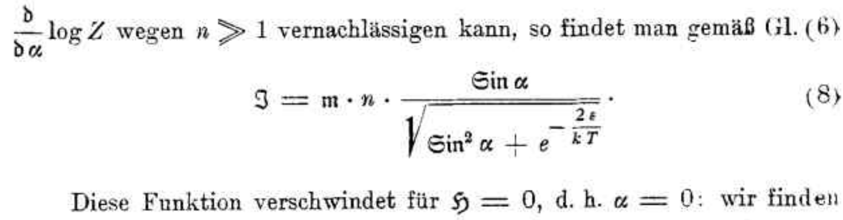
1.3.2 Ising's thesis and his 1925 publication

The goal of Ising's task assigned by Lenz was to explain the appearance of a ferromagnetic state in a three dimensional (3D) solid. In fact this job was a twofold one: First he had to set up the model for the interaction of the elementary magnetic units, which prefer alignment, a problem which belonged to the new and undeveloped quantum mechanics. Second he then had to calculate *analytically* the macroscopic magnetization with the methods of statistical mechanics.

Both tasks were far too big to be solved in a thesis as the development of quantum mechanics and statistical physics later showed. The first problem one could say was answered in 1928 by Heisenberg [22] after the theory of quantum mechanics proceeded far enough and the second in 1941 by Onsager [23] in 2D after special properties of the model had been clarified by Kramers and Wannier and new methods of calculating a partition function had been found [24, 25]. The 3D problem remained analytically unsolved until now (see Section 1.4 for more details).

Therefore Ising had to restrict himself for the first problem to arguments for the model and for the second problem to reductions and approximations. In order to come along with the first problem he refers in the introduction to the paper of E.A. Ewing [26], "where it was shown experimentally and theoretically, that ferromagnetism is caused by a mutual interaction of the elementary magnets." But the interaction is not thought to be the well known interaction between dipoles. In fact "no statement on the nature of this force, which might be of electrical nature [27] can be made, but it is assumed that it decays rapidly with the distance." It is in-

[7] In fact his title was "wissenschaftlicher Hilfsarbeiter".



$\frac{\mathfrak{d}}{\mathfrak{d}\alpha}\log Z$ wegen $n \gg 1$ vernachlässigen kann, so findet man gemäß Gl. (6)

$$\mathfrak{J} = \mathfrak{m} \cdot n \cdot \frac{\mathfrak{Sin}\,\alpha}{\sqrt{\mathfrak{Sin}^2\alpha + e^{-\frac{2\varepsilon}{kT}}}}. \qquad (8)$$

Diese Funktion verschwindet für $\mathfrak{H} = 0$, d. h. $\alpha = 0$: wir finden

Figure 1.6: Ernst Isings result for the magnetization of the chain [28].

teresting that these references are missing in his 1925 publication. So Ising concludes a nearest neighbor interaction is sufficient. He further points out that this is “in crass disagreement” with the hypothesis of a molecular field. We now know that the critical behavior of systems with phase transitions are described by mean field in dimensions high enough otherwise it is an approximation or misleading as in the 1D case here.

In order to attack the second task Ising restricted himself to the 1D case – the famous Ising chain.[8] In the thesis the configurations on the chain are displayed as vectors parallel or antiparallel to the direction of the chain. This presentation in the publication is replaced by the short notation plus and minus restricting to orientations only parallel or antiparallel to the direction of the chain.

The calculation follows the standard methods of equilibrium statistical mechanics. Namely counting the configuration of different energy in order to obtain the partition function and in a next step the mean magnetization $\mathscr{I}$

$$\mathscr{I} = m \cdot n \cdot \frac{\sinh\alpha}{\sqrt{\sinh^2\alpha + e^{-\frac{2\epsilon}{kT}}}}\,, \qquad \alpha = \frac{mH}{kT} \tag{1.1}$$

where m is the elementary magnetic moment, H is the external field, T the

[8]This problem is reconsidered by Kramers and Wannier in the first paper of [24,25] in section 2 as an easy introduction to their new method. In section 3 they explain: “The reduction of the linear chain problem can be described in a qualitative way as follows. It is possible to build up a chain by repeating constantly one and the same operation, namely *adding another spin* beyond the one just placed previously” (emphasis by the authors of this paper). They explain, that the successful mathematical treatment is based on one hand on the fact that no physical change takes place by this procedure, if the chain is very long and on the other hand that the state of the last added spin depends only upon the state of the predecessor.

temperature, k the Boltzmann constant and n the number of elements of the chain (see Fig. 1.6). Thus in zero field no macroscopic magnetization arises at finite temperature.

Ising tried to generalize the model to higher dimensions [9]: "It is imaginable that a spatial model, in which all elements that in some way are neighbors affect each other, brings with it the necessary stability to prevent the magnetization intensity to vanish with H. However, in that case the calculations do not seem to be feasible; at any rate, so far it has not been possible to sort and count the appropriate arrangement possibilities." (translation from [2]$^{(a)}$). Indeed this is not possible (so far) and one had to look for approximations. He considered different kinds of arranging 1D chains. In the publication (section 3. "The spatial model") he assumed the special limit where n_1 identical chains are spacially arranged. He argues that differences in the configuration of the interacting chains are energetically unfavorable. Therefore the result is

$$\mathscr{J} = m \cdot n \cdot n_1 \cdot \frac{\sinh n_1\alpha}{\sqrt{\sinh^2 n_1\alpha + e^{-\frac{2n_1\epsilon}{kT}}}} \tag{1.2}$$

and once again (although not surprising due to the approximation) does not find a finite magnetization in zero magnetic field.

Based on these results in the thesis he concludes [10]: "So, if we do not assume, as P. Weiss did, that also quite distant elements exert an influence on each other and this seems to us not to be allowed under any circumstances we do not succeed in explaining ferromagnetism from our assumptions. It is to be expected that this assertion also holds true for a spatial model in which only elements in the nearby environment interact with each other" (translation from [2]$^{(a)}$).

Ising finished his thesis in 1924 and published in 1925 a short paper [28] with his results. There is not much known about the contact between Ising

[9]"Es ist ja denkbar, dass ein räumliches Modell, bei dem alle irgendwie benachbarten Elemente auf einander wirken, die nötige Stabilität mit sich bringt, um zu verhindern, dass die Magnetisierungsintensität mit H verschwindet. Doch es scheint in diesem Fall die Rechnung nicht durchführbar zu sein; jedenfalls ist es bisher nicht gelungen, die Anordnungsmöglichkeiten geeignet zu sortieren und abzuzählen."

[10]"Wenn wir also nicht annehmen, wie dies P. Weiss tut, dass auch recht entfernte Elemente einen Einfluss aufeinander ausüben – und das scheint uns auf keinen Fall zulässig zu sein – so gelangen wir bei unseren Annahmen nicht zu einer Erklärung des Ferromagnetismus. Es ist zu vermuten, dass diese Aussage auch für ein räumliches Modell zutrifft, bei dem nur Elemente der näheren Umgebung aufeinander wirken."

and Pauli, but Brush [1] reports a letter from Ising to him where he stated "...I discussed the result of my paper widely with Professor Lenz and with Dr. Wolfgang Pauli, who at that time was teaching in Hamburg. There was some disappointment that the linear model did not show the expected ferromagnetic properties...". No further communication is reported apart from a letter from Pauli to Ising found by Sigmund Kobe [3]$^{(d)}$, where Pauli informed Ising about his fate and that of other colleagues in Hamburg after Ising left the institute and which were also known by Ising.

1.3.3 Pauli's struggle with the Bohr-Sommerfeld model of the atom

The Bohr-Sommerfeld model of atoms was only partly successful in explaining the experiments. It fails in cases where more than one electron was present in the shell, but even in the case of one electron discrepancies appeared. The situation in the year 1923 is explained by Landé in a short note [29]. He mentioned that[11] "It turns out that in systems with more than one electron not even the quantum theoretical stationary states and their adiabatic changes are mechanically calculable." He notes as example the helium atom and adds:[12] "The second particularly drastic example for the failure of the mechanical basic principles also in stationary quantum states illustrates the multiplet structure and especially the anomalous Zeeman effect ..."

After his stay in Kopenhagen Pauli gave his "Antrittsvorlesung" where he described the situation of the mechanical atomic theory. He states[13]: "The contents of this lecture appeared very unsatisfactory to me, since the problem of the closing of the electronic shells had been clarified no further. The only thing that was clear was that a closer relation of this problem to the theory of multiplet structure must exist." For another description of

[11]"Es zeigt sich nämlich, daß bei Systemen aus mehreren Elektronen nicht einmal die quantentheoretisch stationären Zustände und ihre adiabatischen änderungen mechanisch berechenbar sind."

[12]"Das zweite besonders drastische Beispiel für das Versagen der mechanischen Grundprinzipien auch in stationären Quantenzuständen gibt die Multiplettstruktur und speziell der anomale Zeemanneffekt ..."

[13]"Der Inhalt dieser Vorlesung schien mir sehr unbefriedigend, da das Problem des Abschlusses der Elektronenschalen noch nicht weiter geklärt war. Das einzige was klar war, war, daß eine engere Beziehung zwischen diesem Problem und der Theorie der Multiplettstruktur bestehen muß."

the desperate situation by 1924 see [30] page 125.

Another severe problem was the understanding of the periodic system although Bohr constructed with help of an additional principle ("Aufbauprinzip") the structure of the shells in the classical atomic model. Pauli tried to connect all these problems and solve them with a new principle by postulating a fourth quantum number for the electron and formulating his exclusion principle (the name was given to it by Paul Dirac [32] p.59) for the electrons. He published those ideas in 1925 $[33]^{(a)}$ p. 385 and $[33]^{(b)}$ p. 765 where he concludes[14]: "According to this point of view the doublet structure of the alkali spectra, as also the piercing of the Larmor theorem, comes about by a peculiar, classically not describable kind of two-valuedness of the quantum mechanical properties of the valence electron." (translation from [34] p. 107). The expression nonclassic "classically not describable" was certified by later development since Bohr was able to show that the spin could not be measured by classically describable experiments [31].

The dramatic story of Pauli's struggle to increase the quantity of quantum numbers from three to four is described by A.I. Miller [32] (see also [35–37]). Immediately afterwards G. Uhlenbeck and S.A. Goudsmit introduced for this two-valuedness the concept of the spin for the electron [38]. Already in 1921 A.K. Compton discussed the possibility that the electron possesses a magnetic moment as a result of its spinning motion. A similar idea was formulated by Ralph Kronig but never published.

After Ising had published his negative result it remained open in the physical community if the higher dimensional cases would lead to spontaneous magnetization or not. Pauli communicated about this question with Heisenberg (see [39] p. 129 ff.) and Heisenberg expressed his belief that if the number of nearest neighbors (i.e. the dimension) is high enough one would succeed finding ferromagnetism.

1.3.4 The formulation of the Hamiltonian for the Ising model

The quantum mechanical foundation of the interaction which might lead

[14]"Die Dublettstruktur der Alkalispektren sowie die Durchbrechung des Larmortheorems kommt gemäß diesem Standpunkt durch eine eigentümliche, klassisch nicht beschreibbare Art von Zweideutigkeit der quantentheoretischen Eigenschaften des Leuchtelektrons zustande."

to ferromagnetism was introduced in 1928 by Heisenberg [22]. It is known as the exchange interaction and is due to the overlap of the wave function of neighboring atoms obeying the exclusion principle. In this way the magnetic moments due to the spin of the electron define the interaction. If the spins are parallel the electrostatic energy is changed so that this configuration is more favorable. He concluded in his paper[15]: "(1) The crystal lattice has to be such, that each atom has at least 8 neighbors. (2) The main quantum number of the electrons, which are responsible for the magnetism has to be $n \geq 3$."

In the year 1930 Pauli was invited to the Solvay conference. His invited talk [40] gave a review of the status of the theory concerning magnetism and its quantum mechanical nature (for a short content of his talk see [34] page 220 ff.) Especially interesting for ferromagnetism is section 5 of [40]. Here for the first time it is mentioned that the phase transition could depend on dimensionality. He also mentions Ising's work in connection with Heisenberg's work and its result for the magnetic moment in molecular field theory

$$\mathscr{M} = N\mu_0\Big[1 - C\Big(\frac{T}{\Theta}\Big)^{\frac{3}{2}}\Big]. \tag{1.3}$$

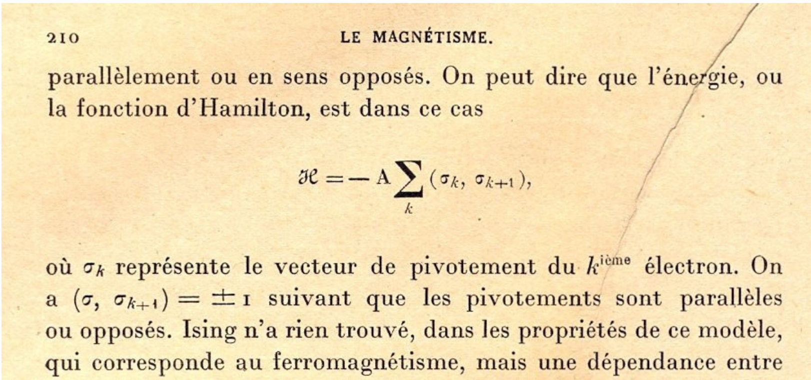
He states:[16] "There is in fact a very close relationship between the problem of Ising and the one we have just treated"([40] p. 209). Pauli's critical appreciation of Ising's model:[17] "In Ising's calculation developed from the point of view of the old quantum mechanics, the components of σ_i that are perpendicular to the field are considered to be zero, whereas in the new quantum theory these components do not commute with the components in the direction of the field." (translation from [2]$^{(a)}$ p. 291 slightly corrected) But Pauli immediately suspects for the classical variant[18] "Irrespective of

[15]"(1) Das Kristallgitter muß von solcher Art sein, daß jedes Atom mindestens 8 Nachbarn hat. (2) Die Hauptquantenzahl der für den Magnetismus verantwortlichen Elektronen muß $n \geq 3$ sein."

[16]"Ce résultat est interéssant en liason avec la discussion d'un modèle semi-classique proposé par Ising."

[17]"Dans le calcul d'Ising, développé au point de vue de l'ancienne théorie des quanta, les composantes des σ_i perpendiculaires à la direction du champ sont considérées comme nulles, tandis que dans la nouvelle mécanique cette composante n'est pas commutable avec celle qui correspond à la direction du champ."

[18]"Malgré cette différence, il est très vraisemblable qu'une extension de la théorie d'Ising au cas d'un réseau à trois dimensions donnerait du ferromagnétisme même au point de vue classique."



210 LE MAGNÉTISME.

parallèlement ou en sens opposés. On peut dire que l'énergie, ou la fonction d'Hamilton, est dans ce cas

$$\mathcal{H} = -\mathrm{A} \sum_k (\sigma_k, \sigma_{k+1}),$$

où σ_k représente le vecteur de pivotement du $k^{\text{ième}}$ électron. On a $(\sigma, \sigma_{k+1}) = \pm 1$ suivant que les pivotements sont parallèles ou opposés. Ising n'a rien trouvé, dans les propriétés de ce modèle, qui corresponde au ferromagnétisme, mais une dépendance entre

Figure 1.7: Part of page 210 of Pauli's contribution to the Solvay conference [40], where he presented the Ising model in the form as it is known nowadays.

this difference, it is quite likely that an extension of the theory of Ising to the case of a lattice of three dimensions would yield ferromagnetism *even from the classical point of view*" (emphasis by the authors of this paper).

Thus it was Pauli himself who introduced the modern notation for the Ising model [40] p. 210, see Fig. 1.7

$$H = -A \sum_k (\sigma_k, \sigma_{k+1}) \tag{1.4}$$

where A gives the strength of the interaction of the spins on the chain position k. Pauli pointed to the difference of the properties of a quantum mechanical spin σ_k and called the "spin" appearing in the Ising model a semiclassical spin.

This compact formulation of the Ising model includes already all the aspects important for the following development: (1) the whole system is described as an interacting many-particle system, (2) these individual particles produce a specific collective behavior leading eventually to phase transitions. The formulation also separates the aspects the strength A of interaction of the interacting units and the properties of the units σ_i themselves. A is dependent on the ferromagnet considered whereas the units are the same for the whole group of ferromagnet. This reflects the important concept of *universality*, at least for ferromagnets, already introduced 1908 by Pierre Curie [15] within mean field theory and comprising phase transitions in liquids and magnets. Future developments like *scaling theory* and *renormalization group theory* show that this universality concept goes beyond mean field theory and the Ising model in three dimensions. Rather it describes the critical behavior of a whole universality class containing liquids, magnets and other physical systems of same dimension, symmetry and type of short ranged interaction.

1.3.5 Comments on Ising's result

The usual explanation for the negative result for permanent magnetization at finite temperature in the 1D case points to the free energy. It consists of two parts, the internal energy and a negative entropic term. This entropic term favors disorder in the 1D case against macroscopic alignment. Another question was, if some kind of long range interaction could change the result. Already from the Curie-Weiss model it was known that taking into account the interaction of all the spins by an effective field a phase transition came about even in the 1D case. However an interaction between two positions in the chain i, j with a decay according to a power law like $1/|i-j|^{1+\alpha}$ leads to a phase transition for a sufficient weak decay, $\alpha < 1$ [41] (see also [42]).

Ising had to struggle with the configurations of the chain. A much more elegant way, used mainly in textbooks, is to calculate the partition function with the method of transfer matrices developed by Kramers and Wannier [24, 25].

Pauli's criticism that in fact quantum mechanics formulates a model where the units are non-classical was taken up in the 1960's. It turned out that (a) there are physical examples for which such a model might be applicable and (b) numerical solutions of the problem could be obtained [43]. It also opened the new field of quantum phase transitions.

1.4 More on Exact Solutions

In the 1920's, the dominant theory for magnetism was that of Pierre Weiss [19]. This was based on the suggestion that ferromagnets comprise domains of parallel-aligned micromagnets. Each micromagnet within a domain is supposed to experience an effective magnetic field (the Weiss mean field) coming from its neighboring magnetic moments. Each magnetic domain is then randomly aligned, up to preferences induced by crystallographic symmetries. Alternatives to Weiss's formulation include the Bragg-Williams approximation [48] as well as Bethe-lattice models [49]. The free energy coming from such mean-field approaches is

$$f(\beta, h) = \frac{qJm^2}{2} - \frac{1}{\beta} \ln\left[2\cosh h + J\beta qm\right], \tag{1.5}$$

where $\beta = 1/kT$, k is the Boltzmann factor, T is the temperature, $h = \beta H$ where H is the strength of an external field, q is the coordination number (number of nearest neighbours of a given site, e.g., $q = 2d$ for a regular lattice of dimensionality d), J is the strength of the inter-site couplings and m is the mean-field magnetization. The model manifests a phase transition at $h = 0$ characterised by non-vanishing and vanishing values of m on either side of a critical temperature $T_c = qJ/k$. It also exhibits discontinuity in the specific heat (the second temperature derivative of the free energy) there.

Following the discovery the specific-heat anomaly of liquid helium at temperatures of around 2.19K, Ehrenfest had introduced a classification system for phase transitions [50]. He christened the anomaly the "lambda point" because of the shape of the experimentally obtained specific-heat curve. He argued that the lambda point is a phase transition, even though it was dissimilar to other known phase transitions in that it did not feature a latent heat or change in volume. Ehrenfest had interpreted the lambda point as a finite discontinuity and he proposed to classify such phase transitions as first- or second-order depending on whether such a discontinuity in the first or second derivative of the free energy. For a recent review of Ehrenfest's scheme and a translation of his original paper [50], see [51]. Thus the mean-field model predicts a second-order phase transition in the original Ehrenfest sense. Moreover this prediction holds for all dimensionalities.

In [28], Ising explicitly highlights the difference between his and Weiss's treatment in that only short-range, nearest-neighbouring interactions are taken into account and the orientation of each micromagnet restricted to only two possibilities. The solution for the free energy is

$$f(\beta, h) = -\frac{1}{\beta} \ln\left[e^{\beta J}\cosh\beta h + \sqrt{e^{2\beta J}(\sinh\beta h)^2 + e^{-2\beta J}}\right], \tag{1.6}$$

and the various thermodynamic functions are easily derived by appropriate differentiation. As we have seen, unlike mean-field theory, the model does not exhibit spontaneous magnetisation.

In 1936, however, Rudolf Peierls showed that the model does manifest ferromagnetism in two dimensions [44] and this problem was investigated by Hendrik Kramers and Gregory Wannier in 1941 [24]. They write in the introduction of their paper: "The problem has a mechanical and a statistical aspect. On the mechanical side we wish to improve our understanding of the responsible coupling forces. On the statistical side we wish to derive with certainty the thermal properties from a reasonable accurate mechanical model. Both aspects have received extensive attention. Quantum theory has explained satisfactorily the origin and nature of the coupling forces. There are also several theories available wich explain in terms of them the thermal behavior of ferromagnets. Not one, however, applies just straight statistics to the mechanical data. Generally some simplifying assumption is introduced to facilitate the evaluation of the partition function. It follows that the results obtained are not necessarily a consequence of the mechanical model, but may well be due to the statistical approximation." In their paper they then introduced the transfer matrix concept and related the free energy of the Ising model for high temperature to a conjugate Ising model at low temperature. By using this relation they were able to calculate the transition temperature of the 2D Ising model. They also developed the transfer matrix method and demonstrated it by re-deriving the results of Ising for the one dimensional chain. By their method they reduced the calculation of the partition function to finding the largest eigenvalue of a two by two matrix. For the two dimensional Ising model the matrix turns out to be a square matrix of infinite dimension and Kramers and Wannier could calculate the finite transition temperature T_c. They showed that the partition function for the infinite system is related to the largest eigenvalue of the matrix. They also discovered a symmetry in the two-dimensional model in that its free energy at low temperature is related to that at high temperature. The exact location for the critical point of the model with square-lattice geometry is then determined as the point which is invariant under this self-duality transformation. It is given by $kT_c/J = 2/\ln(1+\sqrt{2}) \approx 2.269185$. By way of comparison mean-field model theory gives $kT_c/J = 4$ for $d = 2$ and the Bethe approximation gives 2.88.

Onsager solved the model for the square lattice in the absence of an external field (i.e., with $h = 0$) and famously announced his result at the end of a talk by Wannier at the February 1942 meeting of the New York academy of Sciences. He published the result in 1944 in [23]. The free energy for the infinite system in the absence of an external field is

$$\begin{aligned} -\beta f &= \ln 2 + \frac{1}{8\pi^2}\int_0^{2\pi} d\theta_1 \int_0^{2\pi} d\theta_2 \ln\left[\cosh\left(2\beta J_1\right)\cosh\left(2\beta J_2\right)\right. \\ &\quad \left. - \sinh\left(2\beta J_1\right)\cos\theta_1 - \sinh\left(2\beta J_2\right)\cos\theta_2\right], \end{aligned} \tag{1.7}$$

in which J_1 and J_2 are the coupling constants between spins in the two different directions.

This was a milestone achievement in the history of the Ising model in that it was the first exact result for the model with a finite-temperature phase transition and proved that these can be captured by statistical mechanics. Strictly speaking, the phase transition was not of the Ehrenfest type — it has a logarithmic divergence instead of a discontinuity in the specific heat [25]. Nowadays we consider Ehrenfest's classification scheme as extended to include phase transitions with a divergence as well as those with a discontinuity. The significance of Onsager's achievement is reflected in a comment by Wolfgang Pauli to Hendrik Casimir who had inquired about developments in theoretical physics during the second World War: "nothing much of interest has happened except for Onsager's exact solution of the Two-Dimensional Ising Model" [52]. Onsager's solution was simplified by Bruria Kaufman in 1949 [53] and Kaufman and Onsager determined correlation functions in [54].

Onsager made another important announcement at the end of a talk by László Tisza at Cornell University in 1948 [52]. This time he stated that Kaufman and he had derived the spontaneous magnetisation of the two-dimensional Ising model and he wrote the formula on the blackboard. This was important because the non-vanishing of the magnetisation on one side of the transition ($T < T_c$) and its vanishing on the other ($T > T_c$) established the phenomenon as a genuine phase transition. Onsager repeated the claim in May 1949 at a conference of the International Union of Physics in Florence after a talk by George Stanley Rushbrooke [52] but he and Kaufman did not publish the derivation; the first to do so was Chen-Ning Yang in 1952 [55]. Rodney Baxter recently reviewed how the Kaufman-Onsager calculation was developed and added a draft paper giving their result [56]. That result is that the spontaneous magnetisation behaves near the critical point as $(T_c - T)^\beta$ with $\beta = 1/8$. This is different to the mean-field result which is that $\beta = 1/2$. A deviation from mean field exponents was already observed by Verschaffelt [47] in liquids around 1900 and corroborated by further experimental material in different systems. With Kaufman in 1949 Onsager also derived the correlation function at the critical point, and showed that it decays as $1/r^{1/4}$ [54]. In 1965, Alexander Patashinski and Valery Pokrovsky gave the correlation-function exponential decay away from the critical point [57]. The two-dimensional model has still not been solved in the presence of an external field, apart from the $c-$theorem approach of Zamolodchikov [58] using perturbed conformal field theories [59] where the model turns out to be an example of integrable massive field theory.

In [2]$^{(a)}$, Niss discusses the early years of the Ising model within the context of quantum- and statistical-mechanical models of magnetism. From the late 1940's and in the 1950's the model was not believed to provide a good description of magnetic materials due to its lack of physical realism [2]$^{(b)}$. The restriction of the interatomic forces to nearest-neighbouring sites and the further restriction of spins of the Ising model to only two orientations were considered to distance the model from reality wherein the electron spin which can have any direction in three-dimensional space [60]. It was thought that, at best, the Ising model may physically represent anisotropic magnetic materials in which the two spin directions were allowed or binary alloys with spins of each orientation corresponding to one of the two types of atom in the compound. It was also considered a model for lattice gas, in which the presence or absence of a molecule at a point in space was represented by one of the two spin orientations. But as a model of a magnet, it was considered lacking and its interest in this regard was instead as a simplified model of phase transitions in general that has the advantage of being mathematical tractable [2]$^{(b)}$.

