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A consistent description of kinetics and hydrodynamics of systems of interacting particles by means of the nonequilibrium statistical operator method

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A statistical approach to a self-consistent description of kinetic and hydrodynamic processes in systems of interacting particles is formulated on the basis of the nonequilibrium statistical operator method by D.N.Zubarev. It is shown how to obtain the kinetic equation of the revised Enskog theory for a hard sphere model, the kinetic equations for multistep potentials of interaction and the Enskog-Landau kinetic equation for a system of charged hard spheres. The BBGKY hierarchy is analyzed on the basis of modified group expansions. Generalized transport equations are obtained in view of a self-consistent description of kinetics and hydrodynamics. Time correlation functions, spectra of collective excitations and generalized transport coefficients are investigated in the case of weakly nonequilibrium systems of interacting particles.

Key words: *kinetics, hydrodynamics, kinetic equations, transport coefficients, (time) correlation functions*

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1. Introduction

The problem of a self-consistent description of fast and slow processes which are connected with both linear and non-linear fluctuations of observed quantities for various physical systems, such as dense gases, liquids, their mixtures and plasma, remains an actual problem in nonequilibrium statistical mechanics. An important task in this direction is the construction of kinetic equations for dense systems with taking into account collective effects (hydrodynamic contribution) into the collision integrals. On the other hand, there is a problem of the calculation of time correlation functions and collective excitations spectra in the range of intermediate values of wavevectors and frequencies, since low- and high-frequencies regions are described sufficiently well within the frameworks of molecular hydrodynamics and kinetic equations, respectively. In the intermediate region, the kinetic and hydrodynamic processes are connected and must be considered simultaneously.

A significant interest in these problems was exhibited by Prof. D.N.Zubarev in his investigations. One of the approaches to the unification of kinetics and hydrodynamics in the theory of transport phenomena for systems of interacting particles was proposed by Zubarev and Morozov in [1] and developed in further papers [2–12]. This statistical approach is based on a modification of the boundary conditions for the weakening of correlations to the BBGKY hierarchy for many-particle distribution functions and to the Liouville equation for a full nonequilibrium distribution function (nonequilibrium statistical operator – NSO). The modification of the boundary conditions consists in the fact that both the one-particle nonequilibrium distribution function and the local conservation laws for mass, momentum and total energy are included into the parameters for an abbreviated description of a nonequilibrium state of the system. The concept of a self-consistent description of kinetics and hydrodynamics was applied to plasma in an electro-magnetic field [13–15], to quantum systems with taking into account coupled states [9,10,16], quantum Bose systems [17] and the nonequilibrium thermo field dynamics [18]. The questions as to the necessity and possibility of a unified description of kinetic and hydrodynamic processes were discussed in papers by Klimontovich [19–21].

In this paper we present some actual results obtained recently within the concept of a self-consistent description of kinetics and hydrodynamics for dense gases and simple liquids. In section 2 we present the results for kinetic equations of RET [3,7], GDRS [8] and Enskog-Landau theories [3,7], obtained in the polarization approximation for a system of hard spheres and a Lenard-Balescu kinetic equation [11,12] derived from the BBGKY hierarchy with modified boundary conditions. Generalized transfer equations, time correlation functions and transport coefficients are presented in section 3 on the basis of a self-consistent description of kinetics and hydrodynamics within the framework of the nonequilibrium statistical operator method.

2. Conception of a consistent description of kinetics and hydrodynamics for dense gases and liquids

2.1. Overview

Bogolubov [22] proposed a consequent approach to the construction of kinetic equations which is based on the chain of equations for s-particle distribution functions and a weakening correlation principle. There are a lot of theories [23–26] which differ in forms of presentation, but all these approaches use the same weakening correlation condition and are most effective when a small parameter (interaction, density, etc.) is presented.

In the kinetic theory of classical gases we can point out two principal problems. The first problem is connected with the fact that collision integrals can depend nonanalytically on density. Because of this, even in the case of a small-density gas, in order to calculate corrections to transport coefficients it is necessary to perform a partial resummation of the BBGKY hierarchy [27,28]. The second problem is the construction of kinetic equations for the gases of high densities. In this case we cannot restrict ourselves by several terms of the expansion of collision integrals on densities and the analysis of the BBGKY hierarchy becomes very complicated (generally speaking, the density is not a small parameter here). That is why the construction of kinetic equations for dense gases and liquids with model interparticle potentials of interaction has a great importance in the problem under investigation.

