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ON THE PHENOMENOLOGICAL DESCRIPTION OF ELECTROMAGNETIC FLUCTUATIONS IN TURBULENT PLASMAS

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The main aspects of the Langevin approach to the theory of fluctuations in plasma are discussed. It is shown that both dielectric response function and correlation functions of fluctuation sources in plasmas (macroscopically agitated included) may be calculated in terms of the Green functions of evolution equation for the phase density fluctuations in the system without self-consistent electromagnetic interaction. Within the framework of this approach the microscopic theory of electromagnetic fluctuations in plasmas is generalized to the case of a turbulent plasma with random fluid-like motion. General relatiouns for fluctuation spectra in the system under consideration are found

1. Introduction

Theoretical studies of electromagnetic plasma fluctuations are of great importance both for the theory of electromagnetic processes in plasmas and for the development of modern methods for noncontact diagnostics of plasma systems. The theory of electromagnetic fluctuations in stable stationary plasmas is the most advanced today. Fluctuations in such systems are described by many approaches: fluctuation-dissipation theorem and its extension to nonequilibrium plasmas [1,2], method of dressed test particles [3-6], method of microscopic phase density [7,8], method of the inverse fluctuation-dissipation theorem and the probability approach [9-11], etc. These methods have been applied to calculate fluctuation spectra of various electromagnetic quantities in plasmas (both infinite and bounded), to find energy characteristics of fluctuation fields, to derive collision terms and thus to formulate closed kinetic equations for plasmas, to work up the bremsstrahlung theory, to describe wave and particle scattering in plasmas.

The progress of the theory of electromagnetic fluctuations in unstable (turbulent) plasmas appeared to be much slower, and microscopic approaches have been developed only for weakly turbulent plasmas which can be efficiently treated by means of the perturbation theory [2]. In such cases, one manages to calculate the fluctuation spectra and to find relevant stationary levels for the state of saturated turbulence. Renormalization of the plasma dielectric response function with regard to all perturbation orders also proves to be an efficient approach which provides another step towards the theory of strong plasma turbulence [12-15]. As to the theory of electromagnetic fluctuations, the results here are insufficient to be a subject for consistent analysis.

Phenomenological approaches to the description of turbulent plasma fluctuations, which have been intensively developed lately [16-19], turn out to be more promising. As a rule, however, these approaches are insufficiently substantiated because they are based on some intuitive assumptions concerning turbulence effect on the microscopic particle motion.

The purpose of this paper is to work out and substantiate a phenomenological theory of fluctuations in turbulent plasmas in terms of statistical theory of many-particle systems. To do this, we generalize the theory of electromagnetic fluctuations in stable stationary plasmas for the case of turbulent plasma states.

In the second section, we give the main aspects of the fluctuation theory in stable plasmas in terms of the Langevin approach. Both dielectric response and correlation functions for fluctuation sources are calculated with the help of the Green functions of evolution equations for the microscopic phase density fluctuations in a system without electromagnetic interaction.

In the third section, the evolution equations for the microscopic phase density are generalized to the case of turbulent plasmas under the assumption that random particle motion and large-scale plasma perturbations are statistically independent.

2. Electromagnetic fluctuations in stable stationary plasmas

We start from the equation for the microscopic phase density of some particle species

$$\mathcal{F}_\sigma(X,t) = \sum_{i=1}^{N_\sigma} \delta\left(X - X_i^\sigma(t)\right),$$

where $X \equiv (\mathbf{r},\mathbf{v})$, $X_i^\sigma(t) \equiv (\mathbf{r}_i^\sigma(t), \mathbf{v}_i^\sigma(t))$ is the phase trajectory of the i-th particle, $n_\sigma \equiv N_\sigma/V$ is the particle density of the relevant species (V is the volume of the system under consideration). Henceforth we omit the subscribes related to particle species in all cases that do not lead to misunderstanding. The equation for $\mathcal{F}(X,t)$ is given by

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \frac{1}{m}\left(\mathbf{F}^{ext} + \mathbf{F}\right)\frac{\partial}{\partial \mathbf{v}}\right\}\mathcal{F}(X,t) = 0. \tag{1}$$

Here $\mathbf{F}^{ext}$ and $\mathbf{F}$ are the forces produced by the external field and the intrinsic plasma fields, respectively, i.e,

$$\mathbf{F}^{ext} = e\left\{\mathbf{E}^{ext} + \frac{1}{c}[\mathbf{v},\mathbf{B}^{ext}]\right\}, \quad \mathbf{F} = e\left\{\mathbf{E} + \frac{1}{c}[\mathbf{v},\mathbf{B}]\right\},$$

$\mathbf{E}$ and $\mathbf{B}$ are microscopic fields, e and m are particle charge and mass.

Averaging Eq.(1) over the Liouville distribution yields

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \frac{1}{m}\left(\mathbf{F}^{ext} + \langle\mathbf{F}\rangle\right)\frac{\partial}{\partial \mathbf{v}}\right\} f(X,t) = -\frac{1}{m}\frac{\partial}{\partial \mathbf{v}}\langle\delta\mathbf{F}\,\delta f(X,t)\rangle, \tag{2}$$

where $f(X,t) \equiv \langle\mathcal{F}(X,T)\rangle$ is the one-particle distribution function (averaged phase density), $\delta f(X,t) \equiv \mathcal{F}(X,T) - f(X,t)$ is the fluctuation of the distribution function, and $\delta\mathbf{F} \equiv \mathbf{F} - \langle\mathbf{F}\rangle$. If the quantity in the right-hand part of Eq.(2) can be written as a functional of $f(X,t)$, then we may regard it as a collision term, and equation (2) becomes a kinetic (closed) equation for the distribution function. Subtracting (2) from (1), we find the evolution equation for the fluctuations of the one-particle distribution function (averaged phase density) to be given by

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m}\left(\mathbf{F}^{ext}+\langle\mathbf{F}\rangle\right)\frac{\partial}{\partial \mathbf{v}}\right\}\delta f(X,t)+\frac{1}{m}\delta\mathbf{F}\frac{\partial}{\partial \mathbf{v}}f(X,t)$$
$$+\frac{1}{m}\delta\mathbf{F}\frac{\partial}{\partial \mathbf{v}}f(X,t)=\frac{1}{m}\frac{\partial}{\partial \mathbf{v}}\langle\delta\mathbf{F}\delta f(X,t)\rangle. \tag{3}$$

This equation (similarly to (2)) is not closed; in the general case it should be solved together with the relevant equations for the moments $\langle\delta\mathbf{F}\delta f(X,t)\rangle$. However, it is appreciably simplified in the case of stable stationary plasmas. We make two assumptions: i) fluctuations are not large, $|\delta f(X,t)|\ll f(X,t)$, and ii) the characteristic time of fluctuation evolution (correlation time) τ_{cor} is much shorter than the one-particle distribution relaxation time τ_{rel}. The latter suggests the collision terms to be constants rather than functionals of the one-particle distributions, and hence we may disregard them in the analysis of Eq.(3). The first condition enables to neglect the terms nonlinear with respect to fluctuations. Thus, equation (3) reduces to

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m}\left(\mathbf{F}^{ext}+\langle\mathbf{F}\rangle\right)\frac{\partial}{\partial \mathbf{v}}\right\}\delta f(X,t)=-\frac{1}{m}\delta\mathbf{F}\frac{\partial f(X,t)}{\partial \mathbf{v}}. \tag{4}$$

We note that $\langle\mathbf{F}\rangle=0$ for the unbounded plasma if no electromagnetic excitations occur.

