

# THE INFLUENCE OF SPINS' STRUCTURAL FLUCTUATION ON THE SPECTRUM AND DAMPING OF SPIN WAVES IN AMORPHOUS FERROMAGNETS

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## Abstract

The relation between atomic momenta fluctuations and density fluctuations is obtained in frames of mean-field approximation. Using two-time temperature Green functions within Tyablikov approximation the equations for spin excitation energy and damping are obtained. The asymptotics of energy and damping in the long-wave limit are investigated and the anomalous behaviour of spin-wave stiffness constant is discussed.

## Introduction

In amorphous ferromagnets magnetic momenta of atoms do not have fixed values. Structure disorder leads to the differences in the mean momentum  $\langle S_i^z \rangle$  of atoms localized at different sites  $i$ . The first attempt to take into account the structural fluctuations of atomic momenta in order to explain the anomalous behaviour of spin-wave stiffness constant in amorphous ferromagnets, increasing with increasing external field, was done in ref. [1]. Kaneyoshi in [1] assumes that the fluctuations of each momenta are statistically independent of each other and their values are given by the Gauss distribution function. Note, that the atomic momenta fluctuations are entirely connected to the atomic density fluctuations.

The aim of this paper is to determine the relations between the atomic density fluctuations and the atomic momenta fluctuations and to investigate the influence of atomic momenta structural fluctuation on the spectrum and damping of spin waves in amorphous ferromagnets.

## 1 Green Functions of the structurally Disordered Heisenberg Model

Let us consider a structurally disordered system of  $N$  spins in the volume  $V$  which is described by the isotropic Heisenberg Hamiltonian

$$H = -\frac{1}{2} \sum_{i,j} J_{ij} \mathbf{S}_i \mathbf{S}_j - h \sum_j S_j^z, \quad (1.1)$$

where  $J_{ij} = J(|\mathbf{R}_i - \mathbf{R}_j|)$  is the exchange integral describing interaction between the  $i$ -th and  $j$ -th atoms,  $\mathbf{S}_i$  is the spin operator of the  $i$ -th atom,  $h$  is external magnetic field,  $\mathbf{R}_i$  are the coordinates of atoms, that are randomly distributed.

We use the two-time temperature Green function method for the investigation of the spin excitations. The Green function within Tyablikov approximation satisfies the equation of motion

$$(E - h) \ll S_l^+ |S_l^- \gg = 2\delta_{l,l'} \sigma_l + \sum_{j(\neq l)} J_{ij} \Theta(R_{ij} - a) \ll \sigma_j S_l^+ - \sigma_l S_j^+ |S_l^- \gg, \quad (1.2)$$

where  $S_l^\pm = S_x \pm iS_y$ ,  $\sigma_l = \langle S_l^z \rangle$ . We introduce the  $\Theta$ -function in the equation (1.2) to take into account during the operation of configurational averaging the fact that the minimum distance between atoms is  $a$  [2]. Traditionally, the second approximation is specific for disordered systems and means the neglecting of structural fluctuations of spin momentum  $\sigma_l \rightarrow \bar{\sigma}_l = \sigma$  where symbol  $(\dots)$  denotes the random configurational ensemble average. In our paper we do not perform this approximation and therefore we have a possibility to take into account the fluctuations of spin.

The spin excitation spectrum can be obtained from the configurationally averaged Green function  $\overline{\ll S_q^+ | S_{q'}^- \gg}$ , where  $S_q^\pm = \frac{1}{\sqrt{N}} \sum_{j=1}^N S_j^\pm e^{-i\mathbf{q}\mathbf{R}_j}$ .

For the Green function in the momentum space on the basis of (1.2) we obtain the equation

$$(E - E_0(q)) \ll S_q^+ | S_{q'}^- \gg = 2\sigma \delta(\mathbf{q} + \mathbf{q}') + 2 \frac{1}{\sqrt{N}} \Delta \sigma_{\mathbf{q}+\mathbf{q}'} + \quad (1.3)$$

$$+ \frac{N}{V} \sum_{\mathbf{k}} (J(|\mathbf{q} - \mathbf{k}|) - J(k)) \frac{1}{\sqrt{N}} \Delta \sigma_{\mathbf{q}-\mathbf{k}} \ll S_{\mathbf{k}}^+ | S_{\mathbf{q}'}^- \gg,$$

where  $\Delta \sigma_{\mathbf{q}} = \sigma_{\mathbf{q}} - \sigma \sqrt{N} \delta(\mathbf{q})$ ,  $\sigma_{\mathbf{q}} = \frac{1}{\sqrt{N}} \sum_{j=1}^N \sigma_j e^{-i\mathbf{q}\mathbf{R}_j}$ ,  $E_0(q) = \sigma \frac{N}{V} (J(0) - J(q))$ ,  $J(q) = \int d\mathbf{R} \Theta(R - a) J(R) e^{-i\mathbf{q}\mathbf{R}}$ ,  $\delta(\mathbf{q})$  is the Kronecker symbol.