The first theory in this direction is a semi-phenomenological standard Enskog theory (SET) [29,30] of dense gases. Ideas similar to those used at the derivation of the Boltzmann equation were also used in this approach. The Enskog equation plays an essential role in the kinetic theory, next to the Boltzmann one [30]. This equation was obtained at the modelling of molecules by hard spheres for dense gases. As a result, the collision integral was presented in an analytical form. It is obtained by means of a hard sphere model where collisions can be considered as momentary and by the fact that the multiparticle contact at the same time is reputed to be infinitely small. Density correction introduced by Enskog proved to be considerable, as far as transport, due to collisions in a dense system, is the main mechanism of transport. Each molecule is almost localized at one point of space by the surrounding neighbour molecules and, therefore, the flow transport is suppressed. Though this theory properly describes density dependence of the kinetic coefficients, at the same time the suppositions about the structure of the collision integral remain sufficiently rough and phenomenological. Despite approximate assumptions on the collision integral in the kinetic equation for a one-particle distribution function of hard spheres, the Enskog theory very well describes a set of properties for real dense gases [25,31].

Davis, Rise and Sengers [32] proposed a kinetic DRS theory. Within the framework of this theory, the interparticle potential of an interaction is chosen in a square well form. An attractive part of the real potential is approximated here by a finite height wall.

From the point of view of the statistical theory of nonequilibrium processes, SET and DRS contain two essential drawbacks. The first one is that their kinetic equations are not obtained within the framework of some consequential theoretical scheme and one does not know how to improve these theories. And the second one is that the H-theorem has not been proved. Nevertheless, not long ago a way of constructing the SET entropy functional was given [33]. In order to overcome these drawbacks the authors of [34] obtained a kinetic equation for a revised version of the Enskog theory (RET) using the diagram method. Résibois [35,36] proved the H-theorem for it. In 1985, a revised version of the DRS theory (RDRS) was proposed [37]. The kinetic equation of RDRS satisfies the H-theorem as well. A generalized version of RDRS — GDRS was considered in [2,8], being based on the approaches of [1,3]. Here, in order to treat a more realistic model, a kinetic equation for dense classical systems is offered with an interparticle interaction potential in the shape of a multistep function, where its sequential derivation and the normal solution are obtained.

In the paper by Stell et al. [38,39] the authors completed the construction of the kinetic variational theory (KVT) introducing a whole family of theories, namely, KVT I, KVT II and KVT III. The KVT III refers to the local energy constraint in maximizing entropy as distinguished from the global energy constraint (KVT II) or the hard-core constraint (KVT I). According to this classification, the kinetic equation of RDRS [37] is obtained by applying KVT III to the square well potential and KMFT [38] is derived from KVT I as its application to the potential with a smooth tail. In [39], the KVT III version of KMFT was proposed. In this case the quasiequilibrium binary distribution function (QBDF) of hard spheres is to be substituted by the full QBDF, which takes into account the full potential of the interaction. The main conclusion of the KVT I version of KMFT [39,40] is that the smooth part of the potential does not contribute explicitly to transport coefficients. This fact is caused by the approximations of the theory. When the full QBDF is used, as it is in the KVT III version of KMFT, transport coefficients are determined by the soft part of the potential. The case when the potential between the hard sphere and the attractive walls is considered as a smooth tail (instead of the constant potential in DRS) has also been investigated [41]. The kinetic equation was obtained for this potential applying a mean field approximation for a smooth tail.

At this stage of investigations in the kinetic theory of dense gases and liquids, paper [1] by Zubarev and Morozov played a fundamental role. In this paper a new formulation of boundary conditions for the BBGKY hierarchy is presented. Such a formulation took into account correlations connected with local conservation laws. In the binary collision approximation, this modification of the weakening correlation condition by Bogolubov led to a kinetic equation for a system of hard spheres which is close in structure to the usual Enskog kinetic equation. Similar ideas were proposed independently by the authors of paper [37] at the derivation of a kinetic equation when an interaction between the particles is modelled by a square well potential. It is necessary to point out that a somewhat different modification of the Bogolubov approach [22] was presented in the papers by Rudyak [42–44]. This allows one to obtain an Enskog-like kinetic equation and extend it to the systems with a soft potential of interaction between the particles.

The modification of weakening correlation conditions to a chain of equations by Bogolubov was developed in papers [2–12]. An important achievement of the given approach is the fact that the modified boundary condition gives the possibility [4,7,14] to derive consequently the kinetic equation of the RET theory [34] for the first time. On the basis of result [1], a kinetic equation was obtained [2,8] for systems with a multistep potential of interaction (in particular, the H-theorem was proved for this equation in paper [8]), whereas an Enskog-Landau kinetic equation was derived [3,7] for a system of charged hard spheres.