First of all we consider plasma fluctuations assuming that no excitations occur at the initial time instant. We write the solution of Eq.(4) for the general case in terms of the Green functions. We introduce the retarded and advanced Green functions $G^{(+)}(X,t;X',t')$ and $G^{(-)}(X,t;X',t')$. These are governed by the equation

$$\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+\frac{1}{m}\left(\mathbf{F}^{ext}+\langle\mathbf{F}\rangle\right)\frac{\partial}{\partial \mathbf{v}}G^{(\pm)}(X,t;X',t')=\delta(X-X')\delta(t-t'), \tag{5}$$

the causality conditions

$$G^{(\pm)}(X,t;X',t')\Big|_{t\lessgtr t'}=0, \tag{6}$$

and the relevant boundary conditions. We remind the reader that the difference between the retarded and advanced Green functions $W(X,t;X',t')$, given by

$$W(X,t;X',t')=G^{(+)}(X,t;X',t')-G^{(-)}(X,t;X',t'), \tag{7}$$

is the probability density for the particle transition from the point X' to the point X for the time $\tau=t-t'$. The transition probability density $W(X,t;X',t')$ is determined by Eq.(5) with zero right-hand part and the initial condition

$$W(X,t;X',t')\Big|_{t=t'}=\delta(X-X') \tag{8}$$

with the relevant boundary conditions. In the stationary state when the time-dependence of $\mathbf{F}^{ext}+\langle\mathbf{F}\rangle$ may be ignored, the probability density

$W(X,t;X',t')$ depends only on the time difference $t-t'$ and hence, within the context of the initial condition (8), is symmetric with respect to its arguments, i.e.,

$$W(X,t;X',t') = W(X',t';X,t). \tag{9}$$

Moreover, according to (7), the advanced and retarded Green functions are related according to

$$G^{(-)}(X,t;X',t') = -G^{(+)}(X',t';X,t). \tag{10}$$

Making use of the retarded Green function definition, the solution of Eq.(4) may be written as

$$\delta f(X,t) = \tag{11}$$

$$= \delta f^{(0)}(X,t) - \frac{1}{m}\int dt' \int dx' G^{(+)}(X,t;X',t')\delta\mathbf{F}(X',t')\frac{\partial f(X',t')}{\partial \mathbf{v}'},$$

where $\delta f^{(0)}(X,t)$ is the solution of Eq.(4) with zero right-hand part (in the absence of electromagnetic excitations, the distribution function of plasma fluctuations $\delta f^{(0)}(X,t)$ reproduce the one for the system without electromagnetic particle interactions). If we emplay of the relation (7) the solution (11) rewrites as

$$\delta f(X,t) = \tag{12}$$

$$= \delta f^{(0)}(X,t) - \frac{1}{m}\int_{-\infty}^{t} dt' \int dx' W(X,t;X',t')\delta\mathbf{F}(X',t')\frac{\partial f(X',t')}{\partial \mathbf{v}'}.$$

Notice that, using the probability density for particle transition, we can write the solution of the homogeneous equation which corresponds to Eq.(4) with the given initial condition at $t = t_0$, in the form

$$\delta f^{(0)}(X,t) = \int dX' W(X,t;X',t_0)\delta\mathbf{f}(X',t_0), \tag{13}$$

and hence

$$\delta f^{(0)}(X,t_0) = \delta f(X,t_0). \tag{14}$$

The structure of the solution (11) or (12) suggests the two-part structure of the fluctuation densities of the charge $\delta\rho(\mathbf{r},t)$ and the current $\delta\mathbf{J}(\mathbf{r},t)$:

$$\delta\rho(\mathbf{r},t) = \delta\rho^{(0)}(\mathbf{r},t) - \tag{15}$$

$$-\frac{e}{m}\int dt' \int d\mathbf{v} \int dX' G^{(+)}(X,t;X',t')\delta\mathbf{F}(X',t')\frac{\partial f(X',t')}{\partial \mathbf{v}'},$$

$$\delta\mathbf{J}(\mathbf{r},t) = \delta\mathbf{J}^{(0)}(\mathbf{r},t) - \tag{16}$$

$$-\frac{e}{m}\int dt' \int d\mathbf{v} \int dX' \mathbf{v} G^{(+)}(X,t;X',t')\delta\mathbf{F}(X',t')\frac{\partial f(X',t')}{\partial \mathbf{v}'},$$

where

$$\delta\rho^{(0)}(\mathbf{r},t) = e\int d\mathbf{v}\delta f^{(0)}(X,t) \quad \text{and} \quad \delta\mathbf{J}(\mathbf{r},t) = e\int d\mathbf{v}\mathbf{v}\delta f^{(0)}(X,t). \tag{17}$$

The first parts of the charge (15) and the current (16) determines electromagnetic fluctuation sources, whereas the second part depends on the fluctuation fields and characterizes the response of the system to the fluctuation fields. We regard relations (15) and (16) as the constitutive equations specifying the relationship between charge and current fluctuation densities and the fluctuation field. Then the fluctuation electromagnetic field is described by the Maxwell equation with the dielectric permittivities determined by the Green functions and the Langevin sources - by the charge and current densities (17).

Thus we arrive at the Langevin formulation of the fluctuation problem, and the physical meaning of the Langevin sources is known to be charge and current density fluctuations in the system without self-consistent electromagnetic interaction. The latter observation enables us to write the correlation function for such fluctuations as

$$\langle \delta f_\sigma^{(0)}(X,t)\delta f_{\sigma'}^{(0)}(X',t')\rangle = W_\sigma(X,t;X',t')f_\sigma(X',t')\delta_{\sigma\sigma'}, \tag{18}$$

where subscript σ labels particle species. Since the correlation function $\langle \delta f^{(0)}(X,t)\delta f^{(0)}(X',t')\rangle$ is symmetric with respect to its arguments X,t and X',t', in the general case the right-hand part of (18) must be symmetrized. Such result can be immediately obtained by solving the equation

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \frac{1}{m_\sigma}(\mathbf{F}_\sigma^{ext} + \mathbf{F}_\sigma)\frac{\partial}{\partial \mathbf{v}}\right\}\langle \delta f_\sigma^{(0)}(X,t)\delta f_{\sigma'}^{(0)}(X',t')\rangle = 0 \tag{19}$$

with the initial condition

$$\langle \delta f_\sigma^{(0)}(X,t)\delta f_{\sigma'}^{(0)}(X',t')\rangle = f_\sigma(X',t')\delta(X-X')\delta_{\sigma\sigma'}, \tag{20}$$

which follows from the direct calculation of the mean product of simultaneous distribution function fluctuations with the use of the Liouville distribution for uncorrelated particles $D^{(0)}(X_1^\sigma,...,X_{N_\sigma}^\sigma;X_1^{\sigma'},...,X_{N_{\sigma''}}^{\sigma'};t)$. Indeed,

$$\begin{aligned}\langle \delta f_\sigma^{(0)}(X,t')\delta f_{\sigma'}^{(0)}(X',t')\rangle &= \int \left(\Pi\delta X^\sigma\right)\left(\Pi\delta X^{\sigma''}\right) \\ &\times D^{(0)}(X_1^\sigma,...,X_{N_\sigma}^\sigma;X_1^{\sigma'},...,X_{N_{\sigma'}}^{\sigma'}\Big[\sum_{i=1}^{N_\sigma}\delta\Big(X - X_i^\sigma(t')\Big) \\ &- f_\sigma(X,t')\Big]\Big[\sum_{j=1}^{N_{\sigma'}}\delta\Big(X' - X_j^{\sigma'}(t')\Big) - f_{\sigma'}(X',t')\Big] \\ &= f_{\sigma\sigma'}(X,X',t') - f_\sigma(X,t')f_{\sigma'}(X,t') + f_\sigma(X,t')\delta(X-X')\delta_{\sigma\sigma'},\end{aligned} \tag{21}$$

where $f_{\sigma\sigma'}(X,X';t')$ is the two-particle distribution function. Since in the case of uncorrelated particles $f_{\sigma\sigma'}(X,X';t') = f_\sigma(X,t')f_{\sigma'}(X',t')$, Eq.(21) reduces to Eq.(20).