The group around Cyril Domb in King's College London did, however, appreciate the physical importance of the Ising model [2]$^{(b)}$. They worked on different types of 2D lattices, using geometries other than squares and pioneered series-expansion approaches, a strategy also employed by the Rushbrooke group at the University of Newcastle. They gained the crucial insight that the critical exponents describing the phase transition depend strongly on the dimensionality of the system and less so on the geometry of the lattices. This would later be explained by the notion of universality (see Section 1.5). The role of dimensionality in the Ising model was therefore quite different to that in mean-field theories, where it is unimportant for the critical exponents [2]$^{(b)}$.

Comparisons between series expansions and the exact solution in two dimensions lent confidence that the approximate approach may be applied to the three-dimensional version as well as to other models. Indeed, and as discussed in [2]$^{(c)}$, the Onsager solution to the two-dimensional Ising model frequently played (and continues to play) a role analogous to that traditionally played by experiment in that hypotheses were tested against it. This includes the scaling relations between the various critical exponents describing continuous phase transitions. These establish that the critical exponents are not all independent. The development of the scaling relations by figures such as John Essam, Michael Fisher and Benjamin Widom (and the related inequalities derived by Robert Griffiths, Brian Josephson, Rushbrooke and others) were pivotal to the development of more general theories of critical phenomena (for a review, see e.g., [61]). They helped pave the way for Widom's hypothesis that the singular part of the free energy is a homogeneous function of its arguments. The explanation for Widom's form was, in turn, given by Leo Kadanoff who ascribed the singularity in the free energy to the occurrence of large-scale fluctuations in the system as the critical point is approached. These fluctuations cause the correlation length to diverge and a relation between temperature, field and length scales, a concept captured by Kadanoff's block-spin formulation and ultimately by Ken Wilson's renormalisation group. The renormalisation group forms the foundation stone on which the entire modern theory of critical phenomena is built and is of fundamental importance not just for statistical physics but also for high-energy physics and any physical system which can be viewed at different distance scales. This also explained the crucial concept of universality, to use the term coined by Kadanoff in 1971. This means that critical exponents are independent of many details of the Hamiltonian, and are functions instead of the system dimensionality, its internal symmetries and the range of interaction between its constituent entities (spins) (see Section 1.5 for experimental verifications).

The three-dimensional Ising model has proved to be a far tougher problem than its lower-dimensional counterparts and a solution remains elusive, even in the absence of an external field It has a status in statistical physics similar to that which Fermat's last theorem occupied in mathematics, until proof of the latter by Andrew Wiles in 1994; the problem is easily formulated but hard to solve. Already in 1945 Wannier hoped that an analytic solution was imminent and both Onsager and Wolfgang Pauli are believed to have attempted it in the 1950s [2][(b)]. Other notable names worked on a solution [62, 63] "and in the 1950s physicists gradually concluded that a solution was not within reach" [2][(b)].

In 1986, Anders Rosengren reported an attempt to generalise combinatorial considerations of the 2D nearest-neighbour model to the three-dimensional simple cubic case. This led to the "Rosengren conjecture" that the critical temperature for the 3D case is given by $\tanh(J/kT_c) = (\sqrt{5}-2)\cos(\pi/8)$. This gives the value $J/kT_c \approx 0.221\,658\,63$. Although this appears close to the value $0.221\,654\,6(10)$ coming from simulational studies [65], it is still over four standard deviations away. In [66], Fisher showed that Rosengren's form comes from a "critical polynomial". A root of such a critical polynomial delivers the critical point in the 1D and 2D cases and the hope was that one could find the corresponding polynomial in the 3D case, whose vanishing specifies its critical point. Fisher showed that Rosengren's polynomial is a poor candidate; it does not mimic desired features of the $d = 2$ model, is not unique and the resulting estimate for T_c is not convincing.

The question of an exact solution of the 3D model has again come under the spotlight recently and claims to have found the exact exponents were given in Refs. [67, 68]. Rational values for the critical exponents, including $\alpha = 0$ for the specific heat have been given with suggestions of the existence of a multiplicative logarithmic correction there [67, 68]. Such claims are controversial because they are not in agreement with very precise (and presumably accurate) approximations coming from a variety of techniques including series expansions, renormalization group, Monte Carlo simulations and experiment [69]. Additionally, although the values given in Refs. [67,68] obey the standard scaling relations, as they should, a logarithmic term in the specific heat would contradict the scaling relations for logarithmic corrections [72]. The recent claims of [68] and related papers [70] were criticised in Refs. [71].

Exact studies of the Ising model in low dimensions continue apace. Boris Kastening recently presented a simplified version of Kaufman's solution and extended it to various boundary conditions [73, 74]. Alfred Hucht exactly calculated the partition function of the square lattice Ising model on the rectangle with open boundary conditions for arbitrary system size and temperature [75]. For the three-dimensional model, Sheer El-Showk and collaborators produced a series of papers hoped to lead to a solution of the conformal field theory for describing the three dimensional Ising model at the critical temperature [76, 77]. Their bounds are consistent with previous estimates such as from renormalization group, experiments and Monte Carlo simulations. As they say in the final sentence of their first paper: “We have not yet solved the 3D Ising model, but we have definitely cornered it” [76]. In their second paper they ask: “Could it be that the critical 3D Ising model is, after all, exactly solvable?” If not, El-Showk et al. at least have a very efficient method to solve it numerically [77].

Besides these exact results, a vast number of papers appear annually which are related to the Ising model. Indeed, through the renormalization group we now know that the validity of the Ising model and its critical exponents extends far beyond anything that could have been envisaged by Lenz or Ising in the 1920s, by Landau in the 1930s or Onsager in the 1940s. We refer to the second edition of McCoy and Wu's famous book for a discussion of the development in the Ising model since the early 1970s [78].

1.5 Experimental Aspects of the Ising Model

1.5.1 Universality

As discussed in the previous sections, a key theoretical concept of critical phenomena which occur at second order phase transitions is that of universality [79, 80]. According to this concept, among the properties which describe critical singularities in the neighborhood of a second order phase transition, some exhibit a rather robust character, which means that they do only depend on very general — essential — properties of the system under interest. Other — non-essential — characteristics are often called *details* in this context. Among the essential characteristics, one usually mentions space dimensionality, symmetries, range of interactions (see Section 1.4). The very nature of the interactions on the other hand, such as whether they are of magnetic or of electric origin, would they follow from classical or from quantum description of matter, etc, is not essential. This robustness must also be explained in deeper detail. Among the universal properties or characteristics, the critical exponents which describe the leading singularities of the thermodynamic quantities are probably the most famous ones. Certain combinations of the critical amplitudes, these numbers which appear in prefactors of the leading singularities, also are universal. All these are just pure numbers, the set of which defines a universality class. According to the universality argument, let us assume that measurements are performed on some real material which is expected to have the required symmetries to belong to a given universality class. Then, extremely strong predictions can be made for its critical properties. For example if a system is expected to fall in the 2D Ising model universality class, the critical exponent describing, let say its spontaneous magnetization, has to be 1/8. Not another number close to 0.125, but exactly 0.125! And if it is not the case, then, the experiment is wrong! This is the incredibly strong predicting power of the theory of critical phenomena. Of course, our statement that the experiment would be wrong is exaggerated, and reality does not always simply fits mathematical symmetries. Proving that a given material exhibits the correct symmetries may be very challenging, but there are many experimental situations in which the expected universal properties can be measured. Thanks to universality again, although the real material is often only approximately a representative of a given universality class, deviations from the correct symmetry may appear to be non-essential. We will illustrate below the concept of universality with experiments performed on real materials which belong to the Ising model universality class, either in 2D or in 3D. There exist plenty of successful experiments and we will essentially describe two of them which we consider particularly outstanding.

1.5.2 Ising model behaviour in rare-earth materials

The conditions to be fulfilled by real materials in order to be quantitatively described by the Ising model are compelling. Magnetic materials offer obvious candidates which are known to exhibit a rich variety of phase transitions, with transition temperatures ranging from very low to very high, as a result of the wide range of variations of the magnetic interactions. We first have to understand the behaviour of single magnetic ions in a crystalline environment and two preliminary conditions are required. First the ground state has to be a doublet separated energetically from the excited states by a gap which is much larger than $k_B T_c$, where T_c is the transition temperature. Second, in order to keep the ground state degeneracy, the operators involved in the spin-spin interactions should all have vanishing matrix elements between the two Ising states. For example, the exchange interaction $-J\mathbf{s}_i \cdot \mathbf{s}_j$ transforms like a vector and as such, obeys the selection rules $\Delta m = 0, \pm 1$ where m is the angular momentum projection. Both conditions are often satisfied in compounds based on rare-earth and one of the first materials which has been studied in this context is the dysprosium ethyl sulfate, $Dy(C_2H_5SO_4)_3{\cdot}9H_2O$ [81–83] with a doublet ground state in the angular momentum state $|15/2, \pm 9/2\rangle$ with weak superposition of $|15/2, \mp 3/2\rangle$ and $|15/2, \mp 15/2\rangle$. Local anisotropy axes are furthermore parallel to the hexagonal crystal axis. The system is thus well described by a microscopic Hamiltonian

$$H = \frac{1}{2} \sum_{i,j} K_{ij} \sigma_{zi} \sigma_{zj}$$

, where the sum extends over the pairs of spins i and j, presumably decaying with the distance among them. There is no quantitative theory which would allow for a direct calculation of the interaction strength K_{ij}, and these parameters have to be obtained by the comparison between experimental results and theoretical predictions of thermodynamic quantities in regions of the parameters where such theories are asymptotically exact, i.e. when T is either far above T_c or far below T_c. This is for example the case when the susceptibility is expanded in the moments of the spin-spin interaction. Earlier studies then compared experimental results with approximate theories: molecular field models, cluster models, series expansions, etc, which, having no adjustable parameters, were quite conclusive except maybe in the very neighborhood of the transition.

Long power series expansions started to become available in the 1960's and allowed for quantitative agreement in a wider range of parameters, leading to the experimental determination of the 3D Ising model critical exponents. The difficulty with fits to critical point predictions is that the asymptotic range is generally very narrow and limited by rounding effects which broaden the singularities. These effects are described by corrections to scaling, e.g.

$$C(T, H = 0) = A_\pm |t|^{-\alpha_\pm} (1 + D_\pm |t|^{\omega_\pm}) + B_\pm$$

with $t = (T - T_c)/T_c$, which require adjusting the experimental data to non-linear fitting with in our example not less than 11 parameters (if we do not impose theoretical requirements like $\alpha_+ = \alpha_-$, etc)!

Similar studies then extended over half a century (extensive early references can be found in the reviews [84–86]). Many experimental problems were challenging. For example the presence of dipole-dipole interactions lead to demagnetizing factors which result in a sample-shape dependence, or to long-range interactions which modify the upper critical dimension above which mean field exponents become exact (the system under consideration is no longer in the Ising universality class). Other phenomena which can be encountered experimentally are field induced phase transitions (experimentally a non-zero magnetic field is applied to promote one spin orientation and single domain samples), frustration (due to competing local anisotropy axes), disorder (associated to the presence of vacancies or defects). In spite of all these shortcomings which lead to rather large differences between the model Hamiltonian and the experimental situation, the agreement between theory and experiment is relatively unaffected, and this is a result of the extreme robustness of universal quantities in the theory of critical phenomena in general, and of the Ising model in particular which is spectacularly exemplified below.

1.5.3 A beautiful test of 2D Ising model universality

Two-dimensional phase transitions may occur in very different physical systems. The study of two-dimensional matter was initiated in the XIXth century with molecular films of non-soluble molecules on liquid surfaces, and later with physisorbed atoms on solid surfaces. During decades however, investigators were not able to observe experimentally the characteristics of two-dimensional transitions, mainly because of the heterogeneity of the adsorbents with multiple exposed crystal surfaces, defects, or chemisorbed contaminants. In the 70's, lamellar solids, like graphite appeared well suited to such studies and nowadays, 2D adsorbed matter is the subject of numerous works [87]. Reconstruction at crystal surfaces also offer natural candidates to test experimentally two-dimensional universality classes, e.g. the continuous structural transition of Au(110) investigated through LEED experiments, which appears to follow Onsager solution of the two-dimensional Ising model [88].

But we will report here on wonderful experiments performed by C.H. Back, Ch. Würsch, A. Vateriaus, U. Ramsperger, U. Maier and D. Pescia [89], where confirmation of a scaling behaviour belonging to the 2D Ising model universality class was shown to be satisfied over 18 and 32 orders of magnitude in terms of the properly scaled variables!

The experimental system consists in an atomic layer of ferromagnetic iron deposited on a non-magnetic substrate made of single-crystal W(110) surface, and provides a typical two-dimensional system. The epitaxial growth guarantees crystalline order and avoids disorder (as much as possible). The fact that the system obeys Ising symmetry (i.e. typically ± 1 magnetization in normalized units) was confirmed by the square shape of the hysteresis loop, measured by magneto-optic Kerr effect. It also confirms the absence of domains in the sample. In the vicinity of the critical point $t = (T/T_c - 1) = 0$, $H = 0$, the temperature dependence of the spontaneous magnetization $M(t, H = 0)$, the critical isotherm $M(t = 0, H)$ and the zero-field susceptibilty $\chi(t, H = 0)$ were measured, leading to the corresponding critical exponents through $M(t, H = 0) \sim (-t)^\beta$, $M(t = 0, H) \sim |H|^{1/\delta}$ and $\chi(t, H = 0) \sim |t|^{-\gamma}$, $\beta = 0.13 \pm 0.02$, $\delta = 14 \pm 5$ and $\gamma = 1.74 \pm 0.05$. This is a typical illustration of the possible experimental accuracy which can be achieved, where 2D Ising expected exponents are $\beta = 1/8$, $\delta = 15$ and $\gamma = 7/4$. Even more impressive is the determination of the susceptibility amplitudes $\Gamma_\pm$ (via expressions $\chi(t, H = 0) \sim \Gamma_\pm |t|^{-\gamma}$) and their ratio $\Gamma_+/\Gamma_- = 40 \pm 10$, where theory says that $\Gamma_+/\Gamma_- = 37.7$.

Testing universality can be pushed further. The scaling hypothesis [57, 90–93] states that thermodynamic functions can be written in the vicinity of the critical point as generalized homogenous functions, e.g.

$$M(t, H) = b^{-\beta/\nu} \tilde{m}(b^{1/\nu} t, b^{\beta\delta/\nu} H)$$

where b is an arbitrary scaling factor. Fixing $b = 1/M^{\nu/\beta}$ above yields $\tilde{m}(t/M^{1/\beta}, H/M^\delta) = 1$, which then allows to write the parametric equation of state in terms of rescaled variables,

$$H/M^\delta = f(t/M^{1/\beta}).$$

The experiment of Back *et al.*, see Fig.1.8, reported this rescaled equation of state fitted to theoretical results [94] over 18 orders of magnitude in the variable $t/M^{1/\beta}$ and almost 32 orders of magnitude in H/M^δ!

This might be considered as a real achievement and an incredible success of the theoretical prediction, which even raises the opposite question: how is it that all experimental imperfections, inhomogeneities which break translational symmetry, non-localized local magnetic moments (Fe is a broad-band metallic ferromagnet when Ising Hamiltonian is written in terms of localized ones), non-perfect uniaxial local symmetry and possibly other sources of discrepancy do not destroy the 2D Ising model universality

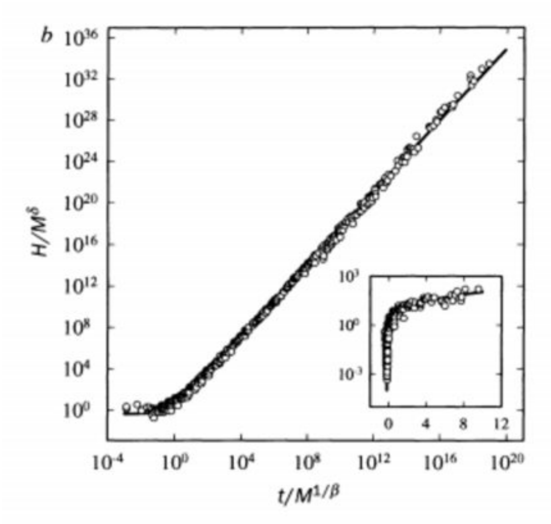


Figure 1.8: Universal plot from [89].

class. Although there cannot be a simple answer to such questions, some of these effects are understood within the frame of universality. Disorder for example can be shown not to change (up to logarithmic corrections) the 2D Ising model universality class [95], and the experimental evidences reported in [89] support a scenario where all imperfections mentioned eventually prove to be non-essential (we say irrelevant in the renormalization group language [96, 97]).

1.6 Simple Model of Complex Systems

But let your communication be, Yea, yea; Nay, nay:
for whatsoever is more than these cometh of evil.
(Matthew 5:37)

It is stated sometimes, that although Ising model is not realistic, its success to a large extent is caused by the fact that it allows analytic treatment (see e.g. [2] and discussions therein). In this sense, it belongs to the "narrow class of models which are balanced (precariously!) between realism and solubility" [99]. This is certainly true, but our belief is that there is another – even more important – reason for the tremendous success of the Ising model. The simplicity of the model not only enables its analytic treatment, it also singles out an essential feature: binarity, i.e. representation of something as a pair of binary oppositions (cf. Umklapp-magnets in Section 1.3). It is this feature that enables a much wider set of applications of the model. Moreover, and as we will see below, this feature has aided the exportation of very notions of physics to other fields, giving rise to science of complex systems [100]. Indeed, the model tailored by the "usual procedure of separating the phenomena till one deals with simple elementary facts" (as Ising himself noted on a different occasion in his other paper [101]) singles out the notion of binarity and enables analytic treatment. This is corroborated by the remark in [102] that such an approach in condensed matter theory "consists of building a model of the system which is simple enough to handle, but rich enough to capture the relevant properties. These simplifications give rise, among others, to classical spin models. A paradigmatic example is the Ising model [26], originally devised to study magnetism." In turn, this enables one to apply the model in almost all fields where binarity plays a core role. Sometimes this role is not obvious from the very beginning and this is the skill of researchers to find a subtle connection between the cause and the consequence.

In the epigraph to this chapter we have chosen probably one of the oldest written references suggesting binary opposition [98]. Indeed, binary variables (plus or minus one, up or down, filled or empty, active or passive) are ubiquitously used in describing various processes occurring in nature and in human society. Quantitative descriptions of such processes, on the one hand, allow us to apply methods developed in one field to another, and on the other hand, it triggers a search for similarities between very different phenomena and ordering them into different classes. It also fosters transfer of knowledge from one branch to another. As we will see from several examples given below, applications of the Ising model for quantitative descriptions and understandings of different phenomena of physical, chemical, biological, or social nature, as well as its application in humanities is based on the fundamental fact that actually the very essence of these phenomena is hidden in their *statistical nature*. In the so-called agent based modeling that lies in the core of such descriptions, one considers a whole system as a set of agents (individuals in social systems; spins in magnets) that are capable of autonomous behaviour. Usually, an agent has a well-defined internal state and interacts with other agents. Allowing such agents to be in one of two possible states leads to the Ising model description.

Currently, there are numerous applications of the Ising model to explain chemical or biological phenomena. An amount of studies and their success lead to the situation when e.g. such typically biological phenomena as dynamics of pattern formation in neural networks [103, 104] or protein folding [105] became conventional and well-established fields of physics. The Ising model is being successfully used to explain properties of living organisms on all scales. Just to give some examples, on a molecular and cellular scale, it is adapted to the analysis of complex genetic models with several genetic effects and with interaction, or epistasis, between the genes (see [106] and references therein) and serves as a framework for phase transitions in multicellular environments [107]. On the other extreme, at the scale of ecosystems, it explains how a critical transition can emerge directly from the dynamics of ecological populations [108]. In ecology, long-range synchronization of oscillations in spatial populations may elevate extinction risk. Therefore, such phenomena may signal an impending catastrophe.

The above examples of Ising-model applications, although outside physics still concern systems that traditionally belong to natural sciences. As a next step, let us illustrate how it is applied in social sciences, where an important topic is to understand the social dynamics of a community, e.g. its transition from an initial disordered state to a configuration that displays at least partial order [5,6,109–114], see Fig.1.9 as an example. Inspired by an idea to exploit binarity in social choice, T.C. Schelling has suggested a model to describe racial segregation in cities [115]. There, in particular, special attention is paid to analysis of the relation between individual and collective states: "But evidently analysis of 'tipping' phenomena wherever it occurs – in neighborhoods, jobs, restaurants, universities or voting blocs – and whether it involves blacks and whites, men and women, French-speaking and English-speaking, officers and enlisted men, young and old, faculty and students, or any other dichotomy, requires explicit attention to the dynamic relationship between individual behavior and collective results" [115]. Although the phenomenon of interest in the above example is rather the phase separation and not an onset of a phase, an analogy with Ising model is obvious but it has not been recognized in the original paper. Only later the similarities between phase separation into domains in the

Figure 1.9: People, similar to magnets, may experience symmetry breaking. At the beginning all of them look in different directions (low order, high symmetry). Then somebody shouts from the other side and all start staring in the same direction (high order, low symmetry). And this is in spite the fact that only one of them has heard the call: curiosity serves as an interaction between the people. (Illustration and caption is taken from the mass media article about phase transitions: *Der Standard*, 02.04.2002, Austria).

Ising model at $T = 0$ and residential segregation in the Schelling model were recognized and the equivalent of the temperature T was introduced into the Schelling model [116].

When the authors of [117] identified binarity of states of social agents to describe the phenomenon of strikes, the analogy with the Ising model was apparent. As Serge Galam recalls in his book: "we developed the idea of using an Ising ferromagnetic system to describe the collective state of an assembly of agents, each being in either one of two distinct individual states, that of working or striking. This produces two collective ordered states: a working state versus a striking state. The ferromagnetic coupling between agents was motivated by the social fact that people have the tendency to reproduce the leading choice of their neighbors, in particular in conflicting situations. We thus implemented the first application of the Ising model to describe the global state of a firm..." [6]. Currently, analysis of opinion dynamics widely exploits agent based modeling with agents being in discrete binary states. The most widely used in this context models are the voter model [118, 119], majority rule models [120], the Sznajd model [121, 122] and other models based on a social impact theory [123] and its extensions [125, 126]. A detailed review of these and other models may be found in [5].

Concepts of phase transition theory, and, more specifically, the Ising model are being also actively used in the field of economics and financial markets, explaining, in particular, statistical properties that are common to a wide range of financial assets [127,128]. In modeling financial markets, the agents are identified with spin variables which can take specific values depending on agents decisions: the $+1$ spin as a buyer and -1 as a seller [129–135]. Considering also the case when an agent may stay inactive ($S = 0$) leads to further generalization [136–140]. Such approaches allow to study inherent features observed in collective behaviour of financial markets: herding, bubbles or crashes and to reproduce main statistical observations of the real-world markets such as fat-tailed distribution of returns or volatility clustering.

In Social Sciences, we can also mention elegant applications of the Ising model to Natural Language Processing, via the ability of magnetic models of statistical physics to extract the essential information contained in texts. Documents are represented as sets of interacting magnetic units (words), and a textual energy is defined as an indicator of information relevance which allows automatic abstract production, information retrieval, document classification and thematic segmentation. The compression of a sentence appears as the ground state of the chain of terms and variants are produced by thermal fluctuations [142].

In almost every example given above the Ising model was used to shed light on the behaviour of systems composed of many interacting agents, which display collective behavior that does not follow trivially from the behaviors of the individual parts. Such systems are currently known as *complex systems* [141]. Their inherent features incorporate self-organization, emergence of new functionalities, extreme sensitiveness to small variations in the initial conditions, power laws governing their statistics (fat-tail behaviour) [143–145]. Their systematic study gave rise to complex system science: the field of knowledge that is actively developed and shaped nowadays.[19] Usually, quantitative description of such systems is achieved by considering agents located on the nodes of a graph called complex net-

[19]It is worth mentioning here words of Wolfgang Pauli from his letter to Herman Levin Goldschmidt (Feb. 19, 1949) [146]: "It seems to me as a philosophical layman that the task of philosophy consists in generalizing the emerging insights of current physics – that is, all its essential elements – in such a way that it can be applied to fields more general than physics. Such an achievement would, in turn, enrich the individual disciplines and prepare future developments."

work [147–149]. Linking between graph nodes corresponds to the interaction between the agents under analysis. For social systems it corresponds to social interactions, for ecological systems it may reflect predator-pray relation between species, for transportation systems it correspond to transportation links, etc. In this sense, treating the Ising model on complex networks has various applications in complex system science. Of special importance are the so-called small-world [150] and scale-free [151] networks. The first are characterized by small characteristic sizes (usually, their typical size ℓ logarithmically grows with number of nodes N: $\ell \sim \ln N$). The second are characterized by the power-law decay of the node degree distribution $P(k) \sim k^{-\lambda}$. Many important natural and man-made networks are small world and scale-free. Examples are given by the internet, world-wide web, some transportation, biological, social networks [147–149]. Properties of the Ising model on such types of networks essentially differ from its properties on d-dimensional lattice. The scale-free networks with slowly decaying node-degree distribution (fat-tailed distributions with small λ) are highly inhomogeneous. It appears, that the decay exponent λ plays a role in some sense similar to that of dimensionality d: Ising model on a scale-free network with $\lambda \leq 3$ is ordered for any finite temperature T whereas it has a finite T second order phase transition for $\lambda > 3$. Moreover, the basic concept of universality is revised: the critical exponents attain λ-dependency in the region $3 < \lambda < 5$ [152–154] and the logarithmic corrections to scaling appear at $\lambda = 5$ [72, 155].

These and many more unusual features of the Ising model on complex networks are currently well established by different approaches (see [156] for a review) and recently revisited by Lee-Yang-Fisher zeros analysis [157, 158].