Normal solutions for the obtained kinetic equations were found using the Chapman-Enskog method. With the help of these solutions, the numerical calculations of transport coefficients (bulk and shear viscosities, thermal conductivity) were carried out for Argon-like systems [2,45], ionized Argon [7] and mixtures [46]. It is important to stress that the RET kinetic equation, the kinetic equation for systems with multistep potentials of interaction and the Enskog-Landau kinetic equation for charged hard spheres were derived from the BBGKY hierarchy with a modified boundary condition in a binary collision approximation. Obviously, the presented equations have a restricted region of application. They cannot be applied to the description of systems with significantly collective effects caused by Coulomb, dipole or other long-range forces of interaction between the particles. To describe the collective effects in systems with a long-range character of interaction it is necessary to consider higher-order approximations for collision integrals. An example of such an equation is the Lenard-Balescu kinetic equation [47–49] for Coulomb plasma.

To analyze solutions to the BBGKY hierarchy with a modified boundary condition in higher-order approximations on interparticle correlations we have applied a concept of group expansions [11,12].

2.2. The Liouville equation and the BBGKY hierarchy with a modified boundary condition

Let us consider a system of N identical classical particles which are enclosed in volume V, with the Hamiltonian:

$$H=\sum_{j=1}^{N}\frac{p_j^2}{2m}+\frac{1}{2}\sum_{\substack{j=1\\ j\neq k}}^{N}\sum_{k=1}^{N}\varPhi\left(|\boldsymbol{r}_{jk}|\right), \tag{2.1}$$

where $\varPhi\left(|\boldsymbol{r}_{jk}|\right)$ is the interaction energy between two particles j and k; $|\boldsymbol{r}_{jk}|=|\boldsymbol{r}_j-\boldsymbol{r}_k|$ is the distance between a pair of interacting particles; $\boldsymbol{p}_j$ is the momentum of jth particle and m denotes its mass. A nonequilibrium state of such a system is described by the N-particle nonequilibrium distribution function $\varrho\left(x^N;t\right)=\varrho\left(x_1,\ldots,x_N;t\right)$ which satisfies the Liouville equation:

$$\left(\frac{\partial}{\partial t}+\mathrm{i}L_N\right)\varrho\left(x^N,t\right)=0, \tag{2.2}$$

where $\mathrm{i}=\sqrt{-1}$, $x_j=\{\boldsymbol{r}_j,\boldsymbol{p}_j\}$ is a set of phase variables (coordinates and momenta), L_N is the Liouville operator:

$$L_N=\sum_{j=1}^{N}L(j)+\frac{1}{2}\sum_{\substack{j=1\\ j\neq k}}^{N}\sum_{k=1}^{N}L(j,k), \tag{2.3}$$

where

$$\begin{aligned} L(j) &= -\mathrm{i}\frac{\boldsymbol{p}_j}{m}\frac{\partial}{\partial \boldsymbol{r}_j}, \\ L(j,k) &= \mathrm{i}\frac{\partial \Phi\left(|\boldsymbol{r}_{jk}|\right)}{\partial \boldsymbol{r}_{jk}}\left(\frac{\partial}{\partial \boldsymbol{p}_j}-\frac{\partial}{\partial \boldsymbol{p}_k}\right). \end{aligned}$$

The function $\varrho\left(x^N;t\right)$ is symmetrical with respect to permutations $x_l \leftrightarrows x_j$ of phase variables for an arbitrary pair of particles. This function satisfies the normalization condition

$$\int \mathrm{d}\Gamma_N \varrho\left(x^N;t\right) = 1, \quad \mathrm{d}\Gamma_N = (\mathrm{d}x)^N/N! = \mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{p}. \tag{2.4}$$

In order to solve the Liouville equation (2.2), it is necessary to introduce a boundary condition. It must be chosen in such a way that the solution to the Liouville equation will correspond to a physical state of the system under consideration. Using the nonequilibrium statistical operator method by D.N.Zubarev [9,10,50,51] (NSO method), we shall search for such solutions to the Liouville equation depending on time explicitly via the values for some set of observable variables, which is sufficient for describing a nonequilibrium state without depending on the initial time t_0.

The solution to the Liouville equation, which satisfies the initial condition

$$\varrho\left(x^N;t\right)|_{t=t_0} = \varrho_{\mathrm{q}}\left(x^N;t_0\right),$$

has the form:

$$\varrho\left(x^N;t,t_0\right) = \mathrm{e}^{-\mathrm{i}L_N(t-t_0)}\varrho_{\mathrm{q}}\left(x^N;t_0\right). \tag{2.5}$$