Averaging (1.3) over possible realization of atomic configurations, the equation for averaged Green function can be written in the following form:

$$(E - E_0(q)) \overline{\ll S_q^+ | S_{q'}^- \gg} = 2\sigma \delta(\mathbf{q} + \mathbf{q}') + \quad (1.4)$$

$$+ \frac{N}{V} \sum_{\mathbf{k}} (J(|\mathbf{q} - \mathbf{k}|) - J(k)) \frac{1}{\sqrt{N}} \overline{\Delta \sigma_{\mathbf{q}-\mathbf{k}} \ll S_{\mathbf{k}}^+ | S_{\mathbf{q}'}^- \gg}.$$

Equation (1.4) contains a higher-order averaged Green function  $\overline{\Delta \sigma G}$ . One can write the equation for this function, multiplying  $\Delta \sigma_{\mathbf{k}'}$  by (1.3) and performing configurational averaging. These equations include  $\overline{\Delta \sigma \Delta \sigma G}$ . To solve these equations the decoupling of configurational averages is used

$$\overline{\Delta \sigma_{\mathbf{q}-\mathbf{k}} \Delta \sigma_{\mathbf{k}-\mathbf{k}'}} \ll S_{\mathbf{k}'}^+ | S_{\mathbf{q}'}^- \gg \approx \overline{\Delta \sigma_{\mathbf{q}-\mathbf{k}} \Delta \sigma_{\mathbf{k}-\mathbf{k}'}} \times \overline{\ll S_{\mathbf{k}'}^+ | S_{\mathbf{q}'}^- \gg},$$

where  $\overline{\Delta\sigma_{\mathbf{q}-\mathbf{k}}\Delta\sigma_{\mathbf{k}-\mathbf{k}'}} = \delta(\mathbf{q}-\mathbf{k}') \times \overline{\Delta\sigma_{\mathbf{q}-\mathbf{k}}\Delta\sigma_{\mathbf{k}-\mathbf{q}}}$ .

Thus, for averaged Green function in the momentum space we have finally

$$\ll S_{\mathbf{q}}^+ | S_{\mathbf{q}'}^- \gg = \delta(\mathbf{q} + \mathbf{q}') \frac{2\sigma + C(q, E)}{E - E_0(q) - \Sigma(q, E)} \quad (1.5)$$

where

$$C(q, E) = \frac{1}{V} \sum_{\mathbf{k}} \frac{(J(|\mathbf{q}-\mathbf{k}|) - J(k))}{(E - E_0(k))} \overline{\Delta\sigma_{\mathbf{q}-\mathbf{k}}\Delta\sigma_{\mathbf{k}-\mathbf{q}}} \\ \Sigma(q, E) = \frac{N}{V^2} \sum_{\mathbf{k}} \frac{(J(|\mathbf{q}-\mathbf{k}|) - J(q))(J(|\mathbf{q}-\mathbf{k}|) - J(k))}{E - E_0(k)} \overline{\Delta\sigma_{\mathbf{q}-\mathbf{k}}\Delta\sigma_{\mathbf{k}-\mathbf{q}}}.$$

## 2 The relation between atomic momenta fluctuations and density fluctuations

We start from the equation for mean atomic momenta  $\sigma_i$

$$\sigma_i = \frac{1}{2} \tanh \left\{ \frac{1}{2T} \left( \sum_j J(R_{ij}) \Theta(R_{ij} - a) \sigma_j + h \right) \right\}. \quad (2.1)$$

Note, that

$$\sum_j J(R_{ij}) \Theta(R_{ij} - a) \sigma_j = \frac{N}{V} J(0) \sigma + \frac{N}{V} \sum_{\mathbf{k}} J(k) e^{i\mathbf{k}\mathbf{R}_j} \frac{1}{\sqrt{N}} \Delta\sigma_{\mathbf{k}} \quad (2.2)$$

In linear approximation over  $\Delta\sigma_{\mathbf{q}}$  using (2.1) and (2.2) we obtain:

$$\Delta\sigma_{\mathbf{q}} = \frac{\sigma}{1 - \left( \frac{1}{4} - \sigma^2 \right) \frac{N}{V} J(q) \frac{1}{T}} \rho_{\mathbf{q}} \quad (2.3)$$

where  $\rho_{\mathbf{q}} = \frac{1}{\sqrt{N}} \sum_j e^{-i\mathbf{k}\mathbf{R}_j} - \sqrt{N} \delta(\mathbf{q})$  is the Fourier transform of the atomic density fluctuations,  $T$  is the temperature,  $\sigma$  satisfies the equation

$$\sigma = \frac{1}{2} \tanh \left\{ \frac{1}{2T} \left( \frac{N}{V} J(0) \sigma + h \right) \right\}. \quad (2.4)$$

Now, using (2.3) we can evaluate

$$\overline{\Delta\sigma_{\mathbf{q}-\mathbf{k}}\Delta\sigma_{\mathbf{k}-\mathbf{q}}} = \sigma^2 \tilde{S}(|\mathbf{k}-\mathbf{q}|) = \frac{\sigma^2}{\left\{ 1 - \left( \frac{1}{4} - \sigma^2 \right) \frac{N}{V} J(|\mathbf{k}-\mathbf{q}|) \frac{1}{T} \right\}^2} S(|\mathbf{k}-\mathbf{q}|) \quad (2.5)$$

where  $S(q) = \overline{\rho_{\mathbf{q}}\rho_{-\mathbf{q}}}$  is the structure factor of the amorphous material.