There are at least two lessons one can learn from the short account given in this chapter. Indeed, exploiting Ising archetype in agent-based modeling of various complex systems of chemical, biological, social, economical origin gives a possibility to quantify them and to understand some of the mechanisms of their behaviour. In this sense the model enables one to single out universal common features of different systems. However, more than this: it would be too trivial to reduce behaviour of these systems just to a single archetype no matter how powerful and general the archetype is. Along with universality in behaviour of many-agent interacting systems, they are characterized by system-specific diversity. Subtle changes in their parameters may lead to crucial changes in their global behaviour: this is another inherent feature of complex systems. In their description, the Ising model plays a role of the main 'course', however these are the spices which make the whole dish tasty.

We have already mentioned Ising's paper [101] at the beginning of this chapter. There, discussing Goethe's approach to analyze nature he says the following: "...his approach to science was that of an artist who thought he could conceive the secrets of nature in all their complexity... He was convinced that translation into language of mathematics was distortion of reality...". Contrary to Goethe's believe, nowadays Ising-like models completed by ideas from complex system science come into play as simple models on the way to "conceive the secrets of nature in all their complexity".

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2 Abstracts of the lectures

In this chapter we present the abstracts of the talks given at the "Ising lectures" starting from 1997 till present time. The abstracts are given chronologically. Some speakers have provided a list of references, useful for understanding the topic. This list, if given, is presented after the abstract.

Abstracts 1997

May 12

- *ERNST ISING: PHYSICIST AND TEACHER (Sigismund Kobe)*
- *ISING MODEL – CALCULATIONS WITHOUT APPROXIMATIONS (Oleg Derzhko)*
- *LATTICE GAS MODEL FOR THE PROTON SUBSYSTEM IN SUPER-IONIC CRYSTALS (Ihor Stasyuk, Natalija Pavlenko)*
- *INVESTIGATION OF DISORDERED ISING-LIKE SYSTEMS USING CLUSTERING METHOD (Petro Sokolovskii, Roman Levitskii)*
- *DE GENNES LIMIT: FROM SPIN STATISTICS TO THE MACROMOLECULES (Yurij Holovatch)*
- *ISING MODEL AND NEURAL NETWORKS (Volodymyr Tkachuk)*

ERNST ISING: PHYSICIST AND TEACHER

Sigismund Kobe
Technical University Dresden, Germany

The Ising model is one of the standard models in statistical physics. Since 1969 more than 12000 publications appeared using this model. In 1996 Ernst Ising celebrated his 96th birthday. Some biographical notes and milestones of the development of the Ising model are given in this lecture.

ISING MODEL – CALCULATIONS WITHOUT APPROXIMATIONS

Oleg Derzhko

Institute for Condensed Matter Physics NAS of Ukraine, Lviv, Ukraine

In this lecture, I wish to emphasize that the model examined by Ernst Ising about 75 years ago, though highly nontrivial, admits exact (i.e., without making any approximation) calculations of many statistical-mechanical quantities. Such exact calculations are very valuable providing an intuition that is worthwhile for more realistic (and more complicated) models. Furthermore, exact results also permit to test approximate approaches and often give hints for new approximations.

More specifically, in my lecture I deal with the initial (zero-field) static susceptibilities of a nonuniform spin-$1/2$ Ising chain [1]. If the chain is uniform (i.e., all exchange couplings are the same), the longitudinal susceptibility can be obtained with the help of the transfer-matrix method, whereas the transverse susceptibility can be obtained by the Jordan-Wigner fermionization method. Interestingly, the initial static susceptibilities can be calculated rigorously for the case of regularly varying exchange couplings or the case of random exchange couplings too [1]. These findings demonstrate how regular alternation or random disorder may lead either to quantitative or to qualitative changes in the temperature dependence of the initial static susceptibilities.

From the pedagogical side, I discuss a number of concepts such as transfer matrix, Jordan-Wigner fermionization, Bogolyubov transformation etc. which play important role in finding the solution of a square-lattice Ising model [2]. We do not know how to solve a three-dimensional Ising model. The search for other exact calculations is encouraged.

[1] O. Derzhko, O. Zaburannyi. Static susceptibilities of nonuniform and random Ising chains. Jour. Phys. Stud. **2** (1998) 128.

[2] T.D. Schultz, D.C. Mattis, E.H. Lieb. Two-dimensional Ising model as a soluble problem of many fermions. Rev. Mod. Phys. **36** (1964) 856.

LATTICE GAS MODEL FOR THE PROTON SUBSYSTEM IN SUPER-IONIC CRYSTALS

Ihor Stasyuk, Natalija Pavlenko
Institute for Condensed Matter Physics NAS of Ukraine, Lviv, Ukraine

It is common knowledge that the application of the Ising model goes far beyond the phenomena of magnetism. In particular, using the simple transformation from pseudospin variables to the occupation numbers this model can be resresented in the form of the so called lattice gas model. In this case the chemical potential which is entered here for the description of systems with a varying number of particles plays the role of external field.

Comparing the results of MFA and CEM, it can be noted that the main features of thermodynamical properties of considered system may be described within the mean-field approximation allowing for the longrange proton interactions. In this case the obtained results agree with the experimental data, but the better quantitative agreement could be obtained by taking into account the molecular field fluctuations. The short-range proton correlations considered in CEM don't change qualitatively the topology of phase diagram but lead to a change in transition temperature and favour the transformation of the II-III transition order from second to first one.

Besides the description of the thermodynamics the proton ordering model can be basic for the further investigations of proton transport processes in this system.

INVESTIGATION OF DISORDERED ISING-LIKE SYSTEMS USING CLUSTERING METHOD

Petro Sokolovskii, Roman Levitskii

Institute for Condensed Matter Physics NAS of Ukraine, Lviv, Ukraine

Some magnetic or ferroelectric compounds with impurities or with random substitution of atoms lead in some approximation to Ising model with random interactions. The theory of disordered Ising model with the nearest neighbour interaction is presented in this lecture. In particular, order parameter and spin correlation functions of the model are calculated and investigated within a two-particle cluster approximation for both quenched and annealed cases. The approach yields the exact results for the one-dimensional system. The long-range interaction is taken into account in the mean field approximation.

DE GENNES LIMIT: FROM SPIN STATISTICS TO THE MACROMOLECULES

Yurij Holovatch

Institute for Condensed Matter Physics NAS of Ukraine, Lviv, Ukraine

In this lecture I will show how statistics of the (generalized) Ising model is connected to statistics of self-avoiding walks (SAWs). This analogy was first realized in early 70-ies mainly to works of P.G. de Gennes [1] and now the limit $n \to 0$ of the n-vector model is often called the polymer of SAW limit. It is well known that the conformation properties of long flexible polymer chains in a good solvent are governed by power laws (scaling laws). The seek for the form of these laws lead not only to the creation of modern theoretical physics and chemistry of polymers, but also enabled to discover a deep connection between the physics of critical phenomena and the physics of macromolecules. These and related questions are the subject of my lecture [2].

[1] P.G. de Gennes. *Scaling Concepts in Polymer Physics.* Cornell University Press, Ithaca, 1979.

[2] Yu. Holovatch. *Lectures on Critical Phenomena: Polymers, Diluted Systems, Magnets.* Linz, 1996.

ISING MODEL AND NEURAL NETWORKS

Volodymyr Tkachuk
Ivan Franko Lviv State University, Ukraine

The talk is the short review of the relation of Ising model with neutral networks. The first model which describes the behavior of neural networks was introduced by J.J. Hopfield. This model was proposed for explanation of the mechanism of associative memory. Formally the Hopfield model is identical to the Ising model of spin glass. Therefore, the methods of statistical mechanics can be used for studies of the properties of the Hopfield model.

Abstracts 1998

April 29

- *HEAT CAPACITY OF LIQUID ^4He NEAR THE λ PHASE TRANSITION (Ivan Vakarchuk)*
- *STATISTICAL FIELD THEORY OF HIERARCHICAL AVALANCHE ENSEMBLE (Alexander Olemskoi)*
- *CRITICAL PROCESSES IN BINARY MIXTURES (Oksana Patsahan)*
- *CRITICAL PHENOMENA ON SURFACES (Mykola Shpot)*
- *OPTICAL-AND-REFRACTIVE INVESTIGATIONS OF CRITICAL INDICES OF PHASE TRANSITIONS IN CRYSTALS (Bohdan Andriyevsky)*

HEAT CAPACITY OF LIQUID ^{4}He NEAR THE λ PHASE TRANSITION

Ivan Vakarchuk
Ivan Franko Lviv State University, Ukraine

The properties of liquid helium are known to have brought about ample literature. Yet due to its unique characteristics, this quantum fluid keeps attracting the attention of specialists in the field of theoretical and experimental physics alike. The first principle microscopic description of thermodynamic and structure functions as well as the phenomenon of Bose-Einstein condensation at a considerable distance from the absolute zero, in particular in the vicinity of the λ-transition, is still an open problem which has not been completely solved. The latter statement can be supported by a fairly instructive study of heat capacity in the vicinity of the λ-transition. The λ-like form of the heat capacity in the vicinity of the phase transition of liquid helium into the superfluid state has been taken as a logarithmic divergence with the critical exponent $\alpha \to 0$. This view has found its way into both text-books and monographs. Precise experiments helped to determine that in fact there is no divergence in the heat capacity even though the exponent α is indeed a small but negative number, $\alpha = -0.01056$[1]. The studies of the λ-transition based on the renormalization group method provide us with a possibility to carry out the correct calculus for the so-called universal characteristics solely, i. e., the critical characteristics of the thermodynamic functions and the relations of the amplitudes of their leading asymptotics at the temperature on either side tending to the phase transition point. Even though the thermodynamic potential functional from the two-component order parameter for liquid He4 was calculated precisely owing to the coherent states depiction, yet the subsequent simplifications necessary for the implementation of the renormalized group approach make it impossible to describe the system's characteristics outside the closest vicinity of the phase transition point using the same method. Notwithstanding the tangible efforts researchers, the renormalization group method has not yielded the logarithmic divergence of the heat capacity (the α exponent was received as a small but still finite positive number, the power divergence having been obtained). Only in subsequent studies, which have made use of the summation procedure of the Borel perturbation theory has a divergent series established the negative value of the exponent: $\alpha = -0.01294$[2], $\alpha = -0.0150$[3], $\alpha = -0.01126$[4].

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STATISTICAL FIELD THEORY OF HIERARCHICAL AVALANCHE ENSEMBLE

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In recent years considerable study has been given to the theory of self-organized criticality (SOC) that explains avalanche dynamics in a variety of systems such as ensemble of grains of sand moving along increasingly tilted surface (sandpile model [1]), intermittency in biological evolution [2], earthquakes and propagation of forest-fires, depinning transitions in random medium and so on (see [3]). The above models had been mostly studied by making use of scalingtype arguments supplemented with extensive computer simulations [4]. By contrast, in this work we put forward the related statistical theory that deals with avalanche ensemble in the course of SOC progressing.

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CRITICAL PROCESSES IN BINARY MIXTURES

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While the phase behaviour of single component fluids is similar, binary mixtures, even of noble gases, demonstrate a vast variety of phase diagram topologies. For example, the liquid-vapour (LV) critical point (CP) of pure components might be connected with a line of critical points of the mixture (Ar-Kr mixture). It might also be the case that the separate critical lines start from the CPs of both pure components (Ne-Kr mixture). The existence of second line of CPs is evidence of the mixing-demixing phase transition (PT). Moreover, it might happen that the liquid-vapour critical line transforms into a critical line of liquid-liquid or vapour-vapour, which are not/none mixing. All theoretical approaches in the investigation of the critical behaviour of binary mixtures might be divided into these classes:

- Mean-field theories (van der Waals approaches)
- Integral equation method
- Phenomenological approach
- Hierarchical reference theory
- Collective variables method

In this lecture the first four methods are discussed.

CRITICAL PHENOMENA ON SURFACES

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An extension of the massive field theory approach of Parisi to systems with surfaces is presented. This approach provides the opportunity to study the surface critical behavior directly in space dimensions $d < 4$ without having to resort to the ϵ expansion, especially in three dimensions. The method is elaborated for the the semi-infinite $|\phi|^4$ n-vector model with a boundary term $c_0 \int_{\partial V} \phi^2$ in the action. To make the theory UV finite in bulk dimensions $d < 4$, a renormalization of the surface enhancement c_0 is required, apart of the standard mass renormalization; required normalization conditions for the renormalized theory are given. As a result, in addition to the the usual bulk 'mass' (the inverse correlation length) m, another mass parameter appears in the theory, the renormalized surface enhancement c. Thus the surface renormalization factors depend on the renormalized coupling constant u and the ratio c/m. The special and ordinary surface transitions correspond to the limits $m \to 0$ with $c/m \to 0$ and $c/m \to \infty$, respectively. The surface critical exponents of the special and ordinary transitions are given to one-loop order in $2 \leq d < 4$ and to two-loop order at $d = 3$. The associated second order series expansions are analyzed by Padé-Borel summation techniques. The resulting numerical estimates for the surface critical exponents are in good agreement with available Monte Carlo simulations. This also holds for the surface crossover exponent Φ, for which the values $\Phi(n=0) \simeq 0.52$ and $\Phi(n=1) \simeq 0.54$ are obtained, considerably lower than the previous ϵ-expansion estimates.

OPTICAL-AND-REFRACTIVE INVESTIGATIONS OF CRITICAL INDICES OF PHASE TRANSITIONS IN CRYSTALS

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Temperature dependences of the refractive indices and linear thermal expansion of TGS in the range of phase transition (PT) were already studied [1–3], but the corresponding critical indices have not been determined. The goals of the present investigation were precise measurements of temperature dependences of interferometric retardation of the sample-air type for the TGS in the range of 2nd order PT at 322 K, calculation of the temperature dependences of refractive indices and linear thermal expansion for the main crystal physics directions, and the study of these dependences using the corresponding critical indices 2β.

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Abstracts 1999
May 20

- *ISING MODELS WITH SPIN 1 (Ostap Baran)*
- *CORRECTION TO SCALING EXPONENT IN DILUTED SYSTEMS (Reinhard Folk, Yurij Holovatch, Taras Yavors'kii)*
- *CRITICAL BEHAVIOR OF THE BINARY SYMMETRIC MIXTURE (Roman Melnyk, Oksana Patsahan)*
- *SOME ASPECTS OF STATISTICAL PHYSICS OF SELF-ASSOCIATED SYSTEMS (Svyatoslav Kondrat, Myroslav Holovko)*
- *POTENTIAL INTERACTION BETWEEN He ATOMS (Andrij Rovenchak)*
- *THERMODYNAMICS OF A PSEUDOSPIN-ELECTRON MODEL (Kyrylo Tabunshchyk)*

ISING MODELS WITH SPIN 1

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Spin-1 Ising model with higher-degree spin terms (of an exchange as well as of a non-exchange origin) in the Hamiltonian is one of the most extensively studied models in condensed matter physics. That is so not only because of the fundamental theoretical interest arising from the richness of the phase diagram that is exhibited due to competition of interactions, but also because versions and extensions of this model can be applied for the description of simple and multi-component fluids [1–3], dipolar and quadrupolar orderings in magnets [3–5], crystals with ferromagnetic impurities [3], ordering in semiconducting alloys [6], etc. Ising model with S=1 has been investigated by different simulation and approximate techniques: using the mean-field approximation [1–4,7], effective field theory [8,9], two-particle cluster approximation [10,11], Bethe approximation [12], high-temperature series expansions [13], renormalization-group theory [14, 15], and Monte-Carlo simulations [12,16,17].

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CORRECTION TO SCALING EXPONENT IN DILUTED SYSTEMS

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From renormalization group (RG) theory one knows that in the asymptotic region the values of the critical exponents are universal and scaling laws between them hold. There the couplings of the model Hamiltonian describing the critical system have reached their fixed point values. In the non-asymptotic region deviations from the fixed point values are present. They die out according to a universal power law governed by the correction to scaling exponent ω. The smaller the exponent, the larger is the region where corrections to the asymptotic power laws have to be taken into account.

The implication of quenched dilution on the critical behavior is a long-standing problem attracting theoretical, experimental and numerical efforts. In the 3d-Ising model quenched disorder changes the asymptotic critical exponents compared to the pure ones [1,2]. In principle this statement should hold for arbitrary weak dilution. But in order to observe this change one should approach the critical point lose enough. The width of this region turns out to be dilution dependent.

[1] A.B. Harris. Effect of random defects on the critical behaviour of Ising models. J. Phys. C **7** (1974) 1671.

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CRITICAL BEHAVIOR OF THE BINARY SYMMETRIC MIXTURE

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Binary mixtures, in contrast to their constituent components, manifest three different types of two-phase equilibrium states: vapour-liquid, liquid-liquid, and gas-gas. Which of these states is realized and to what extent does it depend on both the external conditions and the microscopic parameters of a mixture. The study of the influence of interparticle interactions on the critical properties of a binary mixture is an interesting and relevant problem. During the last decade, this problem has been intensively studied by integral-equation methods. However, this approach, although it reproduces different types of phase diagrams under variation of the microscopic parameters, it only gives a qualitative picture of the phenomenon under consideration.

In the present lecture, we talk about a microscopic approach to studying the vapour-liquid critical point of a symmetric binary mixture. This approach is based on the method of collective variables, which was effective in describing the second-order phase transition of the 3D Ising model and the vapour-liquid critical point of a one-component fluid. Both universal and nonuniversal quantities are obtained on the basis of this approach. The phase diagram of the symmetric mixture is examined within the framework of the Gaussian approximation.

SOME ASPECTS OF STATISTICAL PHYSICS OF SELF-ASSOCIATED SYSTEMS

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The focus of this lecture is on self-assembling systems, which we discuss from a perspective of the statistical theory. Self-assembling systems emerge e.g. in water-oil mixtures with the addition of surfactants, i.e. amphiphilic molecules consisting of polar (water-like) heads and oil-like tails. Surfactants prefer to position themselves between water and oil molecules, reducing the oil-water surface tension, which leads to a spontaneous formation of various structures, such as micelles, lamellar phases, cubic phases, ordinary microemulsions, etc. Three approach to these systems will be discussed. In microscopic models, a Hamiltonian is formulated on a lattice and the thermodynamic properties are described using the standard techniques of the statistical physics. In continuous approaches, a mesoscopic Landau-Ginsburg-like functional is constructed as a functional of some 'order parameters' (e.g. densities); this construction is typically based on symmetry considerations. Finally, in the effective membrane theories, the microscopic details of the system are ignored and the system is described in terms of the membrane curvatures, where the membrane is the water-oil interface. Advantages and disadvantages of these approaches are also discussed.

POTENTIAL INTERACTION BETWEEN He ATOMS

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In this work, the pair potential of interatomic interaction between helium atoms is established from first principles without using fitting parameters. Previously obtained from Schrödinger's equation, is the connection between the potential Fourier image and the coefficient functions. An expansion of the many-boson system ground state wave function logarithm by the "degrees" of density fluctuations is used. RPA as the zero approximation and the approximation of "two sums over wave vector" as the first one are considered. The results of this work have lead to the value of the first sound velocity bieng 231 m/s, while the experiment gives 237 m/s. The fit of our potential with the Aziz model potential is quite good: the well depths are -10.58 K and -11.04 K respectively, with the minimum positions being 3.34 Å and 2.99 Å.

THERMODYNAMICS OF A PSEUDOSPIN-ELECTRON MODEL

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Pseudospin-electron (PE) model is one of theoretical models which considers the interaction of electrons with local lattice vibrations where an anharmonic variables are represented by pseudospins. The theoretical investigation of the PE model is an enduring subject of interest at the quantum statistics department. The model is used to describe the strongly correlated electrons of CuO_2 sheets coupled with the vibrational states of apex oxygen ions O_{IV} (which move in the double-well potential) in YBaCuO type high-T super conductors (HTSC) [1]. Recently a similar model has been applied for investigation of the proton-electron interaction in molecular and crystalline systems with hydrogen bonds [2].

The purpose is to present the thermodynamics of the PE model in the case of the different type interactions between pseudospins. First, we provide an overview of the results of works which deal with the theoretical investigation of the PE model with the inclusion of the direct pseudospin-pseudospin interaction (but without the electron transfer ($t_{ij} = 0$)). Second, we present the results of the investigation of the model in the case of the absence of the direct pseudospin-pseudospin interaction and Hubbard correlation ($J_{ij} = 0$, $U = 0$), when interaction between pseudospins via conducting electron is done.

[1] K.A. Muller. On the oxygen isotope effect and apex anharmonicity in high-T_c cuprates. Z. Phys. B **80** (1990) 193.

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Abstracts 2000
May 24

- *PHASE TRANSITIONS IN THE EARLY UNIVERSE (Bohdan Hnatyk)*
- *IMPACT OF THE EXTERNAL FIELD AND HYDROSTATIC PRESSURE ONTO PHASE TRANSITIONS IN SEGNETOELECTRICS (Alla Moina)*
- *MOLECULAR DYNAMICS OF MAGNETIC LIQUIDS (Igor Omelyan, Ihor Mryglod, Reinhard Folk)*

PHASE TRANSITIONS IN THE EARLY UNIVERSE

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According to the Big Bang theory, the early Universe was an extremely hot and dense environment. With time, the universe went through an inflationary period, causing a decrease in temperature. In statistical physics, systems that come from a high temperature state tending toward a low temperature state experience a symmetry breaking phase transition. This concept might be applied to model the evolution of the Universe. It is well known, that electromagnetic interaction described by U(1) symmetry and weak interaction SU(2) can be generalized into one electroweak interaction SU(2)*U(1) at high energies (temperatures). Division of these two interactions occurred earlier in the Universe, causing symmetry to break.

IMPACT OF THE EXTERNAL FIELD AND HYDROSTATIC PRESSURE ONTO PHASE TRANSITIONS IN SEGNETOELECTRICS

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A microscopic model of the influence of the conjugate to the order parameter external fields-electric field E_3 and shear stress σ_6-on deuterated KD_2PO_4-type ferroelectrics is presented. The major mechanisms for this influence are the splitting of the Slater-Takagi energies of the short-range correlations and the effective field created by piezoelectric coupling with shear strain σ_6. The T_c vs σ_6 phase diagram of KD_2PO_4 is constructed, and the stress dependences of the dielectric, piezoelectric, and elastic characteristics associated with strain σ_6 are discussed.

MOLECULAR DYNAMICS OF MAGNETIC LIQUIDS

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The computer experiment remains an important tool for the prediction and theoretical understanding of various phenomena in magnetic materials. The methods of Monte-Carlo (MC) and molecular dynamics (MD) were intensively exploited over the years for the investigation of phase diagrams, critical phenomena, scaling, and the dynamic behavior of lattice systems such as the Ising, the XY, and the Heisenberg model [1–3].

The necessity to extend these studies to disordered models of magnetic liquids was motivated by a great amount of additional physical properties arising when both spin (orientational) and liquid (translational) degrees of freedom are taken into account [4–9]. Computer experiments for such systems have been restricted to MC simulations [5,7] in which only static quantities could be calculated. Dynamic phenomena, in particular, spin and density relaxations, and the effects connected with the mutual influence of magnetic and liquid subsystems an be investigated in MD simulations.

Until now, there have been no attempts to simulate spin liquids within the MD approach. This an be explained by the absence of an MD algorithm for handling the corresponding equations of motion. The traditional numerical methods [10] for solving differential equations are unsuitable be cause they become highly unstable on time s ales used in MD simulations. As has been well established for pure liquid systems [11,12], even standard predictor-corrector schemes are not efficient because of poor total energy conservation.

The properties of an acceptable algorithm for long-time observations of a many-body system should be: stability, accuracy, speed and ease of implementation. There exists only a small group of integrators satisfying these criteria. An important one is the velocity Verlet (VV) algorithm [13,14] which allows a high accuracy with minimal costs in terms of timeconsuming for e evaluations. However, the VV and other similar schemes [11,15] were designed to simulate pure liquid dynamics. In the case of magnetic liquids the situation is more complicated since the translational positions and momenta are coupled with spin orientations in a characteristic way and, hence, all these dynamical variables must be considered simultaneously. This requires substantial revision of the liquid dynamic algorithms.

Recently, new algorithms have been devised for spin dynamics simulations of lattice systems [16]. They are based (like the VV integrator) on the Suzuki-Trotter (ST) decomposition method and appear to be much more efficient than predictor-corrector schemes. These algorithms are applicable to spin systems if the de composition on two (or several) noninteracting sublattices is possible. However, they cannot be used for models with arbitrary spatial spin distributions and, therefore, not for spin liquids.

In the present study we develop the idea of using ST-like decompositions for spin liquid dynamics and derive the desired MD algorithm. This allows quantitative measurement of dynamical structure fa tors of a Heisenberg ferrofluid. The main result obtained (reflecting the influence of the liquid sybsystem on spin dynamics) is the identification of a new propagative sound-like mode in the spectrum of collective longitudinal spin excitations.

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Abstracts 2001
April 19

- *THE CRASH OF OCTOBER 1987 SEEN AS A PHASE TRANSITION: AMPLITUDE AND UNIVERSALITY (Orest Gera)*
- *EXOTIC STATISTICAL PHYSICS: APPLICATION TO BIOLOGY, MEDICINE, AND ECONOMICS (Ievgeniia Stashkova)*
- *PHASE TRANSITION, UNIVERSALITY AND SUPER UNIVERSALITY IN MORTALITY EVOLUTION (Olena Butrij)*
- *SCALING RELATIONS FOR DIVERSITY OF LANGUAGES (Oleg Fareniuk)*
- *CRITICAL EXPONENTS OF NiFeV ALLOYS FOR THE FERROMAGNETIC-PARAMETRIC PHASE TRANSITION (Uliana Buchko)*

THE CRASH OF OCTOBER 1987 SEEN AS A PHASE TRANSITION: AMPLITUDE AND UNIVERSALITY

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"The evolution of several financial indices before the crash of October 1987 had been analysed. The amplitude of the crash varies from one index to another. However, assuming that the crash is similar to a phase transition and particularly to a specific heat jump, was found that the crash amplitude can be well estimated by assuming a simple background which differs from market to market. It was shown that the divergence near the crash event is logarithmic and extends between 2 weeks and 4 years before the October 1987 crash on both S&P500 and Dow Jones indices. The behavior is like that found for the $d = 2$ Ising model specific heat. The latter result is in contrast to previous works which have considered a power law behavior of the index near the crash. Finally, it had confirmed the presence of log-periodic oscillations and had discussed briefly their origin." [1].