We shall consider such times $t \gg t_0$ when the details of the initial state of the system can be neglected. Then, to avoid the dependence on t_0, let us average (2.5) with respect to the initial times from t_0 to t and perform the boundary transition $(t-t_0) \to \infty$. As a result, one obtains [9,10,49]:

$$\varrho\left(x^N;t\right) = \varepsilon \int\limits_{-\infty}^{0} \mathrm{d}t'\, \mathrm{e}^{\varepsilon t'}\mathrm{e}^{\mathrm{i}L_N t'}\varrho_{\mathrm{q}}\left(x^N;t+t'\right), \qquad t' = t_0 - t, \tag{2.6}$$

where ε tends to $+0$ after the thermodynamic limit transition. It can be shown by straightforward differentiations that solution (2.6) satisfies the Liouville equation with an infinitesimal source in the right-hand side:

$$\left(\frac{\partial}{\partial t} + \mathrm{i}L_N\right)\varrho\left(x^N,t\right) = -\varepsilon\Big(\varrho\left(x^N,t\right) - \varrho_{\mathrm{q}}\left(x^N,t\right)\Big). \tag{2.7}$$

The source breaks the symmetry of the Liouville equation with respect to time inversion and selects retarded solutions which correspond to an abbreviated description of a nonequilibrium state of the system. The auxiliary function $\varrho_{\mathrm{q}}\left(x^N;t\right)$ denotes a quasiequilibrium distribution function that is defined from an extreme condition for the informational entropy of the system, provided the normalization condition is preserved and the average values for variables of the abbreviated description are fixed.

The choice of $\varrho_{\mathrm{q}}\left(x^N;t\right)$ depends mainly on a nonequilibrium state of the system under consideration. In the case of low density gases, where times of free motion are essentially larger than collision times, higher-order distribution functions of particles become dependent on time only via one-particle distribution functions [22,52,53]. It does mean that an abbreviated description of nonequilibrium states is available, and the total nonequilibrium distribution function depends on time via $f_1(x;t)$. In such a case, the quasiequilibrium distribution function $\varrho_{\mathrm{q}}\left(x^N;t\right)$ reads [52,53]:

$$\varrho_{\mathrm{q}}\left(x^N;t\right) = \prod_{j=1}^{N} \frac{f_1(x_j;t)}{\mathrm{e}}, \tag{2.8}$$

where e is the natural logarithm base. Then, the Liouville equation with a small source (2.7) in view of (2.8) corresponds to the abbreviated description of time evolution of the system on a kinetic stage, when only a one-particle distribution function is considered as a slow variable. However, there are always additional quantities which vary in time slowly because they are locally conserved. In the case of a one-component system, the mass density $\rho(\boldsymbol{r};t)$, momentum $\boldsymbol{j}(\boldsymbol{r};t)$ and total energy $\mathcal{E}(\boldsymbol{r};t)$ belong to such quantities. At long times they satisfy the generalized hydrodynamics equations. Generally speaking, the equation for $f_1(x;t)$ must be conjugated with these equations. For low density gases, such a conjugation can be done, in principle, with an arbitrary precision in each order on density. In high density gases and liquids, when a small parameter is absent, the correlation times corresponding to hydrodynamic quantities become commensurable with the characteristic times of varying one-particle distribution functions. Therefore, in dense gases and liquids, the kinetics and hydrodynamics are closely connected and should be considered simultaneously. That is why many-particle correlations, related to the local conservation laws of mass, momentum and total energy, cannot be neglected [1,5,9,10]. The local conservation laws affect the kinetic processes due to an interaction of the selected particles group with other particles of the system. This interaction is especially important in the case of high densities, and it must be taken into consideration. Then, at the construction of kinetic equations for high densities, it is necessary to choose the abbreviated description of a nonequilibrium system in the form to satisfy the true dynamics of conserved quantities automatically. To this end, the densities of hydrodynamic variables should be included together with the one-particle distribution function $f_1(x;t)$ into the initial set of parameters of the abbreviated description [1,5,9,10]. The next phase functions correspond to the densities of the hydrodynamic variables $\rho(\boldsymbol{r};t)$, $\boldsymbol{j}(\boldsymbol{r};t)$ and $\mathcal{E}(\boldsymbol{r};t)$:

$$\begin{aligned}
\hat{\rho}(\boldsymbol{r}) &= \int \mathrm{d}\boldsymbol{p}\, \hat{n}_1(x) m, \\
\hat{\boldsymbol{j}}(\boldsymbol{r}) &= \int \mathrm{d}\boldsymbol{p}\, \hat{n}_1(x) \boldsymbol{p}, \\
\hat{\mathcal{E}}(\boldsymbol{r}) &= \int \mathrm{d}\boldsymbol{p}\, \hat{n}_1(x) \frac{p^2}{2m} + \frac{1}{2} \int \mathrm{d}\boldsymbol{r}'\, \mathrm{d}\boldsymbol{p}\, \mathrm{d}\boldsymbol{p}'\, \hat{n}_2(x,x') \varPhi(|\boldsymbol{r}-\boldsymbol{r}'|),
\end{aligned} \tag{2.9}$$