If there exist binary particle correlations, the two-particle distribution function may be written as

$$f_{\sigma\sigma'}(X,X';t) = \Big\{1 + g_{\sigma\sigma'}(X,X';t)\Big\}f_\sigma(X,t)f_{\sigma'}(X',t), \tag{22}$$

where $g_{\sigma\sigma'}(X,X';t)$ is the binary correlation function. In this case, instead of (20), we obtain the initial condition of Eq.(19) given by

$$\langle\delta f_\sigma^{(0)}(X,t')\delta f_{\sigma'}^{(0)}(X',t')\rangle = f_\sigma(X,t')\Big\{\delta(X-X')\delta_{\sigma\sigma'}$$
$$+g_{\sigma\sigma'}(X,X';t')f_{\sigma'}(X',t')\Big\}.$$

With this initial condition, instead of (18) we have

$$\langle\delta f_\sigma^{(0)}(X,t')\delta f_{\sigma'}^{(0)}(X',t')\rangle = W_\sigma(X,t;X',t')f_\sigma(X',t')\delta_{\sigma\sigma'}$$
$$+\int dX''W_\sigma(X,t;X'',t')f_\sigma(X'',t')g_{\sigma\sigma'}(X'',X',t')f_{\sigma'}(X',t'). \quad (23)$$

It is necessary to remind the reader that by virtue of the assumption $\tau_{cor} \ll \tau_{rel}$, the quantity $\langle\mathbf{F}\rangle$ depends on the slow time since it is determined by the distribution function $f(X,t)$. This means that we may neglect the time dependence of $\langle\mathbf{F}\rangle$ and $f(X,t)$ in the calculation of $G^{\pm}(X,t;X',t')$ and $W(X,t;X',t')$. Inasmuch as in the stationary case the Green functions and the transition probability densities depend only on the time difference $\tau = t-t'$, we find the plasma dielectric permittivity tensor to be given by

$$\varepsilon_{ij}(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}')\delta_{ij} + 4\pi\sum_\sigma \kappa_{ij}^\sigma(\mathbf{r},\mathbf{r}',\omega), \quad (24)$$

$$\kappa_{ij}(\mathbf{r},\mathbf{r}',\omega) = -\frac{e^2}{m\omega^2}\int d\mathbf{v}\int dX'v_iG_\omega^{(+)}(X,X')\times$$
$$\times\left\{\left(\omega+i\mathbf{v}'\frac{\partial}{\partial\mathbf{r}'}\right)\delta_{jk} - iv_j'\frac{\partial}{\partial r_k'}\right\}\frac{\partial f(X')}{\partial v_k'},$$

where

$$G_\omega^{(+)}(X,X') = \int d\tau e^{i\omega\tau}G^{(+)}(X,t;X',t'), \quad \tau = t-t'.$$

The source correlation functions are specified by the relations

$$\langle\delta\rho^{(0)}(\mathbf{r},t)\delta\rho^{(0)}(\mathbf{r}',t')\rangle = e^2\int d\mathbf{v}\int d\mathbf{v}'\langle\delta f^{(0)}(X,t)\delta f^{(0)}(X',t')\rangle, \quad (25)$$
$$\langle\delta J_i^{(0)}(\mathbf{r},t)\delta J_j^{(0)}(\mathbf{r}',t')\rangle = e^2\int d\mathbf{v}\int d\mathbf{v}'v_iv_j'\langle\delta f^{(0)}(X,t)\delta f^{(0)}(X',t')\rangle.$$

For an infinite homogeneous system $\langle F\rangle = 0$ and both the Green functions and the transition probability depend only on the difference $\mathbf{r}-\mathbf{r}'$. Therefore

$$\langle\delta\rho^{(0)}(\mathbf{r},t)\delta\rho^{(0)}(\mathbf{r}',t')\rangle = \langle\delta\rho^2\rangle^{(0)}_{\mathbf{r}-\mathbf{r}',t-t'},$$
$$\langle\delta J_i^{(0)}(\mathbf{r},t)\delta J_j^{(0)}(\mathbf{r}',t')\rangle = \langle\delta J_i\delta J_j\rangle^{(0)}_{\mathbf{r}-\mathbf{r}',t-t'}. \quad (26)$$

The Maxwell equations in the $\mathbf{k},\omega$ representation loox as follows

$$\hat{\Lambda}(\omega,\mathbf{k})\mathbf{E}_{\mathbf{k}\omega} = -\frac{4\pi i}{\omega}\delta\mathbf{J}^{(0)}_{\mathbf{k}\omega},$$
$$\Lambda_{ij}(\omega,\mathbf{k}) = \varepsilon_{ij}(\omega,\mathbf{k}) + \left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)\frac{k^2c^2}{\omega^2}, \quad (27)$$

where

$$\varepsilon_{ij}(\omega,\mathbf{k}) = \delta_{ij} + 4\pi\sum_{\sigma} k_{ij}^{\sigma}(\omega,\mathbf{k}),$$

$$\kappa_{ij}(\omega,\mathbf{k}) = -i\frac{e^2}{m\omega^2}\int d\mathbf{v}\int d\mathbf{v}' v_i G_{\mathbf{k}\omega}^{(+)}(\mathbf{v},\mathbf{v}')\left[\delta_{jk}(\omega-\mathbf{k}\mathbf{v}') + v_j' k_k\right]\frac{\partial f(\mathbf{v}')}{\partial v_k'},$$

$$G_{\mathbf{k}\omega}^{(+)}(\mathbf{v},\mathbf{v}') = \int d\mathbf{R}e^{-i\mathbf{k}\mathbf{R}} G_{\omega}^{(+)}(X,X'), \quad \mathbf{R} = \mathbf{r}-\mathbf{r}'. \tag{28}$$

For the potential field, we have

$$\varepsilon_{ij}(\omega,\mathbf{k}) = 1 + 4\pi\sum_{\sigma}\kappa_{\sigma}(\omega,\mathbf{k}),$$

$$\kappa(\omega,\mathbf{k}) = -i\frac{e^2}{mk^2}\int d\mathbf{v}\int d\mathbf{v}' G_{\mathbf{k}\omega}^{(+)}(\mathbf{v},\mathbf{v}')\mathbf{k}\frac{\partial f(\mathbf{v}')}{\partial v'}. \tag{29}$$

The spectral densities of the source correlation functions are given by

$$\langle\delta\rho^2\rangle_{\mathbf{k}\omega}^{(0)} \equiv e^2\langle\delta n^2\rangle_{\mathbf{k}\omega}^{(0)} = e^2\int d\mathbf{v}\int d\mathbf{v}' W_{\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')f(\mathbf{v}'),$$

$$\langle\delta J_i\delta J_j\rangle_{\mathbf{k}\omega}^{(0)} = e^2\int d\mathbf{v}\int d\mathbf{v}' v_i v_j' W_{\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')f(\mathbf{v}'). \tag{30}$$