This talk was a review of:

[1] N. Vandewalle, Ph. Boveroux, A. Minguet and M. Ausloos. The crash of October 1987 seen as a phase transition: amplitude and universality. Physica A **255** (1998) 201–210.

EXOTIC STATISTICAL PHYSICS: APPLICATION TO BIOLOGY, MEDICINE, AND ECONOMICS

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"This lecture is designed to offer a brief and somewhat parochial overview of some "exotic" statistical physics puzzles of possible interest to biophysicists, medical physicists, and econophysicists. These include the statistical properties of DNA sequences, heartbeat intervals, brain plaques in Alzheimer brains, and fluctuations in economics. These problems have the common feature that the guiding principles of scale invariance and universality appear to be relevant." [1].

This talk was a review of:

[1] H.E. Stanley. Exotic statistical physics: applications to biology, medicine, and economics. Physica A **285** (2000) 1–17.

PHASE TRANSITION, UNIVERSALITY AND SUPER UNIVERSALITY IN MORTALITY EVOLUTION

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"Mortality is the most universal and the best quantified phenomenon in biology. Human probability to survive to given age is well documented in developed countries for the last two centuries. The implications of the presented results for evolution and genetics of mortality are discussed." [1].

This talk was a review of:

[1] M.Ya. Azbel'. Phase transitions, universality and superuniversality in mortality evolution. Physica A **269** (1999) 564–569.

SCALING RELATIONS FOR DIVERSITY OF LANGUAGES

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“The distribution of living languages is investigated and scaling relations are found for the diversity of languages as a function of the country area and population. These results are compared with data from Ecology and from computer simulations of fragmentation dynamics where similar scalings appear. The language size distribution is also studied and shown to display two scaling regions: (i) one for the largest (in population) languages and (ii) another one for intermediate-size languages. It is then argued that these two classes of languages may have distinct growth dynamics, being distributed on the sets of different fractal dimensions.” [1].

This talk was a review of:

[1] M. Gomes, G.L. Vasconcelos, I.J. Tsang and I.R. Tsang. Scaling relations for diversity of languages. Phys. A **271** (1999) 489–495.

CRITICAL EXPONENTS OF NiFeV ALLOYS FOR THE FERROMAGNETIC-PARAMETRIC PHASE TRANSITION

Uliana Buchko

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"A systematic study of the magnetic critical exponents has been made in Ni-rich NiFeV ternary alloys of compostions $Ni_{77}Fe_7V_{16}$, $Ni_{79}Fe_5V_{16}$ and $Ni_{78}Fe_4V_{18}$ in the critical region of ferromagnetic to paramagnetic (FM–PM) transition through detailed AC-susceptibiltiy and DC-magnetization measurements. In all the alloys, we have found a well-defined critical temperature T_c below which the spontaneous magnetization increases with decreasing temperature. The critical exponents and critical amplitudes associated with the transitions are obtained. Their values show drastic changes from those of pure Ni." [1].

This talk was a review of:

[1] G. Mukherjee, et al. Critical exponents of NiFeV alloys for the ferromagnetic–paramagnetic phase transition. J. Magn. Magn. Mater. **214** (2000) 185–194.

Abstracts 2002

March 12–14

PHASE TRANSITIONS IN STRONGLY CORRELATED ELECTRON SYSTEMS. EXACTLY SOLVABLE MODELS

Ihor Stasyuk

Institute for Condensed Matter Physics NAS of Ukraine, Lviv, Ukraine

Some problems of the theory of strongly correlated electron systems are discussed in the lecture. A brief review of the history of the main ideas and model development (from the Bogoliubov polar model of the metal, Hubbard model and its extensions to the Falicov-Kimball and pseudospin-electron models) is given.

The dynamical mean field theory (DMFT) approach, which is exact in the limit of the infinite dimension of space, is presented on the example of the binary alloy lattice model. It provides a derivation of equations for the coherent potential and electron Green's function in an analytic form as well as expressions for the grand canonical potential and static susceptibilities in the cases of the exactly solvable models.

Besides the binary alloy model the pseudospin-electron model (PEM) and Falicov-Kimball (FK) one belong to the models of this kind. The results of recent investigations of the FK model performed by various groups are discussed. The main features of the energy spectrum and thermodynamics of the model as well as phase transitions into modulated or segregated phases are considered.

Special attention is paid in the lecture to the pseudospin-electron model which appeared in the last few years in connection with the investigation of the high-T superconductors and systems with hydrogen bonds (the model is closely related to the FK model but differs by the regime of the thermodynamical averaging procedure). The results of investigation of the equilibrium states of PEM (using its various versions) within the DMFT scheme and by means of the generalized random phase approximation are analyzed and compared. The possibilities of application of the PEM to description of the inhomogeneous states and structure instabilities in the high-T superconducting systems are discussed.

THE RANDOM POTTS MODEL

Bertrand Berche, Christophe Chatelain
IJL, Lorraine Univercity, Nancy, France

Influence of uncorrelated, quenched disorder on the phase transition of two dimensional Potts models will be reviewed. After an introduction where the conditions of relevance of quenched randomness on phase transitions are exemplified by some experimental measurements, the results of perturbative and numerical investigations in the case of the Potts model will be presented. The Potts model is of particular interest, since it an have in the pure case a second-order or a first-order transition, depending on the number of states per spin. In 2D, transfer matrix calculations and Monte Carlo simulations are used in order to check the validity of conformal invariance methods in disordered systems. These techniques are then used to investigate the universality class of the disordered Potts model, in both regimes of the pure model phase transitions. A test of replica symmetry is made possible through a study of multiscaling properties.

TWO-DIMENSIONAL POLYMERS, THE EDWARDS MODEL AND O(n=0) FIELD THEORY

Christian von Ferber
University of Freiburg, Germany

In this lecture, we discuss the scaling properties of long flexible polymer chains in two dimensions. We compare perturbative expansions of the Edwards model, lattice Monte Carlo simulations, and exact results using conformal invariance and 2D quantum gravity for the (scaling) properties of random walks with self and/or mutual avoidance interactions. We are especially interested in the question of the universality of the problem of self and mutually avoiding walks in two dimensions (2D), as well as in validating multifractality found in these situations by field theoretic methods based on the Edwards model and by a conformal theory.

We focus on model star copolymers in two dimensions: walks or polymers of different species with a common starting point; the species avoid each other mutually.

In our field theoretical approach we mapped the problem of finding the scaling properties of the copolymer star to that of determining the anomalous dimensions of appropriate local field operator products. Ressummation of the perturbation series for these dimensions provides reliable numeric values for a family of exponents that displays multifractal behavior.

A recent extension of the conformal theory for 2D polymers to random graphs using methods of 2D gravity has revealed an exact derivation of this multifractal spectrum which is in remarkable coincidence with the perturbative results for a number of situations.

To further investigate this coincidence with respect to universality and moreover to uncover the reasons for deviations, we have undertaken a series of MC simulations on the lattice where the implementation of avoidance and topological restrictions of 2D polymers is most natural. While we confirm the universality of the 2D star copolymer problem of walks with topological avoidance it appears to constitute a lass separate from the 2D Edwards and $O(n)$ models with repelling interactions.

FIELD THEORETICAL APPROACHES IN THE SUPERCONDUCTING PHASE TRANSITION

Flavio Nogueira
Free University of Berlin, Germany

Several field theoretical approaches to the superconducting phase transition are discussed. Emphasis is given to theories of scaling and renormalization group in the context of the Ginzburg-Landau theory and its variants. Also discussed is the duality approach, which allows the access to the strong coupling limit of the Ginzburg-Landau theory.

RELAXATION IN QUANTUM SPIN CHAINS

Dragi Karevski

IJL, Lorraine University, Nancy, France

The aim of this lecture is to give a pedagogical introduction to the exact equilibrium and nonequilibrium properties of free fermionic quantum spin chains. In a first part we present in full details the canonical diagonalisation procedure and review quickly the equilibrium dynamical properties. The phase diagram is analysed and possible phase transitions are discussed.

The remaining part is devoted to the nonequilibrium dynamical behaviour of such quantum chains relaxing from a nonequilibrium pure initial state. In particular, a special attention is made on the relaxation of transverse magnetization. Two-time linear response functions and correlation functions are also considered, giving insights on the nature of the final nonequilibrium stationary state. The possibility of aging is also discussed.

SOME FACTS ABOUT THE MATHEMATICAL THEORY OF THE ISING MODEL AND ITS GENERALIZATIONS

Yuri Kozitsky

Marie Curie-Skłodowska University, Lublin, Poland

The first part of the lecture gives an outlook of the main aspects of the mathematical theory of the Ising model. The existance and differentiability of the infinite volume free energy density, including the properties connected with the Lee-Yang theorem, are discussed. Then the equilibrium state of the model as a probability measure on the space of configurations is introduced, a number of its properties are described. In particular, the nonuniqueness/phase transitions properties are discussed on the base of Dobrushin's criterium, as well as of the Lebowitz/Martin-Löf analiticity results. In the second part of the lecture, the above scheme is applied to the Ising model with a transverse field (De Gennes model), which contains non-comutative operators. Here the Euclidean approach, in which quantum states are represented by probability measures, is employed.

QUANTUM PHASE TRANSITION IN ALTERNATING TRANSVERSE ISING CHAINS

Oleg Derzhko

Institute for Condensed Matter Physics NAS of Ukraine, Lviv, Ukraine

We start from re-calling generally known topics of the phase transition theory: phase transitions of the first and the second order in classical systems at nonzero temperature, the Onsager solution of the square-lattice Ising model, critical behaviour of the physical quantities, universality, scaling, renormalization group. Then we turn to the basic concepts of quantum phase transition theory discussing the experiment of Bitko, Rosenbaum and Aeppli (1996) on Ising system (LiHoF4) placed in a magnetic field transverse to the magnetic axis and the phase diagram of the Ising spin model in the plane temperature – transverse field. Onedimensional spin- 1/2 Ising model in a transverse field is a simplest model exhibiting the second-order quantum phase transition. We discuss a relation of that model to the square-lattice Ising model and present the 'old' results of rigorous calculation derived by Pfeuty (1970). The essential tool in this solution is the Jordan-Wigner fermionization. We briefly explain how the fermionic description can be introduced and thus how the results of Pfeuty (and some other results) were derived. We contrast quantum and classical transverse Ising chains emphasizing that the zerotemperature continuous phase transition driven by the transverse field occurs in the quantum chain only.

The second part of the lecture deals with the effects of regular alternation of the Hamiltonian parameters (i.e., the intersite exchange interaction and on-site field) on the quantum phase transition. We elaborate an approach based on continued fractions for rigorous calculation of the thermodynamic quantities. The spin correlation functions for regularly alternating transverse Ising chains an be obtained numerically. We discuss in detail the case of a chain of period 2 comparing exact analytical and exact numerical results for the ground state properties. Moreover, we demonstrate how the ground state (and therefore all spin correlation functions) an derived for special parameter values. We complete the analysis of the effects of regular alternation examining the low-temperature behaviour of the specific heat. We sketch the phase diagram for a chain of period 3. We end up with conclusions emphasizing the effects of regular alternation on the second-order quantum phase transition in the transverse Ising chain.

Abstracts 2003
April 22–24

- *AN INTRODUCTION TO THE NON PERTURBATIVE RENORMALIZATION GROUP (Bertrand Delamotte)*
- *CRITICAL DYNAMICS (Reinhard Folk)*

AN INTRODUCTION TO THE NON PERTURBATIVE RENORMALIZATION GROUP

Bertrand Delamotte

University Pierre and Marie Curie, Paris, France

The non perturbative renormalization group of Wilson is introduced in the context of (statistical) field theory. We first derive the exact renormalization group equation for the coarse grained free energy. Then, we focus on the para/ferro-magnetic phase transition and within the simplest truncation of the exact renormalization group equation we study the critical physics in all dimensions between two and four.

See also:

B. Delamotte. A hint of renormalization. Am. J. Phys. **72** (2004) 170–184.

CRITICAL DYNAMICS

Reinhard Folk
Johannes Kepler University, Linz, Austria

An introduction into dynamical critical phenomena and their explanation will be given. Apart from a very short summary of the understanding of static critical behavior, in the first lecture we give an overview of the experimental situation in dynamics with concentration on liquids (transport coefficients, light scattering and sound), ferromagnets (neutron scattering) and the lambda transition in liquid He4 (thermal conductivity, second sound). We then discuss Van Hove theory and the concept of dynamic scaling theory. For a certain class of models the dynamical critical exponent z can be found from applying this scaling concept below T_c. We end this part by introducing universality classes and its classification (models A – J). The second lecture contains technical points: We follow step by step the set up of the dynamical model for a liquid, and the transformations leading to a Lagrangian in order to match with static field theory. We discuss the perturbation theory and the structure of dynamical vertex functions, which have to be renormalized. The third lecture presents the comparison of the theoretical results with experiments. The main topic in this comparison will be the explanation of the observation as crossover phenomena: from background to asymptotics, from the hydrodynamic region to the critical region. This is demonstrated by temperature and wave vector dependence of the characteristic frequency in light scattering in liquids, and the frequency dependence of the shear viscosity, on the shape crossover of the scattering function in ferromagnets and the non asymptotic behavior of the thermal conductivity in He4 at the superfluid transition.

Abstracts 2004
March 23–25 and May 4–7

- *INTRODUCTION TO SELF-ORGANIZED CRITICALITY (Alexander Olemskoi)*
- *GEOMETRICAL APPROACH TO PHASE TRANSITIONS (Adriaan Schakel)*

INTRODUCTION TO SELF-ORGANIZED CRITICALITY

Alexander Olemskoi
Sumy State University, Ukraine

In the first part of lectures, an introduction to the phenomena of self-organized criticality, which offers considerable insight into a wide range of phenomena from earthquakes to traffic jams is given. In standard critical phenomena, there is a control parameter which an experimenter can vary to obtain the radical change in behaviour. Self-organized critical phenomena, by contrast, is exhibited by driven systems which reach a critical state by their intrinsic dynamics, independently of the value of any control parameter.

The archetype of a self-organized critical system is a sand pile. Sand is slowly dropped onto a surface, forming a pile. As the pile grows, avalanches occur which carry sand from the top to the bottom of the pile. The slope of the pile becomes independent of the rate at which the system is driven by dropping sand. This is the (self-organized) critical slope. In the second part of lectures we present the theory of a flux steady-state related to avalanche formation for the simplest model of a sand pile within the framework of the Lorenz approach. The stationary values of sand velocity and sand pile slope are derived as functions of a control parameter (driven sand pile slope). The additive noise of above values are introduced for building a phase diagram, where the noise intensities determine both avalanche and non-avalanche domains, as well as mixed one. Corresponding to the SOC regime, the last domain is crucial to affect of the noise intensity of the vertical component of sand velocity and especially sand pile slope.

To address to a self-similar behaviour, we use a fractional feedback as an efficient ingredient of the modified Lorenz system. In the spirit of Edwards paradigm, an effective thermodynamics is introduced to determine a distribution over an avalanche ensemble with negative temperature. Steady-state behavior of the moving grains number, as well as nonextensive values of entropy and energy is studied in detail. The power law distribution over the avalanche size is described within a fractional Lorenz scheme, where the energy noise plays a crucial role. This distribution is shown to be a solution of both fractional and nonlinear Fokker-Planck equation. As a result, we obtain new relations between the exponent of the size distribution, fractal dimension of phase space, characteristic exponent of multiplicative noise, number of governing equations, dynamical exponents and nonextensivity parameter.

See also:

A.I. Olemskoi, A.V. Khomenko, D.O. Kharchenko. Self-organized criticality within generalized Lorenz scheme. cond-mat/0104325.

GEOMETRICAL APPROACH TO PHASE TRANSITIONS

Adriaan Schakel
Institute of Theoretical Physics, Leipzig University, Germany

Using percolation theory as a paradigm, a geometrical approach to phase transitions is developed. The theory is worked out explicitly for the two-dimensional Ising model – one of the simplest statistical models exhibiting non-trivial critical behavior. Other systems considered include Bose-Einstein condensates (BEC) and superfluid He4. It is shown that the fractal dimensions of the relevant geometrical objects (Peierls domain walls in the Ising model, worldlines in BEC, vortex loops in superfluid He) encode the critical exponents. The lectures are physically intuitive and non-technical in nature.

See also:

[1] A. Schakel. Entangled Vortices: Onsager's geometrical picture of superfluid phase transitions. J. Low Temp. Phys **129** (2002) 323.

[2] W. Janke, A.M.J. Schakel. Geometrical vs. Fortuin-Kasteleyn clusters in the two-dimensional q-state Potts model. Nucl. Phys. B **700** (2004) 385.

Abstracts 2005

May 17–20

- *MONTE CARLO SIMULATIONS IN STATISTICAL PHYSICS (Wolfhard Janke)*

MONTE CARLO SIMULATIONS IN STATISTICAL PHYSICS

Wolfhard Janke

Institute of theoretical physics, Leipzig University, Germany

The aim of this lecture series is to give an overview on the current state-of-the-art of Monte Carlo computer simulations and to illustrate them in the first two lectures with simple applications to the Ising model of statistical physics. After reviewing in the first lecture importance sampling Monte Carlo schemes based on Markov chains and standard local update rules such as the Metropolis and heat-bath algorithm, statistical error analyses of simulation data and critical slowing down at a second-order phase transition will be discussed. As an important tool for finite-size scaling analyses, histogram reweighting techniques are introduced. Next advanced update algorithms will be considered which, for certain classes of models, can drastically improve the performance of simulations. This will be illustrated with cluster-update algorithms, reducing critical slowing down at second-order phase transitions, and multicanonical simulations, greatly improving simulations at first-order phase transitions and, in general, for systems with rare-event states. A few other useful methods will be briefly mentioned. Mainly intended as an outlook, the third lecture will be devoted to more advanced applications to disordered systems such as diluted ferromagnets, random lattices and spin glasses which in general require especially tailored algorithms for their successful simulation. Focussing mainly on the basic concepts, the lecture series is addressed to a broad audience of students, whose main focus may range from applied to theoretical physics. Small exercises referring mainly to the first and partly the second lecture will be assigned, that should be worked out by the students.

Lecture I – Introduction to Monte Carlo simulations: This lecture introduces the basic concepts underlying Monte Carlo simulations and their statistical analysis. The power of the method will be illustrated for the Ising model.

- Importance sampling Monte Carlo
- Local update procedures (Metropolis, heat-bath)

- Statistical error analyses (critical slowing down)
- Histogram reweighting techniques
- Applications to the Ising model

Lecture II – Improved algorithms and generalised ensembles: For certain classes of models the simulations can be drastically improved by using more advanced algorithms. This will be illustrated with cluster-update algorithms, drastically reducing critical slowing down at second-order phase transitions, and multicanonical simulations, greatly improving simulations at first-order phase transitions and, in general, for systems with rare-event states. Other useful methods will be only briefly mentioned.

- Cluster algorithms
- Multigrid methods
- Generalized ensembles (multicanonical simulations etc.)
- Simulated and parallel tempering

Lecture III – Applications to disordered systems: Numerical simulations of quenched, disordered systems (e.g. random-bond or diluted ferromagnets, random lattices or graphs, spin glasses) in general require especially tailored algorithms in order to achieve reliable results in reasonable computing times (which are usually still large, even on supercomputers). Methodological similarities to the problem of protein folding will be sketched. The objective of this lecture is to give an outlook to computer experiments for such systems and to illustrate them by specific examples without going too much into the details.

- Diluted ferromagnets
- Random lattices or graphs
- Spin glasses
- Protein folding

Recent textbooks on the subject include:

[1] M.E.J. Newman, G.T. Barkema. *Monte Carlo Methods in Statistical Physics.* Clarendon Press, Oxford, 1999.

[2] D.P. Landau, K. Binder. *Monte Carlo Simulations in Statistical Physics.* Cambridge University Press, Cambridge, 2000.

[3] K. Binder, D.W. Heermann. *Monte Carlo Simulations in Statistical Physics: An Introduction* (4th edition). Springer, Berlin, 2002.

[4] B.A. Berg. *Markov Chain Monte Carlo Simulations and Their Statistical Analysis.* World Scientific, Singapore, 2004.

A few review articles covering the material of the lectures are:

[5] W. Janke. Nonlocal Monte Carlo Algorithms for Statistical Physics Applications. Math. Comput. Simulat. **47** (1998) 329.

[6] W. Janke. Statistical Analysis of Simulations: Data Correlations and Error Estimation, invited lecture notes. In: *Proceedings of the Euro Winter School Quantum Simulations of Complex Many-Body Systems: From Theory to Algorithms.* Edited by J. Grotendorst, D. Marx, and A. Muramatsu, John von Neumann. Institute for Computing, Jülich, NIC Series, Vol. 10 (2002) 423–445.

[7] W. Janke. Multicanonical Monte Carlo Simulations. Physica A **254** (1998) 164.

[8] W. Janke. Histograms and All That. In: *Computer Simulations of Surfaces and Interfaces, NATO Science Series, II. Mathematics, Physics and Chemistry (Vol. 114). Proceedings of the NATO Advanced Study Institute, Albena, Bulgaria, 9 – 20 September 2002.* Edited by B. Donweg, D.P. Landau, A.I. Milchev. Kluwer, Dordrecht, **114** (2003) 137–157.

[9] W. Janke. First-Order Phase Transitions. In: *Computer Simulations of Surfaces and Interfaces, NATO Science Series, II. Mathematics, Physics and Chemistry. Proceedings of the NATO Advanced Study Institute, Albena, Bulgaria, 9 - 20 September 2002.* Edited by B. Donweg, D.P. Landau, A.I. Milchev. Kluwer, Dordrecht, 2003. – Vol. 114, Pp. 111–135.

[10] W. Janke, et al. Phase transitions in disordered ferromagnets. In: *Proceedings NIC-Symposium 2004.* Edited by D. Wolf, et al. Institute for Computing, Jülich, NIC Series, **20** (2003) 241-250.

[11] W. Janke, B.A. Berg, A. Billoire. Multi-overlap simulations of spin glasses. In: *Proceedings NIC-Symposium 2004.* Edited by D. Wolf, et al. Institute for Computing, Jülich, NIC Series **20** (2003) 301–314.

[12] W. Janke, M. Weigel. Monte Carlo studies of connectivity disorder.

In: *High Performance Computing in Science and Engineering, Munich 2004, transactions of the Second Joint HLRB and KONWIHR Result and Reviewing Workshop, March 2nd and 3rd, 2004, Technical University of Munich.*Springer-Verlag, Berlin, Heidelberg, New York, 2004, Pp. 363–373.

Abstracts 2006

May 10–12

- *SCALING RELATIONS – OLD AND NEW (Ralph Kenna)*
- *SPIN GLASSES WITH LONG- AND SHORT-RANGE INTERACTIONS (Serhiy Sorokov)*

SCALING RELATIONS – OLD AND NEW

Ralph Kenna
AMRC, Coventry University, UK

By the early 1960's theoretical and experimental advances in Statistical Mechanics had established the existence of a range of universality classes for systems with second-order phase transitions. These universality classes are characterized by critical exponents which are different to the mean-field (classical) ones. The next crucial theoretical achievement was the discovery (advanced especially by Essam and Fisher) of four (now famous) scaling relations between the six critical exponents describing second-order criticality. These scaling relations are of fundamental importance and now form a cornerstone of Statistical Mechanics and other areas. In the first part of these lectures, the scaling relations will be introduced and a history of their discovery will be given. Separately, in the 1950's Lee and Yang introduced a theoretical way to conceptualize phase transitions. This involves allowing the parameters controlling the system (the temperature or magnetic field) to become complex. In the 1960's and 1970's, Abe and Suzuki used the fact that the even (temperature) and odd (magnetic) scaling fields can be linked by Lee-Yang zeros to re-derive the scaling relations. In the second part of these lectures, the theory of complex Lee-Yang zeros will be discussed and Abe's and Suzuki's achievements will be highlighted. Very recently, new theories concerning scaling relations have been established. These theories were established through the medium of Lee-Yang analyses. These theories will be outlined and a famous long-standing puzzle of Statistical Mechanics will be resolved.

Historical Articles:

[1] J.W. Essam, M.E. Fisher. Padé approximant studies of the lattice gas and Ising ferromagnet below the critical point. J. Chem. Phys. **38** (1963) 802.

[2] C.N. Yang, T.D. Lee. Statistical theory of equations of state and phase transitions. I. Theory of condensation. Phys. Rev. **87** (1952) 404; Statistical theory of equations of state and phase transitions. II. Lattice gas and Ising model. ibid. 410.

[3] R. Abe. Note on the critical behavior of Ising ferromagnets. Prog.

Theor. Phys. **38** (1967) 72.

[4] M. Suzuki. A theory on the critical behaviour of ferromagnets. Prog. Theor. Phys. **38** (1967) 289; A theory of the second order phase transitions in spin systems. II: Complex magnetic field. ibid. 1225.

Reviews:

[5] M.E. Fisher. Renormalization group theory: its basis and formulation in statistical physics. Rev. Mod. Phys. **70** (1998) 653.

[6] F. Ravndal. *Scaling and Renormalization Groups.* Nordisk Inst. for Teoretisk Atomfysik, 1976.

Recent Developments:

[7] W. Janke, D.A. Johnston, R. Kenna. Properties of higher-order phase transitions. Nucl. Phys. B **736** (2006) 319.

[8] R. Kenna, D.A. Johnston, W. Janke. Scaling Relations for Logarithmic Corrections. Phys. Rev. Lett. **96** (2006) 115701.

SPIN GLASSES WITH LONG- AND SHORT-RANGE INTERACTIONS

Serhiy Sorokov

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1. Introduction. Main models (Sherrington-Kirkpatrick, Edwards-Anderson, p-spin spherical model), quantities (spin-glass parameter, overlap distribution function, complexity) and techniques (especially replica trick) used in the spin-glass theory are reviewed.