where $\hat{n}_1(x)$ and $\hat{n}_2(x,x')$ are one- and two-particle microscopic phase densities by Klimontovich [24]:

$$\hat{n}_1(x) = \sum_{j=1}^{N} \delta(x-x_j) = \sum_{j=1}^{N} \delta(\boldsymbol{r}-\boldsymbol{r}_j)\delta(\boldsymbol{p}-\boldsymbol{p}_j), \tag{2.10}$$

$$\hat{n}_2(x,x') = \sum_{j\neq k=1}^{N} \delta(x-x_j)\delta(x'-x_k). \tag{2.11}$$

Relations (2.9) show a distinctive role of the potential interaction energy. Contrary to $\rho(\boldsymbol{r};t) = \langle\hat{\rho}(\boldsymbol{r})\rangle^t$ and $\boldsymbol{j}(\boldsymbol{r};t) = \langle\hat{\boldsymbol{j}}(\boldsymbol{r})\rangle^t$, the nonequilibrium values of the total energy $\mathcal{E}(\boldsymbol{r};t) = \langle\hat{\mathcal{E}}(\boldsymbol{r})\rangle^t$ cannot be expressed via the one-particle distribution function $f_1(x;t) = \langle\hat{n}_1(x)\rangle^t$ only, because in order to evaluate the potential part of $\mathcal{E}_{\mathrm{int}}(\boldsymbol{r};t) = \langle\hat{\mathcal{E}}_{\mathrm{int}}(\boldsymbol{r})\rangle^t$ it is necessary to involve the two-particle distribution function $f_2(x,x';t) = \langle\hat{n}_2(x,x')\rangle^t$. Here

$$\hat{\mathcal{E}}_{\mathrm{int}}(\boldsymbol{r}) = \frac{1}{2} \int \mathrm{d}\boldsymbol{r}'\, \mathrm{d}\boldsymbol{p}\, \mathrm{d}\boldsymbol{p}'\, \hat{n}_2(x,x') \varPhi(|\boldsymbol{r}-\boldsymbol{r}'|) \tag{2.12}$$

is the density of the potential energy of interaction. The next conclusion can be formulated as follows. If the one-particle distribution function $f_1(x;t)$ is chosen as a parameter of an abbreviated description, then the density of the interaction energy (2.12) can be considered as an additional independent parameter. One can find the quasiequilibrium distribution function $\varrho_{\mathrm{q}}\left(x^N;t\right)$ from the condition of extremum for a functional of the informational entropy $S_{\mathrm{inf}}(t) = -\int \mathrm{d}\varGamma_N\ \varrho\left(x^N;t\right) \ln\varrho\left(x^N;t\right)$ at fixed average values of $\langle\hat{n}_1(x)\rangle^t = f_1(x;t)$, $\langle\hat{\mathcal{E}}_{\mathrm{int}}(\boldsymbol{r})\rangle^t = \mathcal{E}_{\mathrm{int}}(\boldsymbol{r};t)$, including the normalization condition for $\varrho_{\mathrm{q}}\left(x^N;t\right)$ [5,9,10,49]: $\int \mathrm{d}\varGamma_N \varrho_{\mathrm{q}}\left(x^N;t\right) = \int \mathrm{d}\varGamma_N \varrho\left(x^N;t\right)$ $= 1$. This is equivalent to finding an unconditional extreme for the functional

$$L(\varrho) = \int \mathrm{d}\varGamma_N \varrho\left(x^N;t\right) \left\{1 - \ln\varrho\left(x^N;t\right) - \Phi(t) - \int \mathrm{d}\boldsymbol{r} \beta(\boldsymbol{r};t)\hat{\mathcal{E}}_{\mathrm{int}}(\boldsymbol{r}) - \int \mathrm{d}x\, a(x;t)\hat{n}_1(x)\right\},$$

where $\Phi(t)$, $\beta(\boldsymbol{r};t)$, $a(x;t)$ are the Lagrange multipliers.