### 3 Energy spectrum and damping of spin waves

The equation for energy spectrum of spin waves can be obtained from the pole of averaged Green function (1.5)

$$E = E_0(q) + \frac{N}{V^2} \sum_{\mathbf{k}} \frac{(J(|\mathbf{q} - \mathbf{q}|) - J(k))(J(|\mathbf{q} - \mathbf{k}|) - J(k))}{E - E_0(k)} \sigma^2 \tilde{S}(|\mathbf{k} - \mathbf{q}|). \quad (3.1)$$

Extracting the damping in a standard way we have

$$\Gamma = \frac{N}{V^2} \sum_{\mathbf{k}} (J(|\mathbf{q} - \mathbf{q}|) - J(k))(J(|\mathbf{q} - \mathbf{k}|) - J(k)) \sigma^2 \tilde{S}(|\mathbf{k} - \mathbf{q}|) \delta(E - E_0(k)). \quad (3.2)$$

Beside the nonregularity in atoms' displacement equations (3.1) and (3.2) take into account the structural fluctuations of mean atomic momenta that is involved in the denominator of right hand side of (2.5).

The long-wave asymptotic of energy spectrum is given by

$$E = h + Dq^2, \quad q \rightarrow 0 \quad (3.3)$$

where  $D$  is the spin-wave stiffness constant

$$D = D_0 + \Delta \tilde{D}, \quad D_0 = -\sigma \frac{N}{V} J'(0), \quad (3.4)$$

$$\Delta \tilde{D} = -\sigma \frac{1}{2\pi^2} \int_0^\infty dk k^2 \tilde{S}(k) \left\{ J'(k) + \frac{2}{3} k^2 J''(k) + \frac{4}{3} k^2 \frac{(J'(k))^2}{J(0) - J(k)} \right\}, \quad (3.5)$$

$$\text{where} \quad J'(k) = \frac{\partial J(k)}{\partial k^2}, \quad J''(k) = \frac{\partial}{\partial k^2} \frac{\partial}{\partial k^2} J(k).$$

Let us assume that  $J(k)$  is localized in the vicinity of zero. Then we can perform the following approximation

$$\tilde{S}(k) \rightarrow \frac{1}{\left\{ 1 - \left( \frac{1}{4} - \sigma^2 \right) \frac{N}{V} J(0) \frac{1}{T} \right\}^2} S(k) \quad (3.6)$$

Substituting (3.6) into (3.5) we obtain

$$D = D_0 + \frac{1}{\left\{ 1 - \left( \frac{1}{4} - \sigma^2 \right) \frac{N}{V} J(0) \frac{1}{T} \right\}^2} \Delta D, \quad (3.7)$$

where  $\Delta D$  is given by (3.5) after  $\tilde{S}(k)$  is changed to  $S(k)$ .

As it follows from (3.7), the spin-wave stiffness constant will increase while applied field increases in the case  $\Delta D < 0$ . Thus, taking into account the structure fluctuations allows to explain the anomalous behaviour of spin-wave stiffness constant in amorphous ferromagnets.

Let us write the expression for the long-wave asymptotics of damping ( $q \rightarrow 0$ )

$$\Gamma = \frac{7}{12\pi} \frac{V}{N} D \frac{S(0)}{\left\{1 - \left(\frac{1}{4} - \sigma^2\right) \frac{N}{V} J(0) \frac{1}{T}\right\}^2} q^5. \quad (3.8)$$

Beside the scattering on the structure inhomogeneities spin excitations scatter on the spin momenta fluctuations as well, this fact leads to increase of damping. Momenta fluctuations disappear for  $T \rightarrow 0$  or  $h \rightarrow \infty$ , then  $\sigma \rightarrow 1/2$  and  $\tilde{S}(k) \rightarrow S(k)$ . That is why the contribution to damping at  $T = 0$  comes only from the scattering processes of excitations on inhomogeneities of structure.

## References

- [1] Kaneyoshi T. On an Anomalous Behaviour of Spin-Wave Stiffness Constant in Amorphous Ferromagnets. Phys. Stat. Sol. (b), 1983, v. 118, p. 751-755.
- [2] Vakarchuk I. A. and Tkachuk V. M. Energy Spectrum and Elementary Excitation Damping in the Structurally Disordered Ising Model in a Transverse Field. Phys. Stat. Sol. (b), 1990, v. 160, p. 321-327.