2. Theory of spin-glasses with infinite radius of interaction. We will consider replica symmetry solution and 1-step replica symmetry breaking for p-spin spherical model. The contribution of metastable states into global equilibrium free energy will be analyzed. We will discuss the structure of energy states and its relation to the ergodic breaking.

3. Theory of spin-glasses with essential short-range interactions. We will review the simulation results for some models with nearest neighbor interaction within the replica symmetry approach and the 1-step replica symmetry breaking. Systems of equations for the distribution functions of static effective fields and linear dynamic susceptibility will be derived and analyzed. The phase diagrams constructed on the basic static and dynamic susceptibility will be discussed. The role of the weak long-range interaction will be illustrated.

4. The proton-glasses of $\mathbf{Rb_{1-x}(NH4)_xH_2PO_4}$–type. The main experimental data (phase diagram, dynamic permittivity) will be reviewed. We will discuss the applicability of some models for description of the proton glasses.

The review article:

[1] T. Castellani, A. Cavagna. Spin-glass theory for pedestrians. J. Stat. Mech. (2005) P05012.

Main historical references:

[2] D. Sherrington, S. Kirkpatrick. Solvable model of spin glass. Phys. Rev. **35** (1975) 1792–1796.

[3] S. Kirkpatrick, D. Sherrington. Infinite-ranged model of spin-gla-

sses. Phys. Rev. B **17** (1978) 4384.

[4] G. Parisi. The order parameter for spin glasses: a function on the interval 0–1. J. Phys. A **13** (1980) 1101–1112.

[5] S.F. Edwards, P.W. Anderson. Theory of spin glasses. J. Phys. F. Metal. Phys. **5** (1975) 965–974.

Main articles about the spin-glass models with nearest neighbor interaction:

[6] F. Matsubara, M. Sakata. Theory of random magnetic mixture. III. Glass-like phase. Progr. Theor. Phys. **75** 1976) 672.

[7] M. Sasaki, Sh. Katsura. The distribution function of the effective field of the Ising spin glass on the Bethe lattice for the coordination number $z = 4, 5, 6$. Physica A **155** (1989) 206–220.

[8] M. Mezard, G. Parisi. The Bethe lattice spin glass revisited. Eur. Phys. B **20** (2001) 217–233.

[9] F. Liers, M. Palassini, A.K. Hartmann, and M. Junger. Ground state of the Bethe-lattice spin glass and running time of an exact optimization. Phys. Rev. B **68** (2003) 094406.

Selected articles on proton glasses:

[10] R. Pirc, B. Tadic, R. Blinc. Random-field smearing of the proton-glass transition. Phys. Rev. B **36** (1987) 8607–8615.

[11] Z. Trybula, V.H. Schmidt, J.E. Drumheller. Coexistence of proton-glass and ferroelectric order in $Rb_{1-x}(NH_4)_xH_2AsO_4$. Phys. Rev. B **43** (1991) 1287.

[12] S.I. Sorokov, R.R. Levitskii, A.S. Vdovych. Spin-glass model with essential short-range competing interactions. Condens. Matter Phys. **8** (2005) 603–622.

Abstracts 2007

June 5–7

- *POWER LAWS IN CRITICAL AND OFF-CRITICAL SYSTEMS (Józef Sznajd)*
- *PHASE AND CRITICAL BEHAVIOUR OF IONIC FLUIDS: EXPERIMENT, THEORY AND COMPUTER SIMULATIONS (Oksana Patsahan)*

POWER LAWS IN CRITICAL AND OFF-CRITICAL SYSTEMS

Józef Sznajd

Institute of Low Temperature and Structure Research, Polish Academy of Sciences, Wroclaw, Poland

1. Phase Transitions: From early Universe to Ice Cube. The evolution of the description of the phase transitions from singularities in the thermodynamic potentials via symmetry breaking to the divergence of the scale-length is presented. The origin of the power laws at the critical state is discussed.

2. Power laws – Saint Graal of the complex systems science. The power laws occurring in economy, sociology, and biology are reviewed.

3. Linear perturbation renormalization group (LPRG). The linear renormalization-group transformation is proposed to study critical temperatures and temperature dependence of the thermodynamic values (free energy, specific heat) of the weakly interacting spin chains. The transition temperature of the uniaxial Heisenberg ferromagnet in the field perpendicular to the easy axis is found. It is shown that only for very small fields and the anisotropy strong enough this temperature is shifted according to h2 as predicted within mean-field-approximation.

4. Power Laws in Off-Critical Systems: a. Ising strips (finite system); b. Spinless fermion chain; c. Social validation model.

See also:

[1] J. Sznajd. Introduction to the modern theory of phase transitions. In: *Patterns of Symmetry Breaking, NATO Science Series, II Mathematics, Physics and Chemistry*, vol. 127, and ref. therein.

[2] A. Drzewinski, J. Sznajd, K. Szota. Power laws in Ising strips: density-matrix renormalization-group calculations. Phys. Rev. B **72** (2005) 014441.

[3] J. Sznajd, K.W. Becker. Spinless fermion chains with weak interchain hopping. J. Phys. Condens. Matt. **46** (2005) 7359–7370.

[4] K. Sznajd-Weron, J. Sznajd. Who is left, who is right? Physica A **351** (2005) 593–604.

PHASE AND CRITICAL BEHAVIOUR OF IONIC FLUIDS: EXPERIMENT, THEORY AND COMPUTER SIMULATIONS

Oksana Patsahan

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It is known that electrostatic forces determine the properties of various systems: physical as well as chemical or biological. In particular, the Coulomb interactions are of great importance when dealing with ionic fluids, i.e. fluids consisting of dissociated cations and anions. In most cases the Coulomb interaction is the dominant one and due to its long-range character can substantially affect the critical properties and the phase behaviour of ionic systems. Thus, the investigations concerning these issues are of great fundamental interest. Over the last ten years, both the phase diagrams and the critical behaviour of ionic solutions have been intensively studied using experimental and theoretical methods (see for example, [1-4]). The overview of the achievements obtained in this field will be presented in the lectures.

[1] J.M.H. Levelt Sengers, J.A. Given. Mol. Phys. **80** (1993) 899.

[2] M.E. Fisher. The story of coulombic criticality. J. Stat. Phys. **75** (1994) 1–36.

[3] G. Stell. Criticality and phase transitions in ionic fluids. J. Stat. Phys. **78** (1995) 197–238.

[4] W. Schröer. *Ionic Soft Matter: Modern Trends and Applications.* Ed. D. Henderson et al. Dordrecht: NATO ASI Series II, Springer, 2005, P. 143.

Abstracts 2009

April 28–30

- *AN INTRODUCTION TO THE PHYSICS OF COMPLEX NETWORKS (Geoff Rodgers)*
- *PHASE TRANSITIONS IN SIMPLE MODELS OF SOCIAL DYNAMICS (Janusz Hołyst)*

AN INTRODUCTION TO THE PHYSICS OF COMPLEX NETWORKS

Geoff Rodgers
Brunel University, London, UK

The aim of this lecture series is to give an introductory overview of the physics of complex networks. In particular the series will cover the growth and properties of complex networks, processes on complex networks, the connection between the theory of complex networks and random matrix theory and the applications of complex networks. In Lecture I I will give an overview of the different classes of complex networks, the way in which they can be created and the different quantities that can be used to characterise their properties. In particular, basic models of scale free graphs and small world networks will be introduced and some of their properties identified. In Lecture II I will examine diffusion, the dynamics of packet transport and the properties of spin systems, including their phase transitions, on complex networks. In Lecture III the link between theories of complex networks and the properties of random matrices will be made. In addition, some of the major applications of complex networks will be introduced.

Review Articles:

[1] R. Albert, A.–L. Barabasi. Statistical mechanics of complex networks. Rev. Mod. Phys. **74** (2002) 47.

[2] S.N. Dorogovtsev, A.V. Goltsev, J.F.F. Mendes. Critical phenomena in complex networks. Rev. Mod. Phys. **80** (2008) 1275.

[3] D.J. Watts. *Small Worlds.* Princeton, 1999.

[4] J.F.F. Mendes et al. *Science of Complex Networks.* AIP, 2005.

PHASE TRANSITIONS IN SIMPLE MODELS OF SOCIAL DYNAMICS

Janusz Hołyst
Warsaw University of Technology, Poland

Various models of social dynamics will be presented and resulting equilibrium or non-equilibrium phase transitions will be discussed. It will be shown how a presence of a strong leader in a small community can effect in discontinuous and non-reversible jumps of opinion dynamics. It will be presented that a smaller but better organized social group can beat a larger one. Phenomenon of communities isolation will be demonstrated using a random version of the Chinese game Go.

See also:

[1] K. Kacperski, J.A. Hołyst. Opinion formation model with strong leader and external impact: a mean field approach. J. Phys. A **269** (1999) 511–526.

[2] J.A. Hołyst, K. Kacperski, F. Schweitzer. Phase transitions in social impact models of opinion formation. J. Phys. A **285** (2000) 199–210.

[3] A. Aleksiejuk, J.A. Hołyst, D. Stauffer. Ferromagnetic phase transition in Barabasi-Albert networks. J. Phys. A **310** (2002) 260–266.

[4] K. Suchecki, J.A. Hołyst. Ising model on two connected Barabasi-Albert networks. Phys. Rev. E **74** (2006) 011122.

[5] K. Suchecki, J.A. Hołyst. First order phase transition in Ising model on two connected Barabasi-Albert networks. Phys. Rev. E **74** (2006) 011122.

[6] J. Sienkiewicz, J.A. Hołyst. Nonequilibrium phase transition due to social group isolation. Phys. Rev. E **80** (2009) 036103.

- *MINORITY GAME: STOCHASTIC PROCESS WITHOUT EQUILIBRIUM (Frantisek Slanina)*

MINORITY GAME: STOCHASTIC PROCESS WITHOUT EQUILIBRIUM

Frantisek Slanina

Institute of Physics CAS, Prague, Czech Republic

In the series of three lectures we present the basics of the Minority Game, which was introduced in the field of econophysics in 1997 and since then it became a paradigmatic example of a frustrated evolutionary game. In the first lecture, we expose basic principles of the game, properties of its two phases and the phase transition. On the basis of numerical simulations, we show the special features of the game, which allow mapping on a Markov process and subsequent analytical solution. The solution itself is shown in the second lecture. First we explain the replica method, which is the technical tool for the solution, and then we apply it to calculate the properties of Minority Game exactly up to the critical point. The third lecture deals with extension of the Minority Game. We show how the applications come back to the field of economic phenomena, where the inspiration started. We show also another extensions which have more abstract motivations.

Abstracts 2011

April 12–14

- *FROM MR ISING TO THE WONDERFUL WORLD OF SOCIO-PHYSICS (Serge Galam)*
- *SIMPLE MODELS FOR COMPLEX SYSTEMS – TOYS OR TOOLS? (Katarzyna Sznajd-Weron)*

FROM MR ISING TO THE WONDERFUL WORLD OF SOCIOPHYSICS

Serge Galam

Center for Research in Applied Epistemology, École Polytechnique, Paris, France

The Ising model is so simple that nobody would have give it a chance to survive after the "deceptive" work by Ising to check the existence of a phase transition between order and disorder. And indeed, it has allow hundreds of researchers all over the world and for decade to produce thousands of research papers in physics but also in many other fields. On top of that epistemological surprise the Ising model has been at the core of the creation of sociophysics, a field built from statistical physics to describe social and political behavior. The study of yes or no agents has driven hundreds of works, which enlighten from a different prospective the hidden mechanisms behind social dynamics. And it is far from being over. I will present the basic ingredients and main features of the Ising model within physics. I will then illustrate its use to apprehend opinion dynamics. Several real life applications will be also discussed.

See also:

[1] S. Galam, Y. Gefen, Y. Shapir. Sociophysics : A mean behavior model for the process of strike. J. Math. Sociol. **9** (1982) 1–13.

[2] S. Galam, S. Moscovici. Towards a theory of collective phenomena: Consensus and attitude changes in groups. Eur. J. Soc. Psychol. **21** (1991) 49–74.

[3] S. Galam. Fragmentation versus stability in bimodal coalitions. J. Phys. A **230** (1996) 174–188.

[4] S. Galam. Rational group decision making: a random field Ising model at $T = 0$. J. Phys. A **238** (1997) 66–80.

[5] S. Galam. Sociophysics: a review of Galam models. Int. Jour. Mod. Phys. C **19** (2008) 409—440.

SIMPLE MODELS FOR COMPLEX SYSTEMS - TOYS OR TOOLS?

Katarzyna Sznajd-Weron
Wroclaw University of Science and Technology, Poland

The talk is thought of as a personal journey through the wonderful world of simple models designed to describe complex systems. I am not going to make a review, neither talk about the most important models. Instead I will present models and papers that influenced my scientific work, brought me new ideas or surprised me. My talk will be divided into three parts. In the first part I will present simple models of biological evolution and population dynamics. Among them the seminal Bak-Seppen model, particularly important from my perspective. In the second part I will talk about interactions between physics and social sciences. I will start from the history of sociology and "social physics", next introduce several simple models of societies. I will try to answer the question whether they should be regarded as tools or as toys. Finally, in the last part, I will address the question: Can people be treated as particles? I will present several social experiments and show how the results of these experiments were used to build a simple model of opinion dynamics, known as the Sznajd model.

See also:

[1] P. Bak, K. Sneppen. Punctuated equilibrium and criticality in a simple model of evolution. Phys. Rev. Lett. **71** (1993) 4083—4086.

[2] M.E.J. Newman, R.G. Palmer. Models of Extinction. A Review. arXiv:adap-org/9908002v1.

[3] Ph. Ball. The physical modeling of society: a historical perspective. J. Phys. A **314** (2002) 1 – 14.

[4] C. Castellano, S. Fortunato, V. Loreto. Statistical physics of social dynamics. Rev. Mod. Phys. **81** (2009).

Abstracts 2012
May 22–23

- *PATTERN FORMATION IN NATURE (Helena Zapolsky)*

PATTERN FORMATION IN NATURE

Helena Zapolsky
Rouen University, France

Half a century ago, the great mathematician Alan Turing wrote his first and last paper on biology and chemistry, about how a certain type of chemical reaction ought to produce many patterns seen in nature. From this moment, the problem of Turing pattern formation has attracted much attention in different fields of nonlinear science. Striped and mottled Turing patterns are found throughout nature-on a zebra's coat, on a fish's skin, on a sand dune, in a microstructure in alloys and in the ocular dominance columns of the brain. Recently, the Turing's idea was also used to describe the self-organizing systems. In my lectures I will discuss the formation of patterns and structures obtained through numerical simulation of the Turing mechanism in two and three dimensions. The forming patterns are found to depend strongly from the system parameters, however there is some universality in a rich variety of patterns: the stripes and spots in 2D, and lamellae and spherical droplets in 3D arranged in structures of high symmetry, with or without defects or distortions. In my first lecture I will present the continuous description of system using phase field formalism and Swift-Hohenberg equation. The atomistic description of the dynamic in systems is the object of the second lecture. In my last lecture I will discuss about the link between these two approaches as well as the link between the pattern on fish's skin, the spinodal decomposition and sand dune stripes.

See also:

[1] A.M. Turing. The chemical basis for morphogenesis. Phil. Trans. R. Soc. Lond. B **237** (1952) 37–72.

[2] M.C. Cross, P.C. Hohenberg. Pattern formation outside of equilibrium. Rev. Mod. Phys. **65** (1993) 851–1112.

[3] L.Q. Chen. Phase field models for microstructure evolution. Annu. Rev. Mater. Res. **32** (2002) 113–140.

[4] Y. Jin, A.G. Khachaturyan. Atomic density function theory and modeling of microstructure evolution at the atomic scale. J. Appl. Phys. **100** (2006) 013519–013532.

[5] K.R. Elder, et al. Phase-field crystal modeling and classical density functional theory of freezing. Phys. Rev. B **75** (2007) 064107.

[6] N. Provatas, et al. Using the Phase-Field Crystal Method in the Multi-Scale Modeling of Microstructure Evolution. JOM **59** (2007) 83.

Abstracts 2013
May 28-30

- *SCALING ABOVE THE UPPER CRITICAL DIMENSION (Bertrand Berche, Ralph Kenna)*
- *NONPERTURBATIVE RENORMALIZATION GROUP APPROACH TO POLYMERIZED MEMBRANES (Dominique Mouhanna)*
- *ERGODICITY VIOLATION AND AGEING IN ANOMALOUS DIFFUSION (Ralf Metzler)*
- *THE ECONOPHYSICS OF SIZE (Iddo Eliazar)*
- *ACTIVE MICRO-RHEOLOGY: LEARNING ABOUT COMPLEX FLUIDS BY PULLING INTRUDERS (Carlos Mejia-Monasterio)*
- *MONTE CARLO METHODS FOR LATTICE SPIN MODELS AND THEIR APPLICATION FOR NUMERICAL SIMULATION OF CRITICAL CASIMIR FORCES (Oleg Vasilyev)*

SCALING ABOVE THE UPPER CRITICAL DIMENSION

Bertrand Berche[1], **Ralph Kenna**[2]

[1] *IJL, Lorraine University, Nancy, France*

[2] *AMRC, Coventry University, UK*

Above the upper critical dimension, the breakdown of hyperscaling is associated with dangerous irrelevant variables in the renormalization group formalism at least for systems with periodic boundary conditions. While these have been extensively studied, there have been only a few analyses of finite-size scaling with free boundary conditions. The conventional paradigm there is that, in contrast to periodic geometries, finite-size scaling is Gaussian, governed by a correlation length comensurate with the lattice extent. Here, we present analytical and numerical results which indicate that this paradigm is unsupported, both at the infinite-volume critical point and at the pseudocritical point where the finite-size susceptibility peaks. Instead the evidence indicates that finite-size scaling at the pseudocritical point is similar to that in the periodic case. An analytic explanation is offered which allows hyperscaling to be extended beyond the upper critical dimension.

NONPERTURBATIVE RENORMALIZATION GROUP APPROACH TO POLYMERIZED MEMBRANES

Dominique Mouhanna
University Pierre and Marie Curie, Paris, France

Polymerized membranes form a particularly rich domain of statistical physics where two-dimensional geometry and thermal fluctuations meet. This coexistence of geometry and possibly strong fluctuations has led to a lot of unexpected behaviors going from crumpled to flat, tubular or glassy phases. For several reasons that will be explained during these lectures the standard - perturbative - approaches are in particularly difficult position to correctly describe the critical behaviour of the various kinds of membranes known: isotropic or anisotropic, pure or disordered, with or without topological defects, etc. I will show in these lectures how a nonperturbative technique, known as the nonperturbative renormalization group, allows to correctly account for the physics of these systems in a large range of parameters - dimensions, temperature, coupling constants, etc.

ERGODICITY VIOLATION AND AGEING IN ANOMALOUS DIFFUSION

Ralf Metzler
University of Potsdam, Germany

In 1905 Einstein formulated the laws of diffusion, and in 1908 Perrin published his Nobel-prize winning studies determining Avogadro's number from diffusion measurements. With similar, more refined techniques the diffusion behaviour in complex systems such as the motion of tracer particles in living biological cells or the tracking of animals and humans is nowadays measured with high precision. Often the diffusion turns out to deviate from Einstein's laws. This talk will discuss the basic mechanisms leading to such anomalous diffusion as well as point out its consequences. In particular the unconventional behaviour of non-ergodic, ageing systems will be discussed within the framework of continuous time random walks. Indeed, non-ergodic diffusion in the cytoplasm of living cells as well as in membranes has recently been demonstrated experimentally.

THE ECONOPHYSICS OF SIZE

Iddo Eliazar
Holon Institute of Technology, Israel

In this talk we amalgamate ideas and concepts from various scientific disciplines – economics, mathematics, physics, probability, and statistics – to explore a topic of wide scientific interest: the omnipresence of power-laws in the distributions of sizes, commonly referred to as "Zipf's law" and as "Pareto's law". The talk is based on an ongoing collaboration with Morrel Cohen (Princeton & Rutgers), and is split into two parts which are outlined as follows.

Part I. Prolog: Rank distributions and Zipf's Law:

- Lorenz's curve, Pietra's formula, and Gini's index: measuring the distribution of wealth and social inequality
- Pareto's Law: from absolute monarchy to pure communism;
- Lorenzian analysis of rank distributions
- Regular variation
- Lorenzian limit law for rank distributions: the universality classes of absolute monarchy, Pareto's law, and pure communism
- Network's macroscopic topologies: the universality classes of total connectedness ('solid state'), fractal connectedness ('liquid range'), and total disconnectedness ('gas state')
- Oligarchic limit law for rank distributions: the universality classes of totalitarianism, criticality, and egalitarianism
- Interlaced universal macroscopic classification of rank distributions and their phase transitions
- Zipfian epilog: egalitarianism, totalitarianism, and criticality

Part II. Prolog: from the single-exponent Zipf Law to the double-exponent composite Zipf Law:

- Lorenzian analysis of rank distributions
- Macroscopic structures of rank distributions: absolute monarchy and versatility
- Mapping between rank distributions and probability laws, power-law connections
- Oligarchic analysis of rank distributions: the universality classes of totalitarianism, criticality, and egalitarianism
- Totalitarianism: absolute monarchy and monarchic clans
- Heapsian analysis of rank distributions: information streams and innovations
- The Heaps process and the Heaps curve: a Functional Central Limit Theorem
- The Heaps curve and Laplace transforms, power-law connections
- Composite Zipfian epilog: Pareto and Inverse-Pareto limits; egalitarianism, monarchic-clan totalitarianism, and criticality; composite Heapsian structure of innovations.

ACTIVE MICRO-RHEOLOGY:LEARNING ABOUT COMPLEX FLUIDS BY PULLING INTRUDERS

Carlos Mejia-Monasterio

Technical University of Madrid, Spain

and

University of Helsinki, Finland

Rheology is an old and firmly established theory that studies the deformation and flow of matter. With the advent of new imaging technologies and numerical tools, rheological studies at the microscopic scales have gained increasing interest to study the dynamical properties of fluids in low-dimensional or confined geometries. The classical experiment in micro-rheology consists of tracking the motion of a colloidal particle, either due to thermal fluctuations or when it is pulled with a constant force, to infer the properties of the surrounding environment. In this lecture we will discuss the dynamical properties of such pulled intruder in arbitrary dimensions. We will introduce the micro-viscosity in the Stokes regime, when the pulling force is not too large. At larger forces nonlinear contributions cannot be neglected, and we will discover that some of these contributions actually dominate the dynamics of the intruder, leading to anomalous diffusion depending on the effective spatial dimensions.

MONTE CARLO METHODS FOR LATTICE SPIN MODELS AND THEIR APPLICATION FOR NUMERICAL SIMULATION OF CRITICAL CASIMIR FORCES

Oleg Vasilyev

Max Planck Institute for Intelligent Systems, Stuttgart, Germany

The universality hypothesis and the finite size scaling concept form a basis of the modern theory of the second order phase transitions. Monte Carlo simulations of lattice spin models of different universality classes let us to study details of the phase transition and to compute critical indexes and amplitudes of thermodynamic quantities. In the fluctuating media near the critical point (critical binary mixture, liquid helium near the superfluid transition point) long ranged fluctuations of the order parameter arise. These fluctuations produce long-ranged critical Casimir forces acting on confining surfaces or immersed objects. In the first part of the lecture basic algorithms for Monte Carlo simulation of lattice models will be described. In the second part the application of these methods for numerical investigation of the critical Casimir effect will be given.

Abstracts 2014

May 6–8

- *PHASE TRANSITIONS WITH QUENCHED DISORDER: UNIVERSALITY AND NON-SELFAVERAGING (Victor Dotsenko)*
- *SHAPES OF POLYMERS AND POLYMER NETWORKS (Christian von Ferber)*
- *SIMPLEXITY OF GEOMETRICALLY FRUSTRATED ISING ANTIFERROMAGNETS (Taras Yavors'kii)*

PHASE TRANSITIONS WITH QUENCHED DISORDER: UNIVERSALITY AND NON-SELFAVERAGING

Victor Dotsenko
University Pierre and Marie Curie, Paris, France

Long standing problem of the nature of the phase transitions in weakly disordered Ising-like statistical systems [1] is considered from the point of view of the recent developments in the replica method [2]. In particular, non-perturbative [3] and non-selfaveraging [4] phenomena in the critical point are considered, as well as the possibility of the universal probability distribution function for non-selfaveraging free energy critical fluctuations is discussed [5].

[1] V. Dotsenko. Critical phenomena and quenched disorder. Physics-Uspekhi **38**. No.5 (1995) 457; V. Dotsenko. *Introduction to the Replica Theory of Disordered Statistical Systems.* Cambridge University Press, 2001.

[2] V. Dotsenko. One more discussion of the replica trick: the example of the exact solution. Philosophical Magazine, **92** (2012) 16; V. Dotsenko. Replica solution of the Random Energy Model. EPL **95** (2011) 50006; V. Dotsenko. Universal Randomness. Physics-Uspekhi **54** (2011) 259.

[3] V. Dotsenko. Non-pertrurbative states in disordered systems. Phys. A **361** (2006) 463.

[4] S. Wiseman, E.Domany. Lack of self-averaging in critical disordered systems. Phys. Rev. E **52** (1995) 3469; A. Aharony, B. Harris. Absence of self-averaging and universal fluctuations in random systems near critical points. Phys. Rev. Lett. **77** (1996) 3700; S. Wiseman, E. Domany. Finite-size scaling and lack of self-averaging in critical disordered systems. Phys. Rev. Lett. **81** (1998) 22.

[5] V. Dotsenko, B. Klumov. Free Energy Distribution Function of a Random Ising ferromagnet. J. Stat. Mech. (2012) P05027.

SHAPES OF POLYMERS AND POLYMER NETWORKS

Christian von Ferber

AMRC, Coventry University, UK

The discovery and experimental confirmation by light scattering experiments in the early 1920ies that polymers in solution are well described as long flexible chains of high numbers of (often identical) units has given rise to research in modeling their properties in Mathematics and Physics [1]. Subsequently, models using random walks and self-avoiding walks were developed as well as sophisticated experimental and mathematical methods. Light and X-ray scattering allows to measure the mean extensions of the polymer coils in solution due to the correlations of monomers belonging to the same chain.