Taking the variation $\dfrac{\delta}{\delta\varrho}L(\varrho)$, after some simple manipulations one obtains the quasiequilibrium distribution function

$$\varrho_{\mathrm{q}}\left(x^N;t\right) = \exp\left\{-\Phi(t) - \int \mathrm{d}\boldsymbol{r}\ \beta(\boldsymbol{r};t)\hat{\mathcal{E}}_{\mathrm{int}}(\boldsymbol{r}) - \int \mathrm{d}x\ a(x;t)\hat{n}_1(x)\right\}, \tag{2.13}$$

$$\Phi(t) = \ln \int \mathrm{d}\varGamma_N\ \exp\left\{-\int \mathrm{d}\boldsymbol{r}\ \beta(\boldsymbol{r};t)\hat{\mathcal{E}}_{\mathrm{int}}(\boldsymbol{r}) - \int \mathrm{d}x\ a(x;t)\hat{n}_1(x)\right\}. \tag{2.14}$$

Here, $\Phi(t)$ is the Massieu-Planck functional. It is determined from the condition of normalization for the distribution $\varrho_{\mathrm{q}}\left(x^N;t\right)$. Relation (2.13) was obtained for the first time in [1]. To determine the physical meaning of parameters $\beta(\boldsymbol{r};t)$ and $a(x;t)$ let us rewrite $\varrho_{\mathrm{q}}\left(x^N;t\right)$ (2.13) in the form:

$$\begin{aligned}\varrho_{\mathrm{q}}\left(x^N;t\right) &= \exp\left\{-\Phi(t) - \int \mathrm{d}\boldsymbol{r}\;\beta(\boldsymbol{r};t)\hat{\mathcal{E}}'(\boldsymbol{r}) - \int \mathrm{d}x\;a'(x;t)\hat{n}_1(x)\right\}, \qquad (2.15)\\ \Phi(t) &= \ln\int \mathrm{d}\Gamma_N\;\exp\left\{-\int \mathrm{d}\boldsymbol{r}\;\beta(\boldsymbol{r};t)\hat{\mathcal{E}}'(\boldsymbol{r}) - \int \mathrm{d}x\;a'(x;t)\hat{n}_1(x)\right\},\end{aligned}$$

where $\hat{\mathcal{E}}'(\boldsymbol{r})$ is the density of the total energy in a reference frame which moves together with a system element of the mass velocity $\boldsymbol{V}(\boldsymbol{r};t)$ [5]

$$\hat{\mathcal{E}}'(\boldsymbol{r}) = \hat{\mathcal{E}}(\boldsymbol{r}) - \boldsymbol{V}(\boldsymbol{r};t)\hat{\boldsymbol{j}}(\boldsymbol{r}) + \frac{m}{2}V^2(\boldsymbol{r};t)\hat{n}(\boldsymbol{r}). \qquad (2.16)$$

Here $\hat{n}(\boldsymbol{r}) = \int \mathrm{d}\boldsymbol{p}\;\hat{n}_1(x)$ is the density of the particles number. Parameters $\beta(\boldsymbol{r};t)$ and $a'(x;t)$ in (2.15) are determined from the conditions of self-consistency, namely, the equality of quasi-average values $\langle\hat{n}_1(x)\rangle_{\mathrm{q}}^t$ and $\langle\hat{\mathcal{E}}'(\boldsymbol{r})\rangle_{\mathrm{q}}^t$ with their real averages $\langle\hat{n}_1(x)\rangle^t$, $\langle\hat{\mathcal{E}}'(\boldsymbol{r})\rangle^t$:

$$\begin{aligned}\langle\hat{n}_1(x)\rangle_{\mathrm{q}}^t &= \langle\hat{n}_1(x)\rangle^t = f_1(x;t),\\ \langle\hat{\mathcal{E}}'(\boldsymbol{r})\rangle_{\mathrm{q}}^t &= \langle\hat{\mathcal{E}}'(\boldsymbol{r})\rangle^t,\end{aligned} \qquad \text{here}\quad \langle\ldots\rangle_{\mathrm{q}}^t = \int \mathrm{d}\Gamma_N\;\ldots\varrho_{\mathrm{q}}\left(x^N;t\right). \qquad (2.17)$$

In these transformations the parameters $a'(x;t)$ and $a(x;t)$ are connected by the relation

$$a'(x;t) = a(x;t) - \beta(\boldsymbol{r};t)\left\{\frac{p^2}{2m} - \boldsymbol{V}(\boldsymbol{r};t)\boldsymbol{p} + \frac{m}{2}V^2(\boldsymbol{r};t)\right\}.$$

In the case, when the conditions (2.17) take place, one can obtain some relations taking into account self-consistency conditions and varying the modified Massieu-Planck functional $\Phi(t)$ after (2.15) with respect to parameters $\beta(\boldsymbol{r};t)$ and $a'(x;t)$

$$\begin{aligned}\frac{\delta\Phi(t)}{\delta\beta(\boldsymbol{r};t)} &= -\langle\hat{\mathcal{E}}'(\boldsymbol{r})\rangle_{\mathrm{q}}^t = -\langle\hat{\mathcal{E}}'(\boldsymbol{r})\rangle^t,\\ \frac{\delta\Phi(t)}{\delta a(x;t)} &= -\langle\hat{n}_1(x)\rangle_{\mathrm{q}}^t = -\langle\hat{n}_1(x)\rangle^t = -f_1(x;t).\end{aligned} \qquad (2.18)$$