If the binary particle correlations exist, then, for a stationary homogeneous system we have

$$\langle\delta\rho^2\rangle_{\mathbf{k}\omega}^{(0)} = e^2\int d\mathbf{v}\int d\mathbf{v}' W_{\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')f(\mathbf{v}')$$
$$+e^2\int d\mathbf{v}\int d\mathbf{v}'\int d\mathbf{v}'' W_{\mathbf{k}\omega}(\mathbf{v},\mathbf{v}'')f(\mathbf{v}'')g_{\mathbf{k}}(\mathbf{v}'',\mathbf{v}')f(\mathbf{v}'). \tag{31}$$

Equations (27)-(30) are sufficient to calculate correlation functions for any electromagnetic quantities. In particular, the correlation function for charge density fluctuations in the potential case is given by

$$\langle\delta\rho_{\sigma}\delta\rho_{\sigma'}\rangle_{\mathbf{k}\omega} = \frac{1}{|\varepsilon(\omega,\mathbf{k})|^2}\sum_{\sigma''}\gamma_{\sigma\sigma''}(\omega,\mathbf{k})\gamma_{\sigma'\sigma''}^{*}(\omega,\mathbf{k})\langle\delta\rho_{\sigma''}^2\rangle_{\mathbf{k}\omega}^{(0)}, \tag{32}$$

where

$$\gamma_{\sigma\sigma''}(\omega,\mathbf{k}) = \delta_{\sigma\sigma''}\varepsilon(\omega,\mathbf{k}) - 4\pi\kappa_{\sigma}(\omega,\mathbf{k}).$$

For $\sigma = \sigma'$ we have

$$\langle\delta\rho_{\sigma}^2\rangle_{\mathbf{k}\omega} = \left|\frac{1+4\pi\sum\limits_{\sigma'\neq\sigma}\kappa_{\sigma'}(\omega,\mathbf{k})}{\varepsilon(\omega,\mathbf{k})}\right|^2\langle\delta\rho^2\rangle_{\mathbf{k}\omega}^{(0)} +$$
$$+\left|\frac{4\pi\kappa_{\sigma}(\omega,\mathbf{k})}{\varepsilon(\omega,\mathbf{k})}\right|^2\sum_{\sigma'\neq\sigma}\langle\delta\rho_{\sigma'}^2\rangle_{\mathbf{k}\omega}. \tag{33}$$

The formula (33) becomes even simpler in the case of a two- component plasma. For example, the correlation function for the electron density fluctuations takes the form

$$\langle \delta n_e^2 \rangle_{\mathbf{k}\omega} = \left| \frac{1 + 4\pi\kappa_i(\omega, \mathbf{k})}{\varepsilon(\omega, \mathbf{k})} \right|^2 \langle \delta n_e^2 \rangle_{\mathbf{k}\omega} +$$

$$+ \left| \frac{4\pi\kappa_e(\omega, \mathbf{k})}{\varepsilon(\omega, \mathbf{k})} \right|^2 Z_i^2 \langle \delta n_i^2 \rangle_{\mathbf{k}\omega}^{(0)}, \qquad (34)$$

where Z_i is the ionic charge number.

In order to enlarge on the fluctuation spectra, we have to specify the system and to find the Green function $G^{(+)}(X, X', \tau)$ and the transition probability $W(X, X', \tau)$ which together with $f(X, t)$ determine both the dielectric response function and the correlation function of the Langevin sources. Since characteristics of Eq.(5) (in the case when initial excitations occur in the plasma) are particle phase trajectories in the system with external and average fields $\mathbf{F}^{ext}$ and $\langle \mathbf{F} \rangle$ respectively, the solution to this equation with the condition (6) is given by

$$G^{(+)}(X, X'; \tau) = \delta\Big(X - X(X', \tau)\Big)\theta(\tau), \qquad (35)$$

where $X(X', \tau)$ is the phase trajectory of a particle influenced by these fields (X' are initial coordinates of a particle at $\tau = 0$), $\theta(\tau)$ is the step function. Then the transition probability is determined by the formula

$$W(X, X'; \tau) = \delta\Big\{X - X(X', \tau)\Big\},$$

which may be reduced to a more convenient form

$$W(X, X'; \tau) = \delta\Big\{X - X' - \Delta X^{(0)}(X', \tau)\Big\}, \qquad (36)$$

where $\Delta X^{(0)}(\tau) \equiv \Delta X^{(0)}(X', \tau)$ is the change of the phase variable for time τ in course of particle motion under the influence of the external and average fields $\mathbf{F}^{ext}$ and $\langle \mathbf{F} \rangle$.

If there are no external fields and no initial excitations, we have

$$W(X, X'; \tau) = \delta(\mathbf{r} - \mathbf{r}' - \mathbf{v}\tau)\delta(\mathbf{v} - \mathbf{v}'). \qquad (37)$$

If the system is exposed to a uniform external magnetic field $\mathbf{B}^{ext} = B_0\mathbf{e}_z$, then

$$W(X, X'; \tau) = \delta\Big(x - x' - \frac{1}{\omega_B}\Big[v_x'\sin\omega_B\tau + v_y'(1 - \cos\omega_B\tau)\Big]\Big) \times$$
$$\delta\Big(y - y' - \frac{1}{\omega_B}\Big[- v_x'(1 - \cos\omega_B\tau) + v_y'\sin\omega_B\tau\Big]\Big) \times$$
$$\delta(z - z' - v_z\tau)\delta(v_x - v_x'\cos\omega_B\tau - v_y'\sin\omega_B\tau) \times$$
$$\delta(v_y + v_x'\sin\omega_B\tau)(-v_y'\cos\omega_B\tau)\delta(v_z - v_z')\theta(\tau), \qquad (38)$$

where $\omega_B = \frac{eB_0}{mc}$ is the cyclotron frequency.

As follows from relations (29) and (30), the description of fluctuations in the potential case requires the Green function moment and the transition probability moment to be known, i.e.,

$$G^{(+)}_{\mathbf{k}\omega}(\mathbf{v}') \equiv \int d\mathbf{v} G^{(+)}_{\mathbf{k}\omega}(\mathbf{v},\mathbf{v}'), \quad W_{\mathbf{k}\omega}(\mathbf{v}') \equiv \int d\mathbf{v} W_{\mathbf{k}\omega}(\mathbf{v},\mathbf{v}'). \tag{39}$$

In the absence of external fields, we have

$$G^{(+)}_{\mathbf{k}\omega}(\mathbf{v}') = \frac{i}{\omega - \mathbf{k}\mathbf{v}' + i0}, \quad W_{\mathbf{k}\omega}(\mathbf{v}') = 2\pi\delta(\omega - \mathbf{k}\mathbf{v}'), \tag{40}$$

and in the presence of an external magnetic field,

$$G^{(+)}_{\mathbf{k}\omega}(\mathbf{v}') = i\sum_{n,m=-\infty}^{\infty} \frac{J_n\left(\frac{k_\perp v'_\perp}{\omega_B}\right) J_m\left(\frac{k_\perp v'_\perp}{\omega_B}\right)}{\omega - k_\parallel v'_\parallel - n\omega_B + i0} e^{i(n-m)\varphi}, \tag{41}$$

$$W_{\mathbf{k}\omega}(\mathbf{v}') = 2\pi \sum_{n,m=-\infty}^{\infty} J_n\left(\frac{k_\perp v_\perp}{\omega_B}\right) J_m\left(\frac{k_\perp v'_\perp}{\omega_B}\right) \delta(\omega - k_\parallel v_\parallel - n\omega_B) e^{i(n-m)\varphi},$$

where φ is the angle between the vectors $\mathbf{k}$ and $\mathbf{v}'$. Thus, for an isotropic plasma we have