In the present lectures we in turn discuss and explore the possibilities of determining the shapes of these polymers coils in solution: e.g. for such random chains we may determine the deviation of the shape of these coils from the symmetric disk like distribution. We will find that indeed coils of such chains display a non-spherical shape which we may measure by defining an "asphericity" parameter. In turn, three dimensional polymers may display deviations from the spherical shape in terms of either an oblate (pancake like) shapes or a prolate (cigar like) shapes. We will find that the prolate shape prevails for linear chains in three dimensions. The way to measure these shapes is via a gyration tensor (similar to the inertia tensor for rigid bodies).

A simplified intuitive version of measuring the shape of a random-walk coil has been proposed earlier by Kuhn [3]. These lectures intend to introduce various concepts that have been used successfully to tackle the intricate problems of determining polymer shape parameters for polymers with and without self-avoidance.

We start by discussing numerical and analytical methods to determine the shapes of two- and three-dimensional random walks. This research has a long history dating back to R. Koyama [4] as well as Solc, Gobush and Stockmeyer [5]. We further explore the impact of excluded volume on the shapes of linear chain-like polymers following the seminal work by Aronovitz and Nelson [6].

Finally, we will investigate situations involving branched polymers, starting with star-branched polymers, i.e. structures where a number of identical chains are all attached by one end to a central core. Further, we look at other regular branched structures such as 'comb'-like architectures and star-burst dendrimers.

Finally, we demonstrate a semi-numerical methods to calculate the shapes of branched structures building on work by Wei and Eichinger [7]. Applying these methods we investigate random branched networks with different branching statistics such as loop-less networks with different degree distributions.

[1] H. Staudinger. Uber Polymerisation. Berichte der deutschen chemischen Gesellschaft **53** (1920) 1073–1085.

[2] P. Debye. Methods to determine the electrical and geometrical structure of molecules (Nobel Lecture, December 12, 1936); see also P. Debye. Debye function. Ann. d. Phys. **32** (1912) 789.

[3] W. Kuhn. Ueber die Gestalt fadenförmiger Moleküle in Lösungen. Colloid Zeitschrift **68** (1934) 2.

[4] R. Koyama. Excluded volume effect on the radius of gyration of chain polymers. J. Phys. Soc. Jpn. **22** (1967) 973.

[5] W. Gobush, K. Solc, W.H. Stockmayer. Statistical mechanics of random-flight chains. V Excluded volume expansion and second virial coefficient for linear chains of varying shape. J. Chem. Phys. **60** (1974) 12.

[6] J.A. Aronovitz, D.R. Nelson. Universal features of polymer shapes. J. Phys. France **47** (1986) 1445.

[7] G. Wei, B.E. Eichinger. On shape asymmetry of Gaussian molecules. J. Chem. Phys. **93** (1990) 1430.

SIMPLEXITY OF GEOMETRICALLY FRUSTRATED ISING ANTIFERROMAGNETS

Taras Yavors'kii
AMRC, Coventry University, UK

The notion of independent variables is central in mathematics. Sequences of independent random variables made up the setting for the first non-trivial mathematical results of the probability theory. The central limit theorem is perhaps the best known such result [1]. Yet, the beauty, richness and complexity of the physical world has its origin in interactions and correlations between degrees of freedom. For instance, geometrically frustrated spin systems at temperatures, lower than the scale of the leading spin interactions, can form massively degenerate correlated dynamical states, called spin liquids or collective paramagnets. Non-trivial properties of these states are believed behind the many exotic, quantum and classical, phenomena observed in geometrically frustrated condensed matter systems [2], including observed disobedience to the third law of thermodynamics. Since several years, graphics processing units (GPUs) have been increasingly used in physics research as a powerful and value general purpose computational alternative to regular computers [3]. In this talk I use GPUs to study the classical spin liquid states of the geometrically frustrated systems. Specifically, I run GPU-aided Monte Carlo simulations [4] on the nearest-neighbor antiferromagnetic Ising spin models on the two-dimensional kagome and the three-dimensional pyrochlore lattice.

I show that, down to the degenerate ground state manifolds, the spin correlations in such models show features consistent with the picture of independent random variables, usually describing spin models at high temperatures. Statistical physics properties of the models, such as pair correlation function, are shown to be well described by the variational single-particle mean-field theory [5] (MFT) ansatz at all $T \geq 0$, provided the MFT temperature scale Θ, where $\Theta_c < \Theta < \infty$, is mapped onto the physical temperature scale $0 \leq T \leq \infty$ by considering Θ as a suitable function of T. The models are thus completely "transparent" to the paramagnetic MFT treatment deep below the MFT critical temperature $\Theta_c > 0$, making MFT a simple and powerful tool for the study of perturbations at low T [6].

[1] A.N. Kolmogorov. *Foundations of the theory of probability.* Chelsea Publishing Company, 1956.

[2] *Introduction to frustrated magnetism: materials, experiments, theory.* Eds. C. Lacroix, P. Mendels, F. Mila. Springer, 2011.

[3] *Computer simulations on graphics processing units.* Eds. M. Weigel, A. Arnold, P. Virnau. Eur. Phys. J. Special Topics **210** (2012).

[4] The GPU code used here has many common elements with the code described. In: T. Yavors'kii, M. Weigel. Optimized GPU simulations of continuous-spin glass models. p. 159 of Ref. [3].

[5] P.M. Chaikin, T.C. Lubensky. *Principles of condensed matter physics.* Cambridge University Press, Cambridge, UK, 1995.

[6] T. Yavorskii, M. Enjalran, M.J.P. Gingras. Spin Hamiltonian, competing small energy scales, and incommensurate long-range order in the highly frustrated $Gd_3Ga_5O_{12}$ garnet antiferromagnet. Phys. Rev. Lett. **97** (2006) 267203.

Abstracts 2015
May 5–7

- *COMPLEX NETWORKS AND INFRASTRUCTURAL GRIDS (Antonio Scala)*
- *EMERGENCE OF SCALING: FROM INFORMATION-THEORETIC CONSTRAINTS TO SAMPLE SPACE REDUCING PROCESSES (Bernat Corominas-Murtra)*
- *BROWNIAN SYSTEM PRESENTATION (Bohdan Lev)*

COMPLEX NETWORKS AND INFRASTRUCTURAL GRIDS

Antonio Scala

Institute for Complex Systems CNR Italy, Roma, Italy
and
The London Institute for Mathematical Sciences, UK

Electric grids, telecommunication networks, railways, healthcare systems, financial circuits, etc. are infrastructures that are critical for functioning and the welfare of our countries. Most of such infrastructures – for historical reasons – have been developed and designed according to engineering paradigms that are staring to become inadequate to cope with their increasing complexity. Much of this complexity is simply due to increased system size: as statistical physics teaches us, collection of interacting objects exibit emergent phenomena (like phase transitions) that goes beyond to the properties of the single objects and have peculiar characteristics in the infinite size limit. Moreover, the increase of interdependencies among the infrastructures (think as an example of the interdependence among communication networks and electric grids) is adding a further element complexity. Hence, the statistical physics's approach can enlarge the understanding of the fragilities and vulnerabilities of such critical infrastructures.

In these lectures, we will cover some of the current models of infrastructural grids – both isolated and coupled – hinting out the possible and needed development of the field. In the first lecture, will first start introducing the constitutive equations for gas/oil pipelines and for electric grids Acha [1]. We will then describe some applications of the fiber-bundle model Peires [2] and of the cavity method Mezard and Parisi [3] to understanding cascading failures in trasmission power grids Pahwa et al. [4] and limiting such failures in distribution power grids by introducing self-healing capabilities Quattrociocchi et al. [5]. In the second lecture, we will focus on interating networked infrastructure Rinaldi et al. [6], D'Agostino and Scala [7]; we will cover a whole range of models, from the first abstract models of coupled cascading systems Newman et al. [8], Carreras et al. [9], Buldyrev et al. [10] to a recent realistic how energy from renewable sources affect network and markets Mureddu et al. [11].

[1] S. Acha. *Modelling distributed energy resources in energy service networks.* IET Digital Library, 2013.

[2] F.T. Peires. J. Textile Inst. **17** (1926) 355–368.

[3] M. Mezard, G. Parisi. The cavity method at zero temperature. J. Stat. Phys. **111** (2003) 1.

[4] S. Pahwa, C. Scoglio, A. Scala. Abruptness of cascade failures in power grids. Sci. Rep. **4** (2014) 3694.

[5] W. Quattrociocchi, G. Caldarelli, A. Scala. Self-healing networks: redundancy and structure. PLoS ONE **9** (2014) 87986.

[6] S.M. Rinaldi, J.P. Peerenboom, T.K. Kelly. Identifying, understanding, and analyzing critical infrastructure interdependencies. IEEE. Contr. Syst. Mag. **21** (2001) 11–25.

[7] G. D'Agostino, A. Scala.*Networks of networks: the last frontier of complexity, understanding complex systems.* Springer International Publishing, 2014.

[8] D. Newman, et al. In: *System sciences, 2005. HICSS '05. Proceedings of the 38th Annual Hawaii International Conference on (2005)*, pp. 63c–63c.

[9] B.A. Carreras, et al. In: *Proceedings of the 40th Annual Hawaii International Conference on System Sciences (IEEE Computer Society, Washington, DC, USA, 2007)*, HICSS '07, pp. 112.

[10] S.V. Buldyrev,et al. Catastrophic cascade of failures in interdependent networks. Nature **464** (2010) 1025.

[11] M. Mureddu, et al. PLoS ONE, Public Library of Science **10** (2015) e0135312.

EMERGENCE OF SCALING: FROM INFORMATION-THEORETIC CONSTRAINTS TO SAMPLE SPACE REDUCING PROCESSES

Bernat Corominas-Murtra
Medical University of Vienna, Austria

The comprehension of the mechanisms behind scaling patterns has become on of the hot topics of modern statistical physics [1]. From complex networks to critical phenomena, scaling laws emerge in somewhat regular way. In these two lectures we will revise parts of current explanatory proposals for the emergence of such behaviours. We will put a special emphasis on the so-called Zipf's Law [2]. In such probability distribution, the probability of observing the k-th most common event scales as 1/k. Its remarkable ubiquity spans from word frequencies in written texts to the distribution of city sizes, family names, wealth distribution or the size of avalanches in systems exhibiting self-organized criticality. Its origin and the consequences one can extract from its observation in real systems is a matter of an intense debate.

We will start with a short critical review of the scope of the results one can derive from the statistical study of scaling behaviours, both from the fundamental and practical side [3,4]. We will then briefly revise some of the standard approaches for the emergence of such statistical behaviours, such as the 'rich-gets-richer' class of models or the critical exponents appearing at the percolation threshold of a random graph [5].

The main focus will be on two non-standard frameworks, having, however, a huge explanatory potential: Information-theoretic constraints for the evolution of complex codes. This leads to a mathematical formalisation of the so-called "least effort" hypothesis, informally proposed by G.K. Zipf as the origin of the scaling behaviour observed in complex codes [2,6,7]. The increase of complexity under information-theoretic constraints has, however, larger ranges of application than the communication systems alone [8].

The "Sample Space Reducing" (SSR) processes, a recently introduced family of stochastic processes displaying a minimal degree of history dependence [9]. SSR process are a totally new route to scaling which can explain a huge range of power-law exponents thanks to the unique assumption that the sampling space is reduced as long as the process unfolds. The intuitive rationale behind the SSR processes and the surprisingly simple mathematical apparatus needed to understand them makes the SSR process approach a new research area with promising applications.

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[3] G.A. Miller, N. Chomsky. Finitary Models of Language Users. In: *Handbook of Mathematical Psychology, Vol. II.* Ed. R.D. Luce, R.R. Bush & E. Galanter. New York, Wiley, 1963.

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[7] B. Corominas-Murtra, J. Fortuny, R.V. Sole. Emergence of Zipf's law in the evolution of communication. Phys. Rev. E **83** (2011) 036115.

[8] B. Corominas-Murtra, R.V. Sole. Universality of Zipf's law. Phys. Rev. E **82** (2010) 011102.

[9] B. Corominas-Murtra, R. Hanel, S. Thurner. Understanding Zipf's law with playing dice: history-dependent stochastic processes with collapsing sample-space have power-law rank distributions. J. R. Soc. Interface **12** (2015) 20150330.

BROWNIAN SYSTEM PRESENTATION

Bohdan Lev

Bogolyubov Institute for Theoretical Physics NAS of Ukraine, Kyiv, Ukraine

According to basic principles of thermodynamics, when a macroscopic system is brought into contact with a thermal bath, the system evolves in time approaching the equilibrium state in the course of relaxation. The state of equilibrium is well defined only under certain idealized conditions, so that the properties of such system are determined by its peculiarities and characteristics of the thermal bath. In most cases, however, the systems are subjected to non-equilibrium conditions and external constraints. Therefore, it is dificult, if not impossible, to determine the governing parameters that can be held constant.

Nevertheless, there exist stationary states that can be unambiguously defined for certain open systems. Examples of such systems are given by hot electrons in semiconductors, a system of photons on inhomogeneous scatterers, when the difraction coeficient depends on the frequency of photons, a system of high-energy particles in accelerators that originates from collision with macroscopic particles in dusty plasma. Currently, there does not exist a well-developed description method of the nonequilibrium distribution function, which would take into account possible system states. A standard method describing non-equilibrium states is based on the information on the equilibrium state and small deviations from this state. The nonequilibrium in this approach is treated as a small modification of the equilibrium distribution function. Although far-from-equilibrium systems are abundant in nature, there is no unified commonly accepted theoretical approach which determines possible states of such systems.

Hence, it is a fundamentally important task to develop a method for exploring general properties of stationary states of open systems and to establish conditions of their existence. The main idea of this presentation consists in the description of the evolution of a non-equilibrium system as a possible Brownian motion of the system between different states with dissipation energy and diffusion in the energy space. For brevity, such systems will be referred to as Brownian systems. It should be emphasized that the theory of the Brownian motion can be applied to the non-equilibrium systems too. The main goal is to present a simple way to describe the non-equilibrium systems in the energy space and to obtain a new special solution. A few nonlinear models of systems with different processes will be described.

See also:

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Abstracts 2016

May 17–19

- *GENERALIZED ENSEMBLE COMPUTER SIMULATIONS OF MACROMOLECULES (Wolfhard Janke)*
- *SIMULATING SPIN MODELS ON GPU: AN INTRODUCTION (Martin Weigel)*
- *STATISTICAL TOPOLOGY OF RANDOM WALKS (Serguei Nechaev)*
- *POLYMER DYNAMICS: FROM SIMPLE TO COMPLEX (Mikhail Tamm)*
- *PROCESS AND INFORMATION – ENTROPIES FOR COMPLEX SYSTEMS (Rudolf A. Hanel)*
- *SPATIAL PATTERNING IN EPIDEMIOLOGY AND IN MACROMOLECULAR SELF-ASSEMBLY (Jaroslav Ilnytskyi)*

GENERALIZED ENSEMBLE COMPUTER SIMULATIONS OF MACROMOLECULES

Wolfhard Janke

Institute of Theoretical Physics, Leipzig University, Germany

Over the last decade generalized ensemble Monte Carlo simulation studies, especially multi-canonical, Wang–Landau, and replica exchange (or parallel tempering) computer simulations, have emerged as a strong tool to study the statistical mechanics of polymer chains. Many investigations have focused on coarse-grained models of polymers on the lattice and in the continuum. Phase diagrams of chains in bulk as well as chains attached to surfaces were studied. Also, aggregation behavior in solution of these models has been investigated. In these lectures I will first discuss the theoretical background for these simulations, explain the algorithms used and discuss their performance. Implementations of these algorithms on parallel computers will be also briefly described. Additionally these simulation methods are perfectly suited for microcanonical analyses which have been recognised as a powerful tool for investigations of phase transitions in nanoscopic polymer systems. As an illustration of these concepts, I give an overview over the systems investigated with these methods, focusing on studies of coarse-grained polymer models.

SIMULATING SPIN MODELS ON GPU: AN INTRODUCTION

Martin Weigel

AMRC, Coventry University, UK

Over the last couple of years it has been realized that the vast computational power of graphics processing units (GPUs) could be harvested for purposes other than the video game industry. This power, which at least nominally exceeds that of current CPUs by large factors, results from the relative simplicity of the GPU architectures as compared to CPUs, combined with a large number of parallel processing units on a single chip. To benefit from this setup for general computing purposes, the problems at hand need to be prepared in a way to profit from the inherent parallelism and hierarchical structure of memory accesses. In this overview lecture I discuss the performance potential for simulating spin models, such as the Ising or Heisenberg models as well as the Edwards-Anderson spin glass, on GPU as compared to conventional simulations on CPU. Different algorithms, including Metropolis [1,2] and cluster updates [3], as well as computional tricks such as multi-spin coding are taken into account.

[1] M. Weigel. Simulating spin models on GPU. Comput. Phys. Commun. **182** (2011) 1833.

[2] M. Weigel. Performance potential for simulating spin models on GPU. J. Comput. Phys. **231** (2012) 3064.

[3] M. Weigel. Connected-component identification and cluster update on graphics processing units. Phys. Rev. E **84** (2011) 036709.

STATISTICAL TOPOLOGY OF RANDOM WALKS

Serguei Nechaev

Laboratory of Theoretical Physics and Statistical Models, CNRS - University Paris XI, France

We discuss few interlinked topics in statistics of entangled random walks: conformal methods in topology of random path on multi-punctured plane, random walks on graphs and groups (including braid groups), "matrix-valued" Brownian bridges and random walks in Lobachevsky geometry. We explain how all these subjects help in understanding topology and fractal structure of strongly collapsed unknotted ring polymer chain.

[1] S. Nechaev. Statistics of knots and entangled random walks, Lectures presented at Les Houches. In: *Summer School "Topological Aspects of Low Dimensional Systems", July 7–31, 1998.* NATO Advanced Study Institute, session LXIX: EDP Sciences; Springer, 1999.

[2] S. Nechaev, O. Vasilyev. Thermodynamics and topology of disordered knots: correlations in trivial lattice knot diagrams. In: *Physical and Numerical Models in Knot Theory.* Series on Knots and Everything. WSPC: Singapore, 2005, chapter 22, pp. 421–472.

[3] M. Imakaev, L. Mirny, S. Nechaev. Effects of topological constraints on globular polymers. Soft Matter **11** (2015) 665–671.

POLYMER DYNAMICS: FROM SIMPLE TO COMPLEX

Mikhail Tamm
Moscow State University, Russia

This talk is aiming to give an overview of classical polymer dynamics theory with discussion of some important recent developments. The subject dates back to the 1940s when the celebrated Rouse model (a set of beads connected with springs and subject to independent thermal noise at each bead) was proposed. This model is very beautiful and has many important applications, but it is unable to catch some of the effects which might be crucially important for the polymer dynamics both in solutions and in melts. Most important of these effects being hydrodynamic interactions in case of diluted solutions and topological constraints imposed on a chain by the surrounding chains in case of melts and concentrated solutions. In order to address this second problem the reptation theory was developed in the 1970s. Now, both Rouse and reptation models work for polymer states where chains at equilibrium are Gaussian, for reptation model it is also crucial that chains are linear. In the second part of my talk I will address generalizations of these models recently suggested for more exotic non-Gaussian polymer states, in particular the states where polymer conformations are controlled by strong topological interactions, e.g. the so-called crumpled globule and melts on non-concatenating polymer rings.

PROCESS AND INFORMATION – ENTROPIES FOR COMPLEX SYSTEM

Rudolf A. Hanel
Medical University of Vienna, Austria

What entropy, information, complexity, story and the absence of free lunches have to do with each other?! Here we will try to scetch a comprehensive picture of what complexity science is about. We will discuss a fundamental source of uncertainty, frequently ignored. The impossibility to infer process purely from statistical information! Statistical means are fine for estimating system parameters once the class of processes is already known, but become useless and misleading when nothing is known about the process we sample. The work we need to invest into knowing the (defining) rules governing a process class of interest always seems to be independent from the work we have to invest into knowing the statistical properties of a process class. This re-expresses a fundamental observation (Wolpert & Macready 1995): there is no free lunch to be had. In order to know what works we have to know the context in which something works. What does this tell us about our concepts of information and entropy? In the context of equilibrium systems entropy concepts all ultimately take the form of Shannon entropy, but describe distinct properties in the context of complex (e.g. driven, dissipative, path-dependent, or non-ergodic) processes; e.g. Boltzmann entropy characterizing the maximum configuration of a process while extensive entropies describe its phase-space growth.

SPATIAL PATTERNING IN EPIDEMIOLOGY AND IN MACROMOLECULAR SELF-ASSEMBLY

Jaroslav Ilnytskyi

Institute for Condensed Matter Physics NAS of Ukraine, Lviv, Ukraine

Spatial patterning is in the focus of both academic and practical needs: from intricate tiling ornaments in household interiors and image recognition in digital media to clustering of nanoparticles and geography-based population or disease spread. I will be providing some examples from two particular fields: epidemiology and macromolecular physical chemistry. The simplest epidemiology model, the SIS model, provides direct link to the Ising model but with the "spin" updates governed by medically-inspired rules. For the purpose of the epidemiology, both a stationary state and the system evolution towards it are literally vital. I will be considering spatial patterns of the disease spread for the classic SIS model as well as for its generalisations for the case of (i) the coexistence of the ordinary and multi-drug-resistant carriers, (ii) competing carriers with various interaction rules for their interaction, and (iii) self-adapting model with dynamically adjusted curability. The example from the macromolecular chemistry is focused on the aggregation of surface modified gold nanoparticles. These represent a perspective new class of soft matter objects that coined the name of "patchy colloids", the Janus particles being one particular example. I will discuss the merits of their aggregation in selective solvent and in bulk including the case of a photo-controllable aggregation for the azobenzene-modified nanoparticles. Resulting spatial patterns, intrinsic networks and percolation between two adsorbing walls will be considered. Practical applications will be given in relation to the plasmon resonance phenomena and photo-controllable electro-conductivity.

Abstracts 2017

June 14–16

- *GEOMETRY, PHYSICS, FREEDOM... THE WORLD OF CONNECTIONS (Bertrand Berche)*
- *THE SEARCH FOR UNIVERSALITY IN FINITE-SIZE SCALING OF PERCOLATION THEORY IN HIGH DIMENSIONS (Ralph Kenna)*
- *CELL DIVISION DYNAMICS WITH APPLICATIONS TO TUMOR GROWTH (Yuri Kozitsky)*
- *THE COURSE OF THE WORLD AND THE COURSE OF CLOCKS (Reinhard Folk)*
- *SOME OLD AND NEW PUZZLES IN THE DYNAMICS OF FLUIDS (Ihor Mryglod)*

GEOMETRY, PHYSICS, FREEDOM... THE WORLD OF CONNECTIONS

Bertrand Berche

IJL, Lorraine University, Nancy, France

We would like to give an introductory lecture to the geometric approach of physics offered by the introduction of connections. These mathematical objects give a prescription for the comparison of vectors in geometry and they are ubiquitous in the gauge description of interactions in physics. They offer an incredible freedom for the physicist! We will try to elaborate on historical perspectives and to give a personal overview of this rich concept.

THE SEARCH FOR UNIVERSALITY IN FINITE-SIZE SCALING OF PERCOLATION THEORY IN HIGH DIMENSIONS

Ralph Kenna
AMRC, Coventry University, UK

Percolation theory has been the subject of extensive mathematical and simulational studies and is of relevance in a broad range of fields including physics, chemistry, network science, sociology, epidemiology, and geology. It has been reported that 80,000 papers on the topic have appeared in 60 years, including about one per day on the arXiv.

In 1985, Coniglio presented a scaling theory for percolation in high dimensions, suggesting that the proliferation of spanning clusters is associated with the breakdown of hyperscaling in its traditional form there. In the intervening years, mathematicians have verified Coniglio's theory, but only for systems with free boundary conditions at the infinite-volume percolation threshold or critical point. Numerical results, by contrast, are ambiguous. High-dimensional percolation theory was an active topic in statistical physics up to about 2004 when it was declared that "percolation in high dimensions is not understood".

In the meantime, the mathematicians have been busy and have made lots of good progress. In 1997, Aizenman established that Coniglio's predictions hold for bulk boundary conditions and suggested a different, non-proliferating scaling behaviour for periodic boundary conditions. Mathematicians have since verified this, but appear to have diverged from the statistical-physics literature and some recent mathematics reviews don't cite Coniglio's theory at all.

A number of questions arise for the physicist. Firstly, Coniglio's theory is built upon renormalization-group concepts such as Kadanoff rescaling, Fisher's dangerous irrelevant variables and well as Binder's thermodynamic length. Why do these not deliver the same scaling for different boundary conditions? Why does Aizenman's picture depend on boundary conditions? What is the explanation of hyperscaling collapse for PBCs if there is no proliferation there? Here, these questions, and more, will be answered. After discussing the history of the problem and the old theories, I show how a recently developed general theory [1] for scaling in high dimensions recovers Coniglio's physics and Aizenman's mathematical predictions in appropriate regimes. This unifies percolation theory and delivers universality and hyperscaling above the upper critical dimension.

[1] R. Kenna, B. Berche. Universal finite-size scaling for percolation theory in high dimensions. J. Phys. A: Math. Theor. **50** (2017) 235001 and references therein.

CELL DIVISION DYNAMICS WITH APPLICATIONS TO TUMOR GROWTH

Yuri Kozitsky
Maria Curie-Skłodowska University, Lublin, Poland

Nowadays, it is well established that the initiation and progression of tumor is related to cell division mechanisms. In particular, the initiation of tumor is related to (driver) mutations, that may occur in the course of division. In the model which we consider, a finite (random) collection of entities (cells) undergoes a continuum time Markov evolution which amounts to two events: fission and death. The state of an entity is characterized by two variables, x and y, where positive x is time to fission whereas y describes a collection of relevant traits. The evolution is the drift in x towards zero that may be interrupted by a death occurring at random with intensity m(x). If the entity manages to stay alive until x reaches zero, it fisses to produce two new entities with random x and y, the distribution of which depends on the value of y of the mother entity. A detailed analysis of this evolution will be done, and some of its therapeutic-relevant conclusions will be discussed.