It means that parameter $\beta(\boldsymbol{r};t)$ is conjugated to the average energy in an accompanying reference frame, and $a'(x;t)$ is conjugated to the nonequilibrium one-particle distribution function $f_1(x;t)$. To determine the physical meaning of these parameters let us define the entropy of a system taking into account the self-consistency conditions (2.17):

$$S(t) = -\langle\ln\varrho_{\mathrm{q}}\left(x^N;t\right)\rangle_{\mathrm{q}}^t = \Phi(t) + \int \mathrm{d}\boldsymbol{r}\;\beta(\boldsymbol{r};t)\langle\hat{\mathcal{E}}'(\boldsymbol{r})\rangle^t + \int \mathrm{d}x\;a'(x;t)\langle\hat{n}_1(x)\rangle^t. \qquad (2.19)$$

Taking functional derivatives of $S(t)$ (2.19) with respect to $\langle\hat{\mathcal{E}}'(\boldsymbol{r})\rangle^t$ and $\langle\hat{n}_1(x)\rangle^t$ at fixed corresponding averaged values gives the following thermodynamic relations:

$$\frac{\delta S(t)}{\delta\langle\hat{\mathcal{E}}'(\boldsymbol{r})\rangle^t}=\beta(\boldsymbol{r};t),\quad \frac{\delta S(t)}{\delta f_1(x;t)}=a'(x;t). \tag{2.20}$$

Hence, $\beta(\boldsymbol{r};t)$ is an analogue of the inverse local temperature.

Two limiting cases follow from the structure of expression (2.19) for entropy. At $a'(x;t)=-\beta(\boldsymbol{r};t)\mu(\boldsymbol{r};t)$, (2.19) transforms into an expression for entropy which corresponds to the hydrodynamic description of the nonequilibrium state of the system [5,9,10]:

$$S(t)=\Phi(t)+\int \mathrm{d}\boldsymbol{r}\ \beta(\boldsymbol{r};t)\left(\langle\hat{\mathcal{E}}'(\boldsymbol{r})\rangle^t-\mu(\boldsymbol{r};t)\langle\hat{n}(\boldsymbol{r})\rangle^t\right), \tag{2.21}$$

with the following quasiequilibrium distribution function [5,9,10]:

$$\begin{aligned}\varrho_{\mathrm{q}}\left(x^N;t\right)&=\exp\left\{-\Phi(t)-\int \mathrm{d}\boldsymbol{r}\ \beta(\boldsymbol{r};t)\Big(\hat{\mathcal{E}}'(\boldsymbol{r})-\mu(\boldsymbol{r};t)\hat{n}(\boldsymbol{r})\Big)\right\}, \\ \Phi(t)&=\ln\int \mathrm{d}\mathit{\Gamma}_N\ \exp\left\{\int \mathrm{d}\boldsymbol{r}\ \beta(\boldsymbol{r};t)\Big(\hat{\mathcal{E}}'(\boldsymbol{r})-\mu(\boldsymbol{r};t)\hat{n}(\boldsymbol{r})\Big)\right\}\end{aligned} \tag{2.22}$$

and the corresponding self-consistency conditions for the definition of thermodynamic parameters $\beta(\boldsymbol{r};t)$, $\mu(\boldsymbol{r};t)$ (local value of the chemical potential):

$$\langle\hat{n}(\boldsymbol{r})\rangle^t_{\mathrm{q}}=\langle\hat{n}(\boldsymbol{r})\rangle^t,\quad \langle\hat{\mathcal{E}}'(\boldsymbol{r})\rangle^t_{\mathrm{q}}=\langle\hat{\mathcal{E}}'(\boldsymbol{r})\rangle^t.$$

In the case when the contribution of the interaction energy between the particles can be neglected (a dilute gas), the quasiequilibrium distribution function (2.15) has the form:

$$\varrho_{\mathrm{q}}\left(x^N;t\right)=\exp\left\{-\Phi(t)-\int \mathrm{d}\boldsymbol{r}\ \beta(\boldsymbol{r};t)\hat{\mathcal{E}}'_{\mathrm{kin}}(\boldsymbol{r})-\int \mathrm{d}x\ a'(x;t)\hat{n}_1(x)\right\},$$

or, taking into account (2.16) and the relation between $a'(x;t)$ and $a(x;t)$, one obtains:

$$\begin{aligned}\varrho_{\mathrm{q}}\left(x^N;t\right)&=\exp\left\{-\Phi(t)-\int \mathrm{d}x\ a(x;t)\hat{n}_1(x)\right\}, \\ \Phi(t)&=\ln\int \mathrm{d}\mathit{\Gamma}_N\ \exp\left\{-\int \mathrm{d}x\ a(x;t)\hat{n}_1(x)\right\},\end{aligned} \tag{2.23}$$

where

$$\hat{\mathcal{E}}'_{\mathrm{kin}}(\boldsymbol{r})=\hat{\mathcal{E}}_{\mathrm{kin}}(\boldsymbol{r})-\boldsymbol{V}(\boldsymbol{r};t)\hat{\boldsymbol{j}}(\boldsymbol{r})+\frac{m}{2}V^2(\boldsymbol{r};t)\hat{n}(\boldsymbol{r}),$$