$$\kappa(\omega,\mathbf{k}) = \frac{e^2}{mk^2}\int d\mathbf{v} \frac{\mathbf{k}\frac{\partial f(\mathbf{v})}{\partial \mathbf{v}}}{\omega - \mathbf{k}\mathbf{v} + i0},$$
$$\langle \delta n^2 \rangle^{(0)}_{\mathbf{k}\omega} = 2\pi \int d\mathbf{v} f(\mathbf{v}) \delta(\omega - \mathbf{k}\mathbf{v}), \tag{42}$$

and for magnetoactive plasmas with the distribution function $f(\mathbf{v})$ being axially symmetric with respect to $\mathbf{B}_0$,

$$\kappa(\omega,\mathbf{k}) = \frac{e^2}{mk^2}\sum_{n=-\infty}^{\infty}\int d\mathbf{v} \frac{J_n^2\left(\frac{k_\perp v_\perp}{\Omega_B}\right)\left(\frac{n\Omega}{v_\perp}\frac{\partial f(\mathbf{v})}{\partial v_\perp} + k_\parallel \frac{\partial f(\mathbf{v})}{\partial v_\parallel}\right)}{\omega - k_\parallel v_\parallel - n\Omega_\sigma + i0},$$
$$\langle \delta n^2 \rangle^{(0)}_{\mathbf{k}\omega} = 2\pi \sum_{n=-\infty}^{\infty} \int d\mathbf{v} f(\mathbf{v}) J_n^2\left(\frac{k_\perp v_\perp}{\Omega}\right) \delta(\omega - k_\parallel v_\parallel - n\Omega). \tag{43}$$

Together with the general formulas like (33), (34) these relations completely determine spectral distributions of fluctuations in the potential case.

Now let us apply the general expressions to the analysis of fluctuation spectra in nonisothermal electron-ion plasmas. For instance, in the case of nonmagnetized plasmas Eq.(34) yields [1]

$$\langle \delta n_e^2 \rangle_{\mathbf{k}\omega} = \sqrt{6\pi}\frac{n_0}{Ks}\Big[\Big\{\Big[a^2k^2 + Z_i t\Big(1 - \varphi(\mu z)\Big)\Big]^2 + \pi Z_i^2 t^2 \mu^2 z^2 e^{2\mu^2 z^2}\Big\} e^{-z^2}$$
$$+ \Big[\Big(1 - \varphi(z)\Big)^2 + \pi z^2 e^{-2z^2}\Big] Z_i \mu e^{-\mu^2 z^2}\Big] \Big/ \Big\{\Big[a^2k^2 + 1 - \varphi(z)$$
$$+ Z_i t\Big(1 - \varphi(\mu z)\Big)\Big]^2 + \pi z^2 \Big(e^{-z^2} + Z_i t \mu e^{-\mu^2 z^2}\Big)^2\Big\}, \tag{44}$$

where

$$\varphi(z) = 2ze^{-z^2}\int_0^z e^{x^2}dx, \quad z = \sqrt{\frac{3}{2}}\frac{\omega}{ks},$$

$a^2 = \frac{T}{4\pi n_e e^2}$ is the squared Debye radius, $s = \sqrt{\frac{3T}{m}}$ is the electron thermal velocity, t is the electron to ion temperature ratio, and $\mu^2 = \frac{M}{m}t$ (M is the ion mass).

It is clear that the ionic contribution is important only for small ω. In this case the fluctuation spectra strongly depend on the parameter a^2k^2 and the temperature ratio t. If $a^2\mathbf{k}^2 \gg 1$, $\varepsilon(\mathbf{k},\omega) \to 1$ and fluctuations are produced by noninteracting electrons (incoherent fluctuations). On the contrary, for $a^2k^2 \ll 1$, the electronic contribution becomes small and ionic fluctuations dominate. In this case a central maximum occurs for an isothermal plasma. Its Doppler width is determined by the ion velocity and side resonances associated with the fluctuational excitation of plasma oscillations. In a strongly nonisothermal case, two maxima arise due to ion-acoustic oscillations and high-frequency plasma peaks.

Similar results may be obtained for low-frequency fluctuations in a magnetoactive plasma. Indeed, for strong magnetic fields , $\omega < \omega_{Bi}$, Eqs.(43) may be approximated by the formulas

$$\kappa(\omega,\mathbf{k}) = k_0(\mathbf{k}) + \frac{e^2}{mK^2}\int d\mathbf{v} J_0^2\left(\frac{k_\perp v_\perp}{\omega_B}\right)\frac{k_\| \frac{\partial f(\mathbf{v})}{\partial v_\|}}{\omega - k_\| v_\| + io},$$

$$\kappa_0(\mathbf{k}) + \frac{e^2}{mk^2}\int d\mathbf{v}\left[J_0^2\left(\frac{k_\perp v_\perp}{\omega_B}\right) - 1\right]\frac{1}{v_\perp}\frac{\partial f(\mathbf{v})}{\partial v_\perp},$$

$$\langle\delta n^2\rangle^{(0)}_{\mathbf{k}\omega} = 2\pi\int d\mathbf{v} J_0^2\left(\frac{k_\perp v_\perp}{\omega_B}\right) f(\mathbf{v})\delta(\omega - k_\| v_\|). \tag{45}$$

Here we use the cylindrical reference system in the velocity space with the $v_\|$ -axis directed along $\mathbf{B}_0$.

When particle distributions are Maxwellian with temperatures $T_\perp$ and $T_\|$ with respect to the external magnetic field direction, Eqs.(45) yield

$$\kappa(\omega,\mathbf{k}) = \kappa_0(\mathbf{k}) + \frac{e^{-\beta}I_0(\beta)}{4\pi a_\|^2 k^2}\left[1 - \varphi(\tilde{z}) + i\sqrt{\pi}\tilde{z}e^{-\tilde{z}^2}\right],$$

$$\kappa_0(\mathbf{k}) = \frac{1}{a_\perp^2 k^2}\left(1 - e^{-\beta}I_0(\beta)\right),$$

$$\langle\delta n^2\rangle^{(0)}_{\mathbf{k}\omega} = \sqrt{6\pi}\frac{n_0}{k_\| s_\|}e^{-\beta}I_0(\beta)e^{-\tilde{z}_\|^2}, \tag{46}$$

where $I_0(B)$ is the modified Bessel function,

$$a_\|^2 = \frac{T_\|}{4\pi n_e e^2}, \quad a_\perp^2 = \frac{T_\perp}{4\pi n_e e^2}, \quad \beta = \frac{k_\perp^2 T_\perp}{m\omega_B^2}, \quad S_\| = \sqrt{\frac{3T_\|}{m}}, \quad \tilde{z} = \sqrt{\frac{3}{2}}\frac{\omega}{k_\| s_\|}.$$

We note that the spectral distribution of fluctuation sources in this case (strong magnetic field) is specified by the longitudinal temperature only.