THE COURSE OF THE WORLD AND THE COURSE OF CLOCKS

Reinhard Folk

Johannes Kepler University, Linz, Austria

Time is measured by clocks, but what do they really indicate? In fact this question is strongly connected with our understanding of the nature of the universe. Astronomy and physics have changed this understanding from Aristotle over Newton to Einstein and changed the properties of time (in fact the property of duration) from something absolute to a property depending on the movement of the clock and its position. However the universality of our physical laws together with the history of the universe urge for a unification of the theory of gravitation and quantum theory. Some of the theoretical approaches for this unifications create doubt in the fundamental character of time.

SOME OLD AND NEW PUZZLES IN THE DYNAMICS OF FLUIDS

Ihor Mryglod

Institute for Condensed Matter Physics NAS of Ukraine, Lviv, Ukraine

One of the basic concepts of modern physics with a long prehistory is a fluid in the meaning of a substance that continually flows (or deforms) under an applied shear stress. In this sense fluids form a wide subset are the phases of matter and include liquids, dense gases, plasmas, and to some extent even plastic solids. The fluidity is one of the main dynamical characteristics that depends strongly on material properties or details of local structure and parameters of many-particle interactions in the terms of statistical physics. The last ones determines some relevant time that divide the fluid behavior in two main time regimes, characterizing material like as a viscous liquid over a long time period and as an elastic-like solid over a short time period. Such a property is typical for a viscoelastic substance or in other words for a fluid. Another aspect to be important for understanding of the fluid dynamics is connected with the structural ordering in the arrangement of atoms and molecules. Orderliness over distances comparable to interatomic distances is usually treated as short-range order, whereas orderliness repeated over infinitely large distances is called long-range order. Both long-range and short-range order are absent in the ideal gas, but liquids and amorphous solids exhibit short-range order. The physics of phonon in crystalline solids with long-range order is well understood. In liquids, however, the atomic structure is changing with time and the concept of phonon becomes questionable for long time processes. How the phonon-like excitations as well as other collective modes determine the fluid dynamics and are reflected in the response functions? Theoretically the flow of a liquid is commonly described by continuum hydrodynamic theories. Less attention has been paid to the atomic level dynamics because it has been believed that the liquids are so random that details of the atomic motion are irrelevant to the physics of liquid flow. Is it true for fluid systems with short-range order? In this lecture we try to find answers for some old and new questions that make the fluid dynamics still very attractive for the theoretical studies.

3 Lecturers biographies

In present chapter we collect short biographies of the speakers at the "Ising lectures" in Lviv over the last 20 years. The information is given in alphabetical order according to the surname of the speaker. The biographies consist of a brief summaries of the speakers personal background information, education, honours and main interests.

Bohdan Andriyevsky

Bohdan Andriyevsky was born in Kyiv, Ukraine, 29 November 1950. He received his PhD from the Ivan Franko Lviv State University in 1980 and completed his Doctoral of Science degree here in 1996. Since 2001 Andriyevsky has been a professor at the Koszalin University of Technology in Poland.

Between 2005–2011 he was involved in the experimental studies of optical properties of crystals using the ellipsometry method and synchrotron radiation in the synchrotron BESSY II, Berlin, Germany. Between 2007–2017 has was involved in the theoretical density functional theory (DFT) based computational studies of structural stability and physical properties of crystals with scientists from Max-Planck Institute for Solid State Research in Stuttgart, Germany.

His main research interests concern the studies of crystals and surfaces using the DFT computational methods, crystals with phase transitions, optical properties of crystals. He is an author of about 250 articles in journals and conference materials.

Ostap Baran

Ostap Baran was born in Lviv on 30 November, 1967. He graduated from the Faculty of Physics at Lviv State University in 1992.
Since then he has worked at the Institute for Condensed Matter Physics NAS of Ukraine. In 2002 Ostap defended his PhD Thesis "Cluster approach to investigation of pseudospin models". Between 2003 and 2014 he delivered a course of lectures "Computer technology of designing" at Lviv National Academy of Arts.
The main scientific interests of Ostap Baran include the theory of order-disorder type ferroelectrics, Ising models with arbitrary spin, quantum antiferromagnets on 2D frustrated lattices and quantum phase transitions.

Bertrand Berche

Bertrand Berche was born on May 6, 1963 in Metz, France. He obtained his MSc in physics and applied physics in 1987 from Henri Poincaré University of Nancy, later completing his PhD studies here in 1991.
Since then, B. Berche has remained at the same university, which in 2009 became a part of the University of Lorraine, as Maître de Conférences. He habilitated in 1997 becoming a full professor in 1998. Between 2008–2011 B. Berche was President of the Condensed Matter Section of the National Council of Universities, France. In 2008–2013 he was Director of the Physics Department at the University of Lorraine, and since 2007, has been involved in the organization of the French-German Doctoral College which subsequently became the $\mathbb{L}^4$–collaboration between Leipzig, Lorraine, Lviv and Coventry. He has advised and co-advised ten PhD students and has been awarded the degree of Doctor Honoris Causa of the Institute of Condensed Matter Physics of Lviv in 2016. His research concerns mainly statistical physics, theoretical condensed matter physics and history of sciences.

Christophe Chatelain

Christophe Chatelain was born on 23 July, 1974 in Epinal (France). He received a MSc in physics from the university of Nancy, now part of the university of Lorraine. Under the supervision of Bertrand Berche, he studied by Monte Carlo simulation the influence of disorder on the phase transition in lattice spin models, in particular the Potts model. He defended his PhD in June 2000, before leaving to Leipzig to work with Wolfhard Janke. Hired by the university of Lorraine in 2001, Christophe Chatelain developed research activities in statistical physics in various directions, particularly in the study of aging phenomena in quenched systems and of random classical spin models as well as quantum spin chains. During the academic year 2012–2013, he was a visiting professor at the Indian Institute for Science, Education and Research (IISER) at Thiruvananthapuram (India). He defended his habilitation to supervise research in December 2012. He also enjoys giving scientific lectures to a broad audience.

Bernat Corominas–Murtra

Bernat Corominas–Murtra is a polymath who has a degrees in both physics and linguistics. He completed a MSc in mathematics and his PhD at the Universitat de Barcelona. His research activities began at the ICREA-Complex Systems Lab (Universitat Pompeu Fabra), headed by Professor Ricard Sole. He has collaborated with several research institutions: the Fundamental Center for Living Technology (University of Southern Denmark), the Information and Autonomous Systems Research Group (University of the Basque Country) and the Centre for Theoretical Linguistics (Universitat Autónoma de Barcelona). He has also actively contributed to the “Mesomorph” project, the “ComplexDys” project and the NWO project “Dependency in Universal Grammar”. He is currently working at the Section for Complex Systems Research (Medical University of Vienna), headed by Professor Stefan Thurner. His current activities range from the exploration of the behavior of artificial societies to the foundations of complexity and statistical mechanics.

Oleg Derzhko

Oleg Derzhko was born in Lviv, Ukraine, 1960. He graduated from Lviv Ivan Franko State University in 1982. Later, in 1988 he defended his PhD thesis and habilitated in 2004.
During the years between 2003 and 2016 he was the head of the department for the Theory of Model Spin Systems at ICMP NAS of Ukraine, Lviv, which in 2016 amalgamated with the department of Quantum Statistics. Since 2016 he has continued to remain the head of this newly created department. He is also currently lecturing at the Lviv Ivan Franko National University and Lviv Polytechnic National University.
Oleg Derzhko's main research interests lie in classical and quantum spin systems, properties of ferroelectric materials, spin glass and liquid crystals.

Victor Dotsenko

Victor Dotsenko is a Physicist that was born 13 July 1957 in Poltava, Ukraine. He was educated at the Moscow Physical Technical Institute between the years of 1974–1980. He completed his Doctor of Science degree in physics and mathematics in 1994.
After completing his studies at Moscow Physical Technical Institute he began his research career at the Landau Institute for Theoretical Physics in Moscow where he stayed until 1999. He then moved to University Pierre and Marie Curie in Paris, France where he currently holds a position as a leading researcher.
His scientific interests mainly concern the study of statistical physics of disordered systems.

Iddo Eliazar

Iddo Eliazar is a graduate of Tel Aviv University (TAU) where he obtained a BSc Summa Cum Laude in mathematics and statistics; MSc Summa Cum Laude in operations research and a PhD in applied probability. Eliazar has held postdoctoral positions at Cambridge University, faculty positions at TAU and Bar Ilan University. He joined the Holon Institute of Technology in 2006, as an Associate Professor of Stochastics and Operations Research.

His research focuses on stochastics: the quantitative modeling and analysis of complex systems incorporating a high level of randomness. He has published over 100 papers, collaborating with world-renowned scientists. He is the recipient of the BSc Nimrod Lapid prize (1992), MSc Nimrod Lapid prize (1994), PhD Wolf prize (1997), PhD Blecher prize (1998) and the HIT award for excellence in research (2009–2012). He has served as the Academic Secretary of the Israeli Operations Research Society (2006–2008) and was on the Advisory Panel of the Journal of Physics A (2013–2014).

Reinhard Folk

Reinhard Folk was born in Neuendettelsau, Germany, on 29 April 1945. He completed his doctoral degree at Vienna University in 1973.

In 1973 Folk took up a research position within the Institute for Theoretical Physics at the University of Linz. It was here he become a Senior Associate Professor in 1984 eventually retiring from this position on 1 December 2009. Between 1986–1987 he also held the position of Visiting Associate Professor (Chair of Prof. Wölfle) at the Physics Department of the Technical University Munich, Germany.

His research interests lie in condensed matter physics; phase transitions; the history of physics. During his distinguished career he was awarded the Walter Schottky Prize of the German Physical Society (together with Volker Dohm) in 1982 and a Degree 'Doctor honoris causa' by the Institute for Condensed Matter Physics of the National Academy of Sciences of Ukraine, Lviv in 2009.

Serge Galam

Serge Galam is a French physicist specialized in disordered systems. In 1981 he completed his PhD at the University of Tel Aviv. A senior researcher at the National Center for Scientific Research, in 2014 he joined the CEVIPOF becoming the first physicist there. In the eighties he envisioned and initiated sociophysics, which today is a flourishing field of research. Besides major contributions in condensed matter and statistical physics he has produced a series of founding models to investigate social and political behavior. His numerous works include voting in hierarchical systems, group decision making, the stability and fragmentation of alliances among countries, minority opinions spreading, rumor phenomenon, radicalization, terrorism, and opinion dynamics. His models have yielded successful predictions in the 2005 French referendum on the European constitution and the Trump election. In 2002 he warned against the likelihood of the Brexit scenario.

Rudolf Hanel

Rudolf Hanel is an Austrian theoretical physicist that was born on 5 May, 1968 in Allabad, India. He completed his PhD in Theoretical Physics at the University of Vienna in 1999.

Since completing his PhD he has worked at the Department of Diagnostic Radiology, University of Vienna where he developed an experimental Virtual Endoscopy tool and the Department of Theoretical Physics at the KU Leuven working on the statistical properties of neural networks. He has also held positions at the Department of Biomedical Engineering and Physics (University of Vienna) working on medical robotics and with VisionLab, Department of Physics, University of Antwerp, working mainly on MR image enhancement. Since 2007 has been a member of the Complex Systems Research Group at the Medical University of Vienna.

Hanel's current interests aim at a thorough understanding of non equilibrium processes, their thermodynamic properties and associated phase transitions, and tipping phenomena.

Bohdan Hnatyk

Bohdan Hnatyk was born 22 January 1952 in vil. Berezhanka, Ternopil region, Ukraine. After graduating from the Lviv University in 1974 he started his research activities at the Institute for Applied Problems of Mechanics and Mathematics NASU. He obtained both his PhD (1983) and Doctoral degree (1997) from the Main Astronomical Observatory NAS of Ukraine.
Since 2001 Hnatyk has been the Director of the Astronomical Observatory of Taras Shevchenko National University of Kyiv becoming a Leading scientific researcher in 2008. He has been a visiting scientist at the Laboratori Nazionali del Gran Sasso of the Istituto Nazionale di Fisica Nucleare and the Astronomical Observatory of the Jagiellonian University.
His research interests include high energy astrophysics, relativistic astrophysics, astrophysics of cosmic rays, cosmology. He has published more than 70 papers. He was the Laureate of the Barabashov prize of NAS of Ukraine in the field of astronomy in 1999.

Yurij Holovatch

Born on 16 June, 1957 in Lviv, Yurij Holovatch graduated from the Ivan Franko University of Lviv, 1979 and completed his PhD at the Bogolyubov Institute for Theoretical Physics in Kyiv, 1984 under the supervision of Prof. I. Yukhnovskii and Prof. I. Vakarchuk. He received the Doctor of physical and mathematical sciences degree in 1998 and became Professor in 2005. Since 1990 he has worked at the Institute for Condensed Matter Physics, NAS of Ukraine where he founded the laboratory for statistical physics of complex systems (2010). He is Co-Director of the $\mathbb{L}^4$ Collaboration (Leipzig-Lorraine-Lviv-Coventry). He was elected a full member of the Shevchenko Scientific Society in 2006 and a corresponding member of the National Acad. Sci. of Ukraine in 2015. He was invited professor at: Ivan Franko National University of Lviv, Johannes Kepler Universität Linz (Austria), Université Henri Poincare (Nancy, France), Ukrainian Catholic University (Lviv). His scientific interests include the study of complex systems, physics of macromolecules, phase transitions and critical phenomena, complex networks, sociophysics and history of science.

Myroslav Holovko

Myroslav Holovko was born on 29 October 1943 in the village Cherniyevi of Ivano-Frankivsk region. He graduated from Ivano-Frankivsk Pedagogical Institute (1965) and completed his postgraduate studies in Ivan Franko Lviv State University (1969).
Myroslav Holovko worked in the Lviv department and later in the Lviv Division of Statistical Physics of the Institute for Theoretical Physics of the Academy of Sciences of Ukrainian SSR between 1969-1990 (Institute for Condensed Matter Physics NAS Ukraine since 1990). He received his PhD degree in 1970 and his Doctor of Sciences degree in 1981.
Since 1985, Holovko has held a Professorship degree and in 2003 became a corresponding member of NAS Ukraine. He is author and co-author of approximately 450 scientific papers.

Janusz Hołyst

Janusz Hołyst was born 19 November, 1955 in Elblag, Poland. He graduated with a MSc in physics (1979) and completed his PhD in solid state physics (1985) at the Warsaw University of Technology. He later received his Habilitation in theoretical physics from WUT (1993).
Hołyst has remained at WUT holding various research positions becoming a Full Professor in 2007. He is currently the Director of the Center of Excellence for Complex Systems Research, Warsaw University of Technology. He has been a visiting scientist at a number of institutions: Max Planck Institute for Physics of Complex Systems, Dresden, Germany, Institute of Theoretical Physics ETH Zürich and Institute of Solid State Physics, Technical University Darmstadt. He is also member of the Polish Physical Society, German Physical Society, and European Physical Society.
Hołyst has over 120 publications with his research interests lying mainly in the physics of complex systems, nonlinear dynamics, application of physics in social and economical sciences. He is an Associate Editor of European Physical Journal B and a Referee for a number of notable Physical journals.

Jaroslav Ilnytskyi

Jaroslav Ilnytskyi was born on 7 October 1963 in Ukraine. He graduated Lviv National University in 1985 and received PhD degree in 1994 (sup. I.Yukhnoskii). Holder of personal research fellowships in 1995 and 1996. He conducted research in the University of Durham, UK (1999–2003) and Potsdam University, Germany (2005–2008). After habilitation in 2010, he became a Leading Researcher at the ICMP and teaches in “Lviv Polytechnic”, both in Lviv, Ukraine.

Research interests cover computer simulations of lattice models in magnetic systems, liquid crystals and in epidemiology, molecular dynamics simulations of photo-active macromolecules and of simple and complex fluids in pores. He collaborates with Leibniz IPF in Dresden, Potsdam University, M. Curie-Sklodowska University, as well as with his colleagues from both the ICMP and “Lviv Polytechnic”. He is a member of the Editorial Board of the “Condensed Matter Physics” journal, of the Ukrainian Physical Society and of the Shevchenko Scientific Society.

Wolfhard Janke

Wolfhard Janke was born 11 December, 1955 in Berlin, Germany. He studied physics and mathematics at Free University Berlin. In 1985 he received a PhD in physics under the supervision of Prof. Hagen Kleinert. His dissertation was awarded the Joachim-Tiburtius Award. After his Habilitation in 1990 at FU Berlin and subsequent postdoctoral stays at Florida State University, USA, with Prof. B. Berg and at Forschungszentrum Jülich, Germany, with Prof. H. Herrmann he held a Heisenberg Professorship at Mainz University in Prof. K. Binder’s group. Since 1998 he has been a Full Professor for Computer-oriented Quantum Field Theory at Leipzig University. Between 1999–2002 he was the Director of the Institute for Theoretical Physics and Vice-Dean of the Faculty of Physics and Geosciences. Since 2011 he has been the Director of the Institute for Theoretical Physics. He is an “International Visiting Professor” of Coventry University, England, and since 2015 an Adjunct Professor of The University of Georgia, USA. His research interests focus on the statistical physics of polymers, phase transitions and critical phenomena, materials with quenched random disorder, and low-dimensional quantum spin systems.

Dragi Karevski

Dragi Karevski was born 18 December 1970, in France. He studied in Longwy, France, where he received his BSc. He later completed his MSc in theoretical physics (1994) at the Henri Poincaré University of Nancy where he also finished his PhD studies under the supervision of Loïc Turban in 1996.
Since completing his PhD he has been a member of the Groupe de Physique Statistique, at the University of Lorraine, becoming a full professor in 2011, and an associate professor at the University of Luxemburg in 2009. Before this he was an assistant professor at the University Henri Poincaré (1997–2010) and has been a visiting professor at the Free university of Berlin, at the Forschungszentrum in Jülich, at the University of Buenos Aires and the University of San Paulo.
His main scientific interests are in quantum non-equilibrium dynamics, quantum quenches in closed and open systems, quantum phase transitions in inhomogeneous/disordered systems.

Ralph Kenna

Professor Ralph Kenna is a statistical physicist who specialises in critical phenomena and sociophysics. Born in Athlone, Ireland, on 27 August 1964, he completed his PhD at the Karl-Franzens-Universitat Graz, Austria, in 1993.
Ralph Kenna was a Marie Curie Research Fellow at the University of Liverpool (1994- 1997) and Trinity College Dublin (1997-1999) where he lectured until 2002. He joined Coventry University in 2002 where he co-founded the Applied Mathematics Research Centre. He founded the Statistical Physics Group at Coventry and is Co-Director of the $\mathbb{L}^4$ Collaboration (Leipzig-Lorraine-Lviv-Coventry).
His research concerns the statistical physics of phase transitions and complex systems. Ralph Kenna has generated over 100 published papers, has given a similar number of presentations internationally, and been awarded over 1M in grant income. He is an editor for Condensed Matter Physics and Advances in Complex Systems as well as the Springer book-series Simulating the Past.

Sigismund Kobe

Proffesor Sigismund Kobe was born 1940 in Zella-Mehlis, Germany. He completed his undergraduate studies in nuclear physics and physics at the Technical University of Dresden in 1965.
In 1965 he took up a position as an assistant at the Institute of Theoretical Physics, Technical University Dresden completing his PhD thesis in 1971 entitled "Application of Effective Field Methods on Amorphous Ferro- and Antiferromagnetics". He later received habilitation in 1988. He became a professor in the theory of disordered solids in 1992 at the Institute of Theoretical Physics, Technical University Dresden where he retired in 2006.
His research interests included the study of the theory of amorphous magnetism, complex optimization of systems with competing interactions (Ising systems, neural networks, protein folding) and history of the Lenz-Ising model.

Svyatoslav Kondrat

Svyatoslav Kondrat is a theoretical and computational physicist who works in various fields of condensed matter physics, such as critical phenomena, ionic liquids in nano-confinement, diffusion and reactions in biological systems, etc. Born in Lviv, Ukraine, he graduated from Lviv National University, and completed his MSc degree by studying polyelectrolytes and surfactant systems with Professor Holovko. He obtained a PhD at the Institute of Physical Chemistry in Warsaw (Poland) by working with Professor Poniewierski on nematic liquid crystals at inhomogeneous substrates. He worked as a post-doctoral fellow and a research associate at Max-Planck Institute in Stuttgart and Stuttgart University (Germany), Imperial College London (UK) and Forschungszentrum Julich (Germany). Recently he accepted an assistant professor position in the department of complex systems at the Institute of Physical Chemistry in Warsaw, where he focuses on biophysical and chemical physics problems.

Yuri Kozitsky

Born in Kremenchuk, Ukraine on 21 April 1949. Yuri Kozitsky graduated from Physical Faculty of Lviv State University in 1972 and defended his PhD supervised by Prof. I. Yukhnovskii in 1981.
Kozitsky worked at the Chair of Higher Mathematics of Lviv Trade-Economic Institute between 1982 and 1996; where he was the head of Chair from 1984 onwards. In the year of 1992 he received a Doctor of Science degree. Since 1996 he has been a professor at the Institute of Mathematics of Maria Curie-Skłodowska University in Lublin, Poland.
His main scientific interests include the study of mathematical methods of quantum physics; dynamics of interacting particle systems and random structures. He has also coordinated a number of international research project, including EU Project STREVCOMS PIRSES-2013-612669 and coauthored a monograph Quantum Anharmonic Crystal: A Path Integral Approach, EMS Tracts in Mathematics, 8, Zurich, 2009.

Bohdan Lev

Bohdan Lev was born in the Gubakha village of the Permskii Krai of Russia, on 26 August 1952. In 1981, he received a PhD from the Institute of Physics of the NAS of Ukraine. In 1992, he received a Doctor of Science degree in physical and mathematical sciences from the Bogolyubov Institute for Theoretical Physics NAS of Ukraine.
In 2002, Lev achieved Professorship later becoming the Head of the Department of Synergetics at the Bogolyubov Institute for Theoretical Physics of the NAS of Ukraine in 2007. He was elected a Corresponding Member of the NAS of Ukraine in 2009. He has on numerous occasions been invited as a visiting scientist by leading research centers: York University (Canada), Japan Science and Technology Corporation (Japan), University of Helsinki (Finland).
Bohdan Lev has over 140 scientific publications with his main research interests residing within the fields of Statistical Physics, Semiconductor Physics and the physics of colloidal liquid crystals.

Roman Levitskii

Roman Levitskii was born on 6 January, 1943 in the village Cherche, Ivano-Frankivsk region, Ukraine.
He graduated from the Physics Department of Ivan Franko Lviv State University in 1965 and obtained his PhD degree in 1971. In 1990 Levitskii defended his doctoral thesis "Statistical theory of quasi-spin systems with basis taking into account of short-range interactions". In 1997 he was conferred the rank of professor. Between 1990–1995 he was Head of the Laboratory of Theory of Model Spin Systems of Institute for Condensed Matter Physics of the NAS of Ukraine. Between 1995–2003 he was Head of Department of Theory of Model Spin Systems of this Institute. Between 1990 and 1999 he was Head of the Scientific Coordination Council in physics of the Western Scientific Centre of the National Academy of Sciences of Ukraine. Since 2003 Roman Levitskii has been on the position of a leading researcher.
His research interests focus on the theory of pseudo-spin systems and ferroelectric materials.

Carlos Mejia-Monasterio

Carlos Mejia-Monasterio was born in Acapulco, Mexico, on 11 February 1971. He completed his undergraduate studies in theoretical physics at the National University of Mexico and obtained his PhD degree in 2001.
Since 2010 he has been a professor of physics at the School of Agricultural, Food and Biosystems Engineering of the Technical University of Madrid and now has more than 15 years of professional experience.
He has published over 60 scientific articles mainly on non-equilibrium statistical mechanics, transport phenomena in open classical and quantum systems, dynamical systems and stochastic systems. He is member of the National System of Researchers of Mexico, the Royal Spanish Society of Physics and the European Physical Society.

Roman Melnyk

Roman Melnyk was born 14 June, 1973. In 1995 he completed a MSc in physics at Ivan Franko Lviv National University, Ukraine. During 2004 he defended his PhD in Theoretical Physics at the Institute for Condensed Matter Physics of the NAS of Ukraine, Lviv.

He currently holds positions as a senior researcher and scientific secretary at the ICMP. His main research areas and interests include the statistical mechanics of condensed matter and critical phenomena in simple and complex fluids.

Ralf Metzler

Born in Neuenburg/Wurtt, Germany, 13 October 1968, Ralf Metzler completed his PhD (1996) at the University of Ulm. Subsequently R. Metzler has taken up postdoctoral studies at the Tel Aviv University (1998–2000) and MIT (2000–2002). Previous faculty positions include NORDITA, Copenhagen (2002–2006), University of Ottawa (2006–2007), Technical University of Munich (2007–2011). He presently resides as the Chair professor for Theoretical Physics at the Institute of Physics & Astronomy, University of Potsdam.

His scientific interests include the study of stochastic processes, anomalous diffusion, soft matter, biological physics and gene regulation. He has published over 240 publications and is an editor for journals of Physics A, Scientific Reports, Physical Review E, Journal of Biological Physics. Honors include Finland Distinguished Professor (2010–2015), DFG Emmy Noether fellow (2000–2002), MINERVA Amos de Shalit fellow (1998–2000), Alexander von Humboldt Feodor Lynen fellow (1998). He received the 2017 SigmaPhi Prize for 'seminal contributions to statistical physics'.

Alla Moina

Alla Moina was born in 1971 in Boryslav, Lviv region. She graduated from Lviv University in 1993 defending her doctoral dissertation in 1998. Currently she is a senior research fellow in the Institute for Condensed Matter Physics of the NAS of Ukraine, Lviv.
Her current scientific interests include the theory of order-disorder type ferroelectrics, crystals with hydrogen bonds, in particular, the effect external factors such as pressure or electric field have on phase transitions and physical properties in such systems.

Dominique Mouhanna

Dominique Mouhanna is Professor of Physics at the University Pierre et Marie Curie, Paris 6. He was born on 1967 in Paris, France. He received his PhD in physics "Theories des champs des antiferromagnetiques quantiques et classiques" from the University Pierre et Marie Curie in 1994 and his habilitation to conduct researches "Autour des systèmes magntiques frustrés" from the University Pierre et Marie Curie in 2004. He joined the University as an Assistant Professor of Physics in 1994 and was promoted to Professor in 2011. His research concerns the study of the critical and long range behaviours of systems coming from statistical, condensed matter, and soft matter physics. His approach, based on both perturbative and non-perturbative field theoretical techniques, have lead to contributions in solving longstanding problems concerning the physics of frustrated magnets, polymerized membranes and disordered spin systems.

Ihor Mryglod

Ihor Mryglod was born on 26 May 1960 in Kozliv, Ternopil region (Ukraine). He graduated from Physics Department of Ivan Franko Lviv State University in 1982 and obtained his PhD in 1988.
In 2000 Mryglod defended his doctoral thesis "Statistical theory of collective excitations in fluids: Generalized collective mode approach". In 2012 he was elected as a real member of the NAS of Ukraine for specialization "Physics of liquid state".
Since 2006 Ihor Mryglod has been the Director of the Institute for Condensed Matter of the NAS of Ukraine.
His main research interest relates to phase transition phenomena, non-equilibrium statistical theory, and liquid dynamics.

Sergei Nechaev

Sergei Nechaev works at the edge of statistical physics, topology, probability theory and biophysics. He was born 9 July 1962 in Moscow, Russia and graduated from the Moscow State University in 1985 where also he completed his PhD in Physics. He later received his Doctor of Sciences degree in 1996. Sergei has held positons as a researcher at the Institute of Chemical Physics, Russian Academy of Sciences (1985-1991), Landau Institute for Theoretical Physics, RAS (1991-1998), CNRS-Universite Paris XI, lab. LPTMS where he currently holds a position as a Director since 2008.
He is also currently a Director of the CNRS mathematical Laboratory Poncelet at the Independent University (Moscow).
Sergei's main research interests and contributions are on knot statistics in macromolecules which have served as a basis for a new area called "statistical topology". He was also the first who proposed using conformal approach to describe the optimal embedding in 3D of exponentially growing squeezed surfaces, like plants leaves. He has been awarded numerous grants and in 1986 was awarded The Gold Medal of the USSR Ministry of Educations in the All-Union competition in mathematics among postgraduate students.

Flavio Nogueira

Flavio Nogueira is a German-Brazilian physicist specializing on the application of quantum field-theoretic methods to condensed matter systems.
After his PhD in Brazil he worked as a researcher at the Center for Theoretical Physics of the Ecole Polytechnique, Palaiseau, France before joining the Institute for Theoretical Physics of the Free University Berlin in 2000, where he worked as a lecturer and researcher until 2011. After 2011 he has worked at the Faculty of Physics and Astronomy of the Ruin-University Bochum. Since 2015 he has been a research associate at the Leibniz Institute for Theoretical Solid State Physics in Dresden.
His recent publications and research interests deal mostly with topologically protected quantum states of matter.

Alexandr Olemskoi

Alexandr Olemskoi was born on September 19, 1949 in the village of Ekatirinovka, Voronezh province in Russia. He graduated from the Voronezh Polytechnical Institute and defended his PhD in 1977. In 1987 at the Moscow State University he defended his thesis for the degree of a Doctor of physical and mathematical sciences. In 1988 he started working in the Sumy division of the Institute of Physics of Metals, NAS of Ukraine (now – the Institute of Applied Physics). Since 1995 he has been the head of the Chair for Electronics of Sumy State University and in 2006 he was appointed head of the Laboratory of Microstructural Research of Reactor Materials of the Institute of Applied Physics.
Olemkoi was awarded the title of Soros professor and Pekar Prize of the Presidium of the NAS of Ukraine. He received the honorary title of a Recipient of the Order of Merit in Science and Technology. The field of scientific interests of Olemskoi concerns many aspects of statistical physics, such as fractals in condensed matter, spin glasses and complex systems.
He passed away on August 3, 2011.

Igor Omelyan

Igor Omelyan was born 20 December, 1961 in Lviv. In 1984 he graduated from Ivan Franko Lviv State University. He started his scientific work in the Lviv Division of Statistical Physics of the Institute for Theoretical Physics. In 1990 this division transformed into the Institute for Condensed Matter Physics NAS of Ukraine, where Igor currently holds a research position. His scientific interests and collaboration include investigations on non-equilibrium processes in dense gases and liquids (1986–1999, together with D.Zubarev, V.Morozov and M.Tokarchuk); development of the generalized collective mode approach for simple and polar fluids (1994–2000, with I.Mryglod, M.Tokarchuk); construction of numerical algorithms for molecular dynamics (MD) simulations (1998–2012); study of dynamical and critical behaviour in spin liquids (2000–2009, with R.Folk, I.Mryglod); elaboration of the integral equation theory(2003–2005, with F.Hirata, A.Kovalenko); combination of MD with the molecular theory of solvation for computer simulations of proteins (2013–2017, with A.Kovalenko).

Oksana Patsahan

Oksana Patsahan was born 15 February, 1956 in Kremenets, Ternopil region, Ukraine. She graduated from Physical Department of Ivan Franko Lviv State University in 1978 and received her PhD from Kiev State University in 1988. She completed her Doctoral of Sciences degree in theoretical physics in 2008.

Since 1990 she has held a number of research positions at the Institute for Condensed Matter Physics of the NAS of Ukraine, Lviv, as a Leading Researcher since 2009. She is a member of the Learned Council of the Institute for Condensed Matter Physics, the Shevchenko Scientific Society and Ukrainian Physical Society.

Her research interests include statistical mechanics of multicomponent continuous systems, theory of phase transitions and critical phenomena, theory of ionic fluids, theory of liquids under confinement.

Natalija Pavlenko

Natalija Pavlenko was born 18 May, 1968 in Ternopil region, Ukraine. She defended her PhD in theoretical and mathematical physics at ICMP NAS of Ukraine (1999).
Pavlenko was awarded an Alexander von Humboldt Fellowship (Hannover University, 2000-2001). She held Postdoc positions at the Technical University of Munchen, Garching (2001–2003), and the Institute of Physics, University of Augsburg (IPUA) (2003–2006). In 2006–2011 she was a Senior Researcher at the ICMP whilst also being a Research Associate at the IPUA and the Max Planck Institute for Solid State Research at Stuttgart (Germany). In 2014 she was the Principal Investigator of the DFG-TRR80 project at the IPUA.
Natalija Pavlenko's scientific interests were mainly in theoretical condensed matter physics, with particular emphasis on the oxide interfaces, interface/surface magnetism, hydrogen conductors, and surface phenomena.
She passed away on 1 August, 2014.

Geoff Rodgers

Geoff Rodgers gained his BSc in mathematics at Imperial College London and a PhD in disordered systems theory at Manchester University.
In 1988 he won a European Postdoctoral Fellowship from The Royal Society to work at the Service de Physique Théorique, CEN-Saclay, France. He joined Brunel University (London) in 1989 as a Physics lecturer, becoming a Reader in mathematical physics in 2001 and Professor of theoretical physics in 2003. The following year he joined Brunel's new Graduate School and later became its Dean. He was appointed Pro-Vice-Chancellor for Research in 2007 and is currently Deputy-Vice-Chancellor for Research and Innovation.
Rogers has many publications, conference presentations and grants. His research interests lie in the fields of complex networks and the application of statistical mechanics to problems in social science, economics and finance. He is a Fellow of both the Institute of Physics and the Higher Education Academy. He is also a Visiting Professor at the University of Havana.

Andrij Rovenchak

Andrij Rovenchak was born 12 November 1976 in Lviv. He graduated from the Faculty of Physics, Ivan Franko National University of Lviv, in 1998. He completed his PhD in 2003 and later habilitated in 2016, with his thesis titled: Bose-statistics and fractional types of statistics in the many-body theory and related problems.

Rovenchak currently holds a position as an Associate Professor at the Department for Theoretical Physics, Ivan Franko National University of Lviv.

His research interests include statistical physics; Bose–Einstein condensation; systems obeying fractional statistics; quantitative methods in linguistics; studies of writing systems; history of science. He has published 75 articles and 10 books, in particular: Physics of Bose-systems (Lviv University Press, 2015); African writing systems of the modern age: the Sub-Saharan region (Athinkra LLC, 2011; with J. Glavy).

Antonio Scala

Antonio Scala holds a MSc in physics and computer sciences at the University of Napoli “Federico II” and a PhD in condensed matter physics at Boston University.

He is currently a Professor of physics at the CNR Institute for Complex Systems at the University of Roma “La Sapienza”, associate professor at IMT Alti Studi Lucca and research Fellow at LIMS the London Institute for Mathematical Sciences. Together with G. D’Agostino, he is the organizer of the workshop series “Networks of networks” on systemic risk and infrastructural inter-dependencies.

His main areas of research are in statistical and computational physics; he has published papers on percolation, disordered systems, pattern formation, metastable liquids, glassy systems, energy landscapes, protein folding, complex networks, event-driven algorithms, Brownian simulations for hard-bodies, complexity in economics, network medicine, power grids and self-healing networks.

Frantisek Slanina

Born 1962, Frantisek Slanina graduated in solid state physics from Charles University, Prague. After graduating he remained with the University completing his PhD studies.
After completing his PhD he held a position with the University of Rome. He now holds a position within the Institute of Physics, Czech academy of Sciences, Prague.
His research interest revolve around the study of non-equilibrium statistical physics.

Ievgeniia Stashkova (Ivaneiko)

In the period between 1997–2002 Ievgeniia Stashkova studied at the Physical Department of Ivan Franko National University of Lviv. In 2002 she obtained her MSc under the supervision of Professor I.O. Vakarchuk.
Ievgeniia Stashkova has since February 2012, worked as a researcher at the Department of Elastomer in the Institute of Polymer Materials, Dresden, Germany.
Her main research interests include the modeling of dynamic-mechanical behavior of reinforced elastomers with the help of a multiscale approach.

Mykola Shpot

Mykola Shpot was born on August 2, 1960 in Lviv, Ukraine. In 1982 he obtained a MSc in radiophysics and electronics from Lviv State University. In 1990 he completed his PhD in theoretical physics at the Institute for Condensed Matter Physics of NAS Ukraine, Lviv, subsequently obtaining a permanent position within the Institute. Soon after, between February-April in 1991, he visited the CEN Saclay working with C. Bervillier. In 1993–1994 he was awarded a Fellowship by an Alexander von Humbold. This started his collaboration with H. W. Diehl at the University of Essen, which permanently continued until 2011.
Mykola Shpot's current research interests mainly concern the study of mathematics, in particular special functions. He has published papers with prominent contemporary mathematicians such as H.M. Srivastava, R.B. Paris and T.K. Pogány.

Ihor Stasyuk

Ihor Stasyuk was born on 23 September 1938 in Berezhany, Ternopil region, Ukraine. He graduated (1959) and defended his PhD (1963) at the Ivan Franko Lviv State University. He became a Doctor of Sciences in 1986, obtaining Professorship in 1987 and has been a Corresponding Member of NAS of Ukraine since 1995.
He has been the chief scientific researcher at Quantum Statistics Department of Institute for Condensed Matter Physics of NAS of Ukraine, Lviv since 2016. He has received awards from NAS of Ukraine for Scientific Achievements (2008), a Doctor Honoris Causa from the Bogolyubov Institute of Theoretical Physics of NAS of Ukraine (2011), and the Davydov Prize of NAS of Ukraine (2014).
His scientific interests include the study of mathematical methods in quantum statistics, many-particle fermion and boson systems with strong correlations, quantum lattice models, ion (proton) conductors and superionic crystals, phase transitions and field effects in hydrogen bonded ferroelectrics and Jahn-Teller crystals. These interests have resulted in his authoring and co-authoring over 200 scientific papers and four books.

Józef Sznajd

Józef Sznajd was born in Wroclaw, Poland, 21 February 1947. He completed his PhD at the Institute of Low Temperature and Structure Research, Polish Academy of Sciences (ILTSR PAS) in 1973 where he also received his Habilitation (1980) and professorship (1989). He is currently a head at ILTSR PAS. Other notable position include being a Humboldt fellow (1985–1986) and visiting professor (1991–1992) at the University of Cologne.
His research interests include the study of solid state theory, theory of phase transitions, low dimensional spin and fermion systems, applications of the real space renormalization group method to the study of low dimensional quantum spin systems and applications of statistical physics to sociological and political environments. In 1971 and 1978 he received the Award of Scientific Secretary of PAS. He is also a Member of the Presidium of the Physics Committee of the PAS.

Katarzyna Sznajd-Weron

Katarzyna Sznajd-Weron is a Professor in the Department of Theoretical Physics at the Faculty of Fundamental Problems of Technology, Wroclaw University of Science and Technology. Formerly she was head of the Complex Systems and Nonlinear Dynamics Division and UNESCO Chair of Interdisciplinary Complex Systems at the University of Wroclaw.
Her research focuses on applications of statistical physics (mainly simple lattice models and the theory of phase transitions) in a variety of complex systems, including social and biological. A model of opinion dynamics, developed by her in 2000 is known in literature as the Sznajd model and has been cited approximately 600 times (according to Web of Science). In 2007 she was awarded a prestige worldwide Young Scientist Award in Socio- and Econo-physics for her original contribution to the better understanding of open problems in socio-economic systems by means of physical methods.

Mikhail Tamm

Born in Moscow, Russia, 1977 Mikhail Tamm graduated from Moscow State University, 1999 and completed his PhD at the same institution in 2002. Since 2002 he has remained at Moscow State University taking up a position within the Faculty of Physics and is currently a Senior Research Associate within the department. Here, twice in 2010 and 2015 he won MSUs best young scientist award. On numerous occasions he has been a visiting scientist LPTMS, Universite Paris Sud, Orsay, France; LPTMC, University Pierre et Marie Curie, Paris, France; Theoretical physics group, Potsdam University, Potsdam, Germany, and Applied Mathematics Research Center, Coventry University, Coventry, UK.
His research interests concern complex systems, non equilibrium statistical mechanics, statistical physics of macromolecules, fractal globule, dynamics of polymers and biopolymers, anomalous diffusion, asymmetric simple exclusion process, random surface growth, random graphs and complex networks, Lifshitz localization.

Volodymyr Tkachuk

Volodymyr Tkachuk was born 1957 in Ivano-Frankivsk region to a family of teachers. In 1979 he graduated with distinction from Ivan Franko Lviv State University and began his scientific work in the field of phase transitions under the supervision of Professor I.R. Yukhnovskii in the Bogoliubov Institute for Theoretical Physics. In 1985 Volodymyr Tkachuk started working at the Ivan Franko Lviv State University. Here the young scientist studied thermodynamical functions and the dynamics of structurally disordered spin systems with his supervisor Professor I.O. Vakarchuk and obtained his PhD in 1990. His habilitation thesis was on the supersymmetry and exactly solvable problems in quantum mechanics for which he completed in 2005, becoming a Doctor of Science. V. Tkachuk become a Docent of the Department for Theoretical Physics in 1991. In 2006 he obtained his Professor degree. Since 2006 Tkachuk has held the position of Professor of the Department for Theoretical Physics at Ivan Franko National University of Lviv.

Ivan Vakarchuk

Ivan Vakarchuk was born on March 6, 1947 in the vill. of Stari Bratushany of Jedynets' district (now in Moldova). In 1974 he completed his PhD at the Lviv Department of statistical theory of condensed state of the Institute for Theoretical Physics of the NAS of Ukraine SSR (ITP). In 1980 he became one of the youngest doctors habilitated in physics within the USSR.

Between 1980–1984 he was the Head of the quantum statistics department in the Lviv Division of ITP. Between 1984–2015 he was the Head of Chair for theoretical physics of the Ivan Franko Lviv State (since 1999 National) University, being the Rector of this university in 1990-2014. From 2007 to 2010 he held the position of Minister of education and science of Ukraine.

His scientific interests include statistical physics, fundamental problems of quantum mechanics and quantum information, mathematical methods of theoretical physics, general relativity and theory of stellar spectra.

Oleg Vasilyev

Oleg Vasilyev was born on April 18, 1974 in Lipetsk, Russia. In 1997 he graduated from Moscow Institute of Physics and Technology, department of General and Applied Physics. In 2000 he completed his PhD thesis "Numerical Investigation of Diluted Spin Systems" at Landau Institute for Theoretical physics, Russia. Between 2000 and 2004 Vasilyev was a Junior Research Associate at Landau Institute. In 2004–2005 he was a Research associate (CNRS) at the Laboratory of Theoretical Physics of Condensed Matter, University Paris VI, France and in 2005–2006 he was a Research Associate (FNRS) at the Research Center for Molecular Modeling, University Mons-Hainaut, Belgium. Since 2006 he has been a Research Scientist at the Theory of Inhomogeneous Condensed Matter department, Max Planck Institute of Intelligent Systems, Stuttgart, Germany.

His research interests include numerical simulations in the statistical physics of phase transitions and complex systems.

Christian von Ferber

Christian von Ferber was born on May 15, 1961 in Göttingen, Germany. He completed his PhD at the University of Essen in 1993. His subsequent research was performed in several leading scientific centers, including Tel Aviv University, as a Fellow at the Minerva Foundation, and the universities of Dusseldorf and Freiburg. He was also a Guest Professor at the University of Linz and a Marie Curie Fellow at the University of Krakow. In 2003 he earned his Habilitation (Venia Legendi in Physics) at the University of Freiburg and now holds a privatdozent position in the university of Dusseldorf. Since 2006 he has worked at the Applied Mathematics Research Centre in Coventry University (UK), where he now holds a position as a Reader. The degree of Doctor Honoris Causa of the Institute of Condensed Matter Physics was conferred to Christian von Ferber in 2012. His scientific interests mainly concern soft matter physics and complex systems. Subjects of his analyses range from polymers, colloids and disordered magnets to transportation networks.

Martin Weigel

Martin Weigel was born on September 6, 1972 in Neustadt an der Weinstrasse, Germany. He graduated in 1998 with a Diploma from Mainz University, working with Kurt Binder and Wolfhard Janke on predictions of conformal field theory. He then moved to Leipzig for his PhD under the supervision on Wolfhard Janke, where he worked on the dynamical triangulations approach to quantum gravity and its relation to systems with disorder in statistical physics.
After his PhD award in 2002 summa cum laude and post-doc positions in Waterloo, Canada and Edinburgh, in 2007 he returned to Mainz with an Emmy Noether Junior Research Group, funded by the German Research Foundation. Since 2011 he has worked as a Lecturer in the Applied Mathematics Research Centre at Coventry University, becoming a Reader here in 2013.
Martin's research interests revolve around equilibrium and non-equilibrium phase transitions and critical phenomena, the advancement of methods in Monte Carlo simulations and optimization problems, and the physics of disordered systems.

Taras Yavors'kii studied physics at and earned his PhD from National University of Lviv, Ukraine, where he was developing the quantitative theory of critical phenomena in φ-4 field theories with complex symmetry order parameters.
He then worked as a Research Associate and a Post-Doctoral Fellow at Universities in Hanover (Germany), Waterloo (Canada), Mainz (Germany) studying quantum and classical spin models on highly geometrically frustrated lattices. He was also employed as a consultant on high-performance GPU computing in molecular dynamics at the University of Waterloo. In 2013, he was appointed a Lecturer in Mathematics at Coventry University.
He is an author of over 20 publications in peer-reviewed international professional journals garnering over 400 references to his work, including in journals of "Science" and "Nature".

Taras Yavors'kii

Helena Zapolsky is a Professor of theoretical physics at Rouen University in France. She received her PhD in 1991 at Institute for Theoretical Physics in Kiev, Ukraine and at the University of Rouen. In 1992 she was invited to the University of France as a Guest Researcher. In 1996 Helena Zapolsky joined the Group of Material Physics at the University of Rouen. It was here that she completed her Habilitation in 2007. The research area of Zapolsky covers a broad spectrum of fundamental problems within theoretical solid-state physics. Her main research interests lie in the theoretical understanding of the emergence of pattern formation and kinetics of phase transformation in different types of materials. Applying different mathematical and computational approaches to diverse problems in materials science, she developed an original multi-scale tool to model the non-equilibrium dynamics in complex systems.

Helena Zapolsky

4 | Gallery

Here we collect something more than pictures – memories! You can find photos from talks and discussions, coffee and tea breaks, traditional mountain hiking...

Figure 4.1: A group of physicists at the Conference MECO 22 (Szklarska Poreba, Poland, 1997) where an idea for the Ising lectures appeared. From left to right: Volodymyr Tkachuk, Oleh Derzhko, Jaroslav Ilnytskyi, Mai Suan Li, Yurij Holovatch, Rinat Mamin, Anatoliy Zaharov

Figure 4.2: Ising lectures 2003 (22 April 2003).

Figure 4.3: “Shall we start?” Yurij Holovatch, Ihor Mryglod, Andriy Baumketner (from left to right), during “Ising lectures 2014” (6 May 2014).

Figure 4.4: Antonio Scala about modern battle between Physics and Engineering (5 May 2015).

Figure 4.5: Waiting for the next session: Olesya Mryglod, Oksana Dobush, Olesya Krupnitska, Khrystyna Hnatenko and Petro Sarkanych (from left to right) (Lviv, 6 May 2014).

Figure 4.6: The beginning of the Ising lectures 2016 (Lviv, 17 May 2016).

Figure 4.7: Wolfhard Janke is lecturing for us (Lviv, May 17, 2016).

Figure 4.8: Serguei Nechaev and white roses from Victor Dotsenko (Lviv, May 18, 2016).

Figure 4.9: Martin Weigel lecture on using GPU's for MC simulations (Lviv, 17 May 2016).

Figure 4.10: Rudolf Hanel's talk on Entropy (Lviv, 19 May 2016).

Figure 4.11: Tea discussions: Yurij Holovatch, Martin Weigel and Victor Dotsenko (from left to right)(Lviv, 17 May 2016).

Figure 4.12: Dinner at Soborna Square (Lviv, 18 May 2016).

Figure 4.13: Participants of the Ising Lectures 2016 (Lviv, 19 May 2016).

Figure 4.14: Co-authors of Chapter 1: Yurij Holovatch, Reinhard Folk, Bertrand Berche and Ralph Kenna (from left to right) (Lviv, 14 June 2017).

Figure 4.15: Discussions at 303 room: Maxym Dudka, Reinhard Folk, Yurij Holovatch (from left to right) (Lviv, 15 June 2017).

Figure 4.16: Ihor Mryglod introducing Bertrand Berche(Lviv, 14 June 2017).

Figure 4.17: Fruitful scientific discussion between Yuri Kozitsky and Mykhailo Kozlovskii (Lviv, 15 June 2017).

Figure 4.18: During the break it is time to have a chat: Ihor Yukhnovskii speaks to Yuri Kozitsky, and Ihor Stasyuk talks with Ihor Mryglod (from left to right)(Lviv, 16 June 2017).

Figure 4.19: Yurij Holovatch receiving 'AMRC Fellow' Certificate: Robin de Regt, Christian von Ferber, Taras Yavors'kii, Martin Weigel, Yurij Holovatch, Ralph Kenna, Petro Sarkanych (from left to right) (Lviv, 14 June 2017).

Figure 4.20: After the lectures (Lviv, 15 June 2017).

Figure 4.21: Traditional Carpathian hiking (Slavsk, 12 June 2017).

Figure 4.22: Group photo from the Ising Lectures (Lviv, 14 June 2017).

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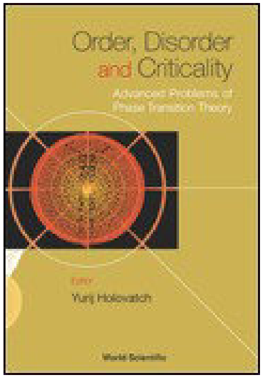

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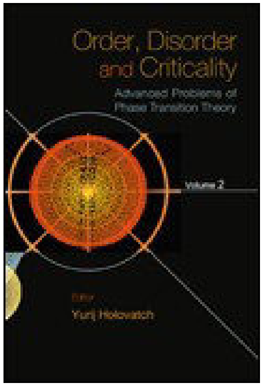

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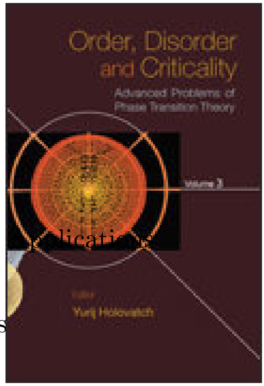

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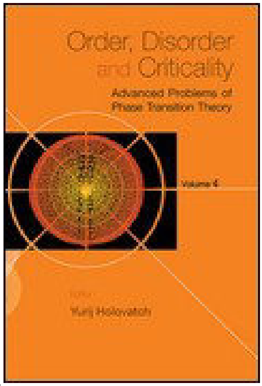
<u>**Contents:**</u>



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Ernst (Ernest) Ising
(10.05 1900, Cologne, Germany – 11.05 1998, Peoria USA)

In 1920 Wilhelm Lenz suggested a model for ferromagnetism to his student Ernst Ising, who solved it in his thesis for a 1D case in 1924. This work of Lenz and Ising marked the start of a scientific direction that, over nearly 100 years, delivered extraordinary successes in explaining collective behaviour in a vast variety of systems, both within and beyond the natural sciences.

In 1997 the first Ising lectures in Lviv were held. These lectures have evolved from a small local workshop on phase transitions and critical phenomena to the point where each year leading scientists from all over the world give talks on subjects related to complex systems with particular emphasis on statistical physics and its interdisciplinary practice. This book summarises the first twenty years of the Ising Lectures in Lviv.

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