Thus,

$$\varepsilon(\omega,\mathbf{k}) = \varepsilon_0(\mathbf{k})\left[1+\frac{1}{\tilde{a}^2k^2}\left(1-\varphi(\tilde{z})+i\sqrt{\pi}\tilde{z}e^{-\tilde{z}^2}\right)\right.$$
$$\left.+\frac{\tilde{Z}_i\tilde{t}}{\tilde{a}^2k^2}\left(1-\varphi(\tilde{\mu}\tilde{z})+i\sqrt{\pi}\tilde{\mu}\tilde{z}e^{-\tilde{\mu}^2\tilde{z}^2}\right)\right], \tag{47}$$

where

$$\varepsilon_0(\mathbf{k}) = 1+4\pi\sum_\sigma \kappa_0^\sigma(\mathbf{K}),$$

$$\tilde{t}=t_\parallel,\quad \tilde{\mu}=\mu_\parallel,\quad \tilde{a}^2=\frac{\varepsilon_0(\mathbf{k})}{e^\beta I_0(\beta)}a_\parallel^2,\quad \tilde{Z}_i=\frac{e^{-\beta/t_\perp z_i}I_0(\beta/t_\perp z_i)}{e^{-\beta}I_0(\beta)}Z_i.$$

Having substituted Eqs.(46) and (47) into (34), we find

$$\langle\delta n_e^2\rangle_{\mathbf{k}\omega}^{(0)} = \sqrt{6\pi}\frac{n_0}{k_\parallel s_\parallel}e^{-\beta}I_0(\beta)\left\{\left\{\left[\frac{1+4\pi\kappa_0^i(\mathbf{k})}{\varepsilon_0(\mathbf{k})}\tilde{a}^2k^2+\tilde{Z}_i\tilde{t}\left(1-\varphi(\tilde{\mu}\tilde{z})\right)\right]^2\right.\right.$$
$$\left.+\pi\tilde{Z}_i^2\tilde{t}^2\tilde{\mu}^2\tilde{z}^2e^{2\tilde{\mu}^2\tilde{z}^2}\right\}e^{-\tilde{z}^2}+\left\{\left[\frac{\kappa_0^e(\mathbf{k})}{\varepsilon_0(\mathbf{k})}\tilde{a}^2k^2+\left(1-\varphi(\tilde{z})\right)\right]^2\right.$$
$$\left.\left.+\pi\tilde{z}^2e^{-2\tilde{z}^2}\right\}\tilde{Z}_i\tilde{\mu}e^{-\tilde{\mu}^2\tilde{z}^2}\right\}\Big/\left\{\left[\tilde{a}^2k^2+1-\varphi(\tilde{z})+\tilde{Z}_i\tilde{t}\left(1-\varphi(\tilde{\mu}\tilde{z})\right)\right]^2\right.$$
$$\left.+\pi\tilde{z}^2\left(e^{-\tilde{z}^2}+\tilde{Z}_i\tilde{t}\tilde{\mu}e^{-\tilde{\mu}^2\tilde{z}^2}\right)^2\right\}. \tag{48}$$

Comparing Eqs.(48) and (44), we observe that these equations are related by the scaling transformations $z\to\tilde{z}$, $a^2\to\tilde{a}^2$, $Z_i\to\tilde{Z}_i$, $n_0\to n_0e^{-\beta}I_0(\beta)$ and $\kappa(\omega,\mathbf{k})\to\kappa(\omega,\mathbf{k})+\kappa_0(\mathbf{k})$. This enables to reproduce the above analysis using the appropriate results for a nonmagnetized plasma [20].

Thus, electromagnetic fluctuations in a stable plasma (that is described by particle distribution functions) may be considered provided one rather simple quantity is known, namely, the particle transition probability with regard to the external field influence. For distribution functions given, just this quantity determines both response functions and correlation functions for the Langevin sources. Though the range of validity of this theory is essentially restricted (the plasma state is stable and stationary, the correlation time is much shorter that the relaxation time), the general relations of the type (14)-(16) hold also in the case when particle collisions are taken into account. It is possible to show that these relations may be generalized for the case of large-scale (kinetic) fluctuations if one allows for in (6) interaction between particles and other dynamical subsystems (including stochastic ones) and thus obtains a modified transition probability.

The theory of fluctuations in a weakly ionized plasma has been developed just in this manner [21]. Now our task is to construct the transition probability for a turbulent plasma under the assumption that turbulent fluid-like motions are chaotic, and to obtain relevant modified microscopic evolution equations for fluctuations. This will be done in the next section.

3. Transition probability for turbulent plasmas

Let us generalize the transition probability (36) for the case of a turbulent plasma state. We consider plasmas with developed turbulence and assume that there occur large-scale turbulent pulsations. This means that microscopic motion of noninteracting particles (and we are interested just in the transition probability of this system) reduces to the motion of particles under the influence of the external field that is averaged over a small macroscopic volume, and the stochastic motion of the latter. We admit thermal motion of individual particles and chaotic (turbulent) large-scale motions to occur independently. In this case, it does not matter whether we first average over microscopic states and then over the ensemble of turbulent pulsations, or vice versa.

In view of above speculations and under the above assumptions, the transition probability for a specific turbulent state may be written in the form (36) but with the replacement

$$\Delta X^{(0)}(X',\tau) \to \Delta X^{(0)}(X',\tau) + \Delta X^T(\tau),$$

where $\Delta X^T(\tau)$ is the phase variable change due to the stochastic motion of the elementary volume for the specific realization of interest, i.e.,

$$W\Big(X, X' + \Delta X^T(\tau); \tau\Big) \equiv \delta\Big\{X - X' - \Delta X^{(0)}(X',\tau) - \Delta X^T(\tau)\Big\}. \quad (49)$$

Inasmuch as we have assumed that microscopic distributions and distributions of turbulent realizations are statistically independent, we can average $W\Big(X, X' + \Delta X^T(\tau); \tau\Big)$ over the ensemble of turbulent motions and carry out further calculations making use of the general relations obtained in the previous section. Thus,

$$W^T(X, X'; \tau) = \int d\Delta X^T P_\tau(\Delta X^T) W(X, X' + \Delta X^T; \tau), \quad (50)$$

where $P_\tau(\Delta X)$ is the probability that phase coordinates of the elementary volume are changed by ΔX for the time τ. Its explicit form depends on the model of the turbulent process. In particular, if the elementary volume is involved in the diffusion-drift motion, then

$$P_\tau(\Delta X) = \int d\mathbf{u} F(\mathbf{u}) W^B(\Delta\mathbf{r}, \mathbf{u} + \Delta\mathbf{v}; 0, \mathbf{u}; \tau). \quad (51)$$

Here $W(X, X'; \tau)$ is the probability of a diffusion-drift transition in the phase space for time τ ($\mathbf{v}' \equiv \mathbf{u}$ and $\mathbf{v} \equiv \mathbf{u} + \Delta\mathbf{v}$), $F(\mathbf{u})$ is the velocity distribution function for turbulent pulsations.

We have already mentioned that the knowledge of the zero-order velocity moment of the transition probability is sufficient to treat the potential case. As follows from (50),

$$W^T(\mathbf{R}, \mathbf{v}'; \tau) \equiv \int d\mathbf{v} W^T(X, X'; \tau) = \int d\Delta\mathbf{r}^T P_\tau(\Delta r^T) W(\mathbf{R} - \Delta\mathbf{r}^T, \mathbf{v}'; \tau), \quad (52)$$

where

$$P_\tau(\Delta r^T) \equiv \int d\Delta\mathbf{v}^T P_\tau(\Delta x^T) = \int d\mathbf{v} \int d\mathbf{v}' F(\mathbf{v}') W^B(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; \tau). \quad (53)$$

Having performed the Fourier transformation, we find the transition probability for a turbulent system to be given by

$$W^T_{\mathbf{k}\omega}(\mathbf{v}') = \int \frac{d\omega'}{2\pi} P_{\mathbf{k}\omega'} W_{\mathbf{k}\omega-\omega'}(\mathbf{v}'), \tag{54}$$

where $P_{\mathbf{k}\omega'}$ is the factor determined by the Brownian particle transition probability, i.e.,

$$P_{\mathbf{k}\omega} = \int d\mathbf{v} \int d\mathbf{v}' F(\mathbf{v}') W^B \mathbf{k}\omega(\mathbf{v}, \mathbf{v}'). \tag{55}$$

Making use of (50), we rewrite the retarded Green function for the turbulent state as

$$G^{(+)T}(X, X'; \tau) = \int d\Delta X^T P_\tau(\Delta X^T) W(X, X' + \Delta X^T; \tau)\theta(\tau). \tag{56}$$

Then, having performed the Fourier transformation, we obtain the spectral representation of the retarded Green function of a turbulent system in the form

$$G^{(+)T}_{\mathbf{k}\omega}(\mathbf{v}') = \int \frac{d\omega'}{2\pi} \tilde{P}_{\mathbf{k}\omega'} G^{(+)}_{\mathbf{k}\omega-\omega'}(\mathbf{v}'), \tag{57}$$

where $\tilde{P}_{\mathbf{k}\omega'}$ is the factor determined by the retarded Green function for the Brownian particle, i.e.,

$$\tilde{P}_{\mathbf{k}\omega} = \int d\mathbf{v} \int d\mathbf{v}' F(\mathbf{v}') G^{(+)B} \mathbf{k}\omega(\mathbf{v}, \mathbf{v}'). \tag{58}$$

Thus, employing of the general definitions of the plasma dielectric permittivity (23) and the spectral correlation function for the spontaneous fluctuation sources (24), we obtain the general relations between these quantities in turbulent and nonperturbed states, i.e.,

$$\kappa^T(\omega, \mathbf{k}) = \int \frac{d\omega'}{2\pi} \tilde{P}_{\mathbf{k}\omega-\omega'}(\omega', \mathbf{k}), \tag{59}$$

$$\langle \delta n^2 \rangle^T_{\mathbf{k}\omega} = \int \frac{d\omega'}{2\pi} P_{\mathbf{k}\omega-\omega'} \langle \delta n^2 \rangle^0_{\mathbf{k}\omega-\omega}. \tag{60}$$

The quantity $W^B(X, X', \tau)$ may be calculated from the Fokker-Planck equation for the Brownian particle transition probability [22]. One can also use simplified model equations, such as the Bhatnagar-Gross-Krooke equation [21,23]

$$\begin{aligned} &\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right\} W^B(X, X', \tau) = \\ &= -\nu\Big\{W^B(X, X'; \tau) - \Phi(\mathbf{v}) \int d\mathbf{v}'' W^B(\mathbf{r}, \mathbf{v}'', X', \tau)\Big\}. \end{aligned} \tag{61}$$

Here $\nu = 1/\tau_T$ is the effective collision frequency, τ_T t is the mean time of free evolution for a fluid element.

The solution to Eq.(61) is given by

$$W^B(X,X',\tau)=\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')-i\omega\tau}W^B_{\mathbf{k}\omega}(\mathbf{v},\mathbf{v}'),$$

where

$$W^B_{\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')=\frac{i\delta(\mathbf{v}-\mathbf{v}')}{\omega-\mathbf{kv}+i\nu}$$
$$-\frac{\nu\Phi(\mathbf{v})}{(\omega-\mathbf{kv}+i\nu)(\omega-\mathbf{kv}'+i\nu)}\left[1-i\nu\int d\mathbf{v}\frac{\Phi(\mathbf{v})}{\omega-\mathbf{kv}+i\nu}\right]^{-1}. \quad (62)$$

This solution describes the Brownian particle motion in the phase space and may be regarded as a possible version of the transition probability calculation for turbulent plasmas, $W^B(X,X',\tau)$.

Further calculations require specification of the transition probability $W^B(X,X',\tau)$ (or, in the potential case, the quantity $W^B(\mathbf{R},\mathbf{u},\tau)$. To do this, we employ formula (62). In the limiting cases $\tau\ll\nu^{-1}$, $|\mathbf{r}-\mathbf{r}'|\ll\langle v\rangle/\nu$, and $\tau\gg\nu^{-1}$, $|\mathbf{r}-\mathbf{r}'|\gg\langle v\rangle/\nu$, such relations become appreciably simpler.

In the first case (large correlation period $\tau_T\equiv\frac{1}{\nu}\gg\tau$, large correlation length $\langle v\rangle\tau_T\gg|r-r'|$), the correlation is weak,

$$W^B(X,X',\tau)=\delta(\mathbf{r}-\mathbf{r}'-\mathbf{v}\tau)\delta(\mathbf{v}-\mathbf{v}'), \quad (63)$$

the particle motion is free, the velocity is conserved, i.e., is equal to the initial velocity v'. Then we have

$$W^B_{\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')=2\pi\delta(\omega-\mathbf{kv})\delta(\mathbf{v}-\mathbf{v}'), \quad (64)$$

$$P_{\mathbf{k}\omega}=2\pi\int d\mathbf{v}F(\mathbf{v})\delta(\omega-\mathbf{kv}). \quad (65)$$

The retarded Green function is given by

$$G^{(+)B}(\mathbf{v},\mathbf{v}')=i\frac{\delta(\mathbf{v}-\mathbf{v}')}{\omega-\mathbf{kv}+io}, \quad (66)$$

and therefore

$$\tilde{P}_{\mathbf{k}\omega}=i\int d\mathbf{v}\frac{F(\mathbf{v})}{\omega-\mathbf{kv}+io}. \quad (67)$$

Making use of formulas (54) and (57), we thus find that for the turbulent plasma

$$G^{(+)T}_{\mathbf{k}\omega}(\mathbf{v}')=\int d\mathbf{v}F(\mathbf{v})_{\mathbf{k}\omega'}G^{(+)}_{\mathbf{k}\omega-\mathbf{kv}}(\mathbf{v}'), \quad (68)$$

We see that in the potential case both the transition probability and the retarded Green function for the turbulent plasma depend only on the velocity distribution function of turbulent pulsations.

If the initial plasma drift is given, $F(\mathbf{v})=\delta(\mathbf{v}-\mathbf{u}_D)$, then

$$P_{\mathbf{k}\omega}=2\pi\delta(\omega-\mathbf{ku}_D),\quad \tilde{P}_{\mathbf{k}\omega}=i\frac{1}{\omega-\mathbf{ku}_D+io}. \quad (69)$$

and the transition probability corresponds to the free drift, i.e.,

$$W^T_{\mathbf{k}\omega}(\mathbf{v}) = W^T_{\mathbf{k}\omega-\mathbf{k}\mathbf{u}_D}(\mathbf{v}'), \quad G^{T(+)}_{\mathbf{k}\omega}(\mathbf{v}') = G^{(+)}_{\mathbf{k}\omega-\mathbf{k}\mathbf{u}_D}(\mathbf{v}'). \tag{70}$$

In the second case (small correlation period $\tau_T \equiv \frac{1}{\nu} \ll \tau$, small correlation length $\langle v\rangle\tau \ll |r - r'|$), the correlation is strong, i.e.,

$$\begin{aligned} W^B_{\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') &= \frac{2\gamma}{(\omega - \mathbf{k}\mathbf{u}_D)^2 + \gamma^2}\Phi(\mathbf{v}), \\ G^{(+)B}_{\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') &= i\frac{\Phi(\mathbf{v})}{\omega - \mathbf{k}\mathbf{u}_D + i\gamma}, \end{aligned} \tag{71}$$

Here $\mathbf{u}_D$ is the drift velocity and D is the diffusion coefficient given by

$$\begin{aligned} \mathbf{u}_D &= \int d\mathbf{v}\mathbf{v}\Phi(\mathbf{v}), \\ D &= \frac{1}{3}\tau_T \int d\mathbf{v}v^2\Phi(\mathbf{v}), \quad \gamma = k^2 D. \end{aligned} \tag{72}$$

The solution of (71) corresponds to the particle motion with regard to diffusion and drift.

$$\begin{aligned} W^B_{\mathbf{k}}(\mathbf{v},\mathbf{v}';\tau) &= e^{-i\mathbf{k}\mathbf{u}_D\tau - k^2 D|\tau|}\Phi(\mathbf{v}), \\ W^B(X,X';\tau) &= \frac{1}{(4\pi D\tau)^{3/2}} e^{-\frac{1}{4D\tau}(\mathbf{r}-\mathbf{r}'-\mathbf{u}_D\tau)^2}\Phi(\mathbf{v}). \end{aligned} \tag{73}$$

By virtue of strong correlation, the distribution over velocities v does not depend on the initial distribution over v'. The strong correlation is caused by strong collisions (i.e., important role of the collision term in Eq.(61)) which determine the distribution over v'. At the same time, this distribution (i.e., strong collisions) determines the drift and diffusion described by (72).

According to the definitions (55) and (58), we have

$$P_{\mathbf{k}\omega} = \frac{2\gamma}{(\omega - \mathbf{k}\mathbf{u}_D)^2 + \gamma^2}, \quad \tilde{P}_{\mathbf{k}\omega} = i\frac{1}{\omega - \mathbf{k}\mathbf{u}_D + i\gamma}, \quad \gamma = k^2 D, \tag{74}$$

and thus we find the transition probability and the retarded Green function of a plasma with drift and diffusion caused by strong collisions, to be given by

$$\begin{aligned} W^T_{\mathbf{k}\omega}(\mathbf{v}') &= \frac{2\gamma}{(\omega - \mathbf{k}\mathbf{u}' - \mathbf{k}\mathbf{u}_D)^2 + \gamma^2}, \\ G^{(+)T}_{\mathbf{k}\omega}(\mathbf{v}') &= \frac{i}{\omega - \mathbf{k}\mathbf{u}' - \mathbf{k}\mathbf{u}_D + i\gamma}. \end{aligned} \tag{75}$$

Formulas (68) and (75) completely determine the dielectric permittivities and the spectral distribution of the spontaneous fluctuation sources in a turbulent plasma. The explicit expressions for the dielectric permittivity and the spontaneous fluctuation sources spectral distribution in a turbulent plasma also may be derived by substituting expressions (65), (67), and (74) for $P_{\mathbf{k}\omega}$ and $\tilde{P}_{\mathbf{k}\omega}$ into (59) and (60).

The results obtained may be reproduced making use of the modified equation (1). One has to separate in $\mathbf{F}$ the forces, responsible for the large-scale turbulent pulsations, and to describe their effect on the microscopic particle motions in terms of interaction with an additional subsystem (turbulent thermostat). Indeed, having assumed that Eq.(1) describes only microscopic particle distributions in the presence of large-scale plasma motions, one has to introduce in the right-hand part of Eq.(1) the collision term with regard to the fact that $\mathcal{F}(X,t)$ must relax towards the distribution averaged over the ensemble of turbulent pulsations. We take the collision term in the form

$$I_T = -\frac{1}{\tau_T}\Big\{\mathcal{F}(X,t) - \int_{V_T} d\Delta\mathbf{r} P_{\tau_T}(\Delta\mathbf{r})\mathcal{F}(\mathbf{r}-\Delta\mathbf{r},\mathbf{v},t)\Big\}, \tag{76}$$

where τ_T and V_T are the characteristic time and characteristic volume of turbulent perturbations, $P_\tau(\Delta r)$ is the probability that the elementary volume coordinate is changed under the influence of the turbulent field. Assuming the distribution to be Gaussian. we write it as

$$P_\tau(\Delta\mathbf{r}) = \frac{1}{(2\pi)^{3/2}V_T} e^{-\frac{(\Delta\mathbf{r}-\mathbf{u}_d\tau)^2}{2V_T^{2/3}}}, \tag{77}$$

where $\mathbf{u}_d$ is the drift velocity of an elementary volume. Having expanded $\mathcal{F}(\mathbf{r}-\Delta\mathbf{r},\mathbf{v},t)$ in terms of $\Delta\mathbf{r}$, we obtain the Fokker-Planck collision term to be

$$I_T = \frac{\partial}{\partial\mathbf{r}}\left[D\frac{\partial\mathcal{F}(X,t)}{\partial\mathbf{r}} - \mathbf{u}_d\mathcal{F}(X,t)\right], \tag{78}$$

where the diffusion coefficient D and the drift velocity $\mathbf{u}_d$ are given by

$$D = \frac{1}{2\tau_T}\int_{V_T} d\Delta\mathbf{r}\Delta\mathbf{r}^2 P_\tau(\Delta\mathbf{r}),$$
$$\mathbf{u}_d = \frac{1}{\tau_T}\int_{V_T} d\Delta\mathbf{r}\Delta\mathbf{r} P_\tau(\Delta\mathbf{r}).$$

Thus, the equation for the microscopic phase density takes the form

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial\mathbf{r}} + \frac{1}{m}(\mathbf{F}^{ext}+\mathbf{F})\frac{\partial}{\partial\mathbf{v}}\right\}\mathcal{F}(X,t) = \frac{\partial}{\partial\mathbf{r}}\left[D\frac{\partial}{\partial\mathbf{r}} - \mathbf{u}_d\right]\mathcal{F}(X,t). \tag{79}$$

This equation may be used in the same manner as Eq.(1). As a result we reproduce all the relations of the first section in which, however, $W(X,X',t)$ satisfies the equation

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial\mathbf{r}} + \frac{1}{m}(\mathbf{F}^{ext}+\langle\mathbf{F}\rangle)\frac{\partial}{\partial\mathbf{v}} - \mathbf{u}_d\frac{\partial}{\partial\mathbf{r}} - D\frac{\partial^2}{\partial\mathbf{r}^2}\right\}W(X,X',\tau) = 0. \tag{80}$$

It is not difficult to show that in the absence of external fields

$$\tilde{G}^{(+)}_{\mathbf{k}\omega}(\mathbf{v}') = \frac{i}{\omega - \mathbf{k}\mathbf{v}' - \mathbf{k}\mathbf{u}_d + ik^2D}$$

or

$$W_{\mathbf{k}\omega}(\mathbf{v}') = \frac{2k^2D}{(\omega - \mathbf{k}\mathbf{v}' - \mathbf{k}\mathbf{u}_d)^2 + (k^2D)^2}.$$

Thus, the model representation of the transition probability (50) corresponds to the microscopic description of the phase density in the system with relaxation. This suggests that one can derive an equation lize (80) more consistently, in terms of the kinetic fluctuation theory making no at-priori assumptions that turbulent motions do not depend on the microscopic distributions.

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ДО ФЕНОМЕНОЛОГІЧНОГО ОПИСУ ЕЛЕКТРОМАГНІТНИХ ФЛУКТУАЦІЙ В ТУРБУЛЕНТНІЙ ПЛАЗМІ

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Обговорюються основні аспекти ланжевенівського підходу в теорії флуктуацій у плазмі. Показано, що як діелектрична функція, так і кореляційна функція флуктуючих джерел (включаючи макроскопічно збуджену плазму) можуть бути можуть бути розраховані з допомогою функцій Гріна рівняння еволюції для фазової густини флуктуацій в системі без самоузгодження електромагнітної взаємодії. В рамках цього наближення мікроскопічну теорію електромагнітних флуктуацій в плазмі узагальнено на випадок турбулентної плазми з випадковим рідиноподібним рухом. Для системи, що розглядається, знайдено загальні співвідношення для флуктуаційного спектру.