$\hat{\mathcal{E}}_{\mathrm{kin}}(\boldsymbol{r})=\int \mathrm{d}\boldsymbol{p}\ \frac{p^2}{2m}\hat{n}_1(x)$ is the density of the kinetic energy. Now, determining in (2.23) parameter $a(x;t)$ with the help of the self-consistency condition $\langle\hat{n}(x)\rangle^t_{\mathrm{q}}=$

$\langle\hat{n}(x)\rangle^t$, one can show [5] that (2.23) for $\varrho_{\mathrm{q}}\left(x^N;t\right)$ transforms into distribution (2.8) when it is assumed that the only parameter of the abbreviated description for a nonequilibrium state of the system is a one-particle distribution function. As it is known [51,53], the quasiequilibrium distribution function (2.23) corresponds to the Boltzmann entropy of a dilute gas:

$$S_{\mathrm{B}}(t) = -\int \mathrm{d}x\ f_1(x;t)\ln\frac{f_1(x;t)}{\mathrm{e}}. \tag{2.24}$$

In a general case, when kinetic and hydrodynamic processes are considered simultaneously, the quasiequilibrium distribution function (2.15) or (2.13) can be rewritten in a somewhat different form. This form is more convenient for the comparison with $\varrho_{\mathrm{q}}\left(x^N;t\right)$ (2.8), obtained in a usual way [22,52], when $f_1(x;t)$ is the only parameter of the abbreviated description. First of all, let us note that one can include parameter $\Phi(t)$ from (2.13) into parameter $a(x;t)$ as a term which does not depend on x. Parameter $a(x;t)$ in $\varrho_{\mathrm{q}}\left(x^N;t\right)$ can be excluded with the help of the self-consistency condition $\langle\hat{n}(x)\rangle^t_{\mathrm{q}} = \langle\hat{n}(x)\rangle^t = f_1(x;t)$. Reduction of $\varrho_{\mathrm{q}}\left(x^N;t\right)$ results in

$$\varrho_{\mathrm{q}}\left(x^N;t\right) = \exp\left\{-U_N\left(\boldsymbol{r}^N;t\right)\right\}\prod_{j=1}^{N}\frac{f_1(x_j;t)}{u(\boldsymbol{r}_j;t)}, \tag{2.25}$$

where functions $u(\boldsymbol{r}_j;t)$ are obtained from the relations

$$\begin{aligned} u(\boldsymbol{r}_j;t) &= \int\frac{\mathrm{d}\boldsymbol{r}^{N-1}}{(N-1)!}\exp\left\{-U_N\left(\boldsymbol{r},\boldsymbol{r}^{N-1};t\right)\right\}\prod_{j=2}^{N}\frac{n(\boldsymbol{r}_j;t)}{u(\boldsymbol{r}_j;t)}, \\ U_N\left(\boldsymbol{r}^N;t\right) &= U_N\left(\boldsymbol{r}_1,\dots,\boldsymbol{r}_N;t\right) = \frac{1}{2}\sum_{j\neq k=1}^{N}\mathit{\Phi}(|\boldsymbol{r}_j-\boldsymbol{r}_k|)\beta(\boldsymbol{r}_k;t), \end{aligned} \tag{2.26}$$

$n(\boldsymbol{r};t) = \langle\hat{n}(\boldsymbol{r})\rangle^t = \int\mathrm{d}\boldsymbol{p}\ f_1(x;t)$ is the nonequilibrium particles concentration. In expression (2.25), $U_N(\boldsymbol{r};t)$ and $u(\boldsymbol{r}_j;t)$ respectively, depend explicitly and implicitly on $n(\boldsymbol{r};t)$ and $\beta(\boldsymbol{r};t)$ (or $\langle\hat{\mathcal{E}}'(\boldsymbol{r})\rangle^t$). To obtain the ordinary Bogolubov scheme [22,52], it is necessary to put $U_N(\boldsymbol{r};t) = 0$ in (2.25) and (2.26). Then, one can define $u = \mathrm{e}$, and (2.25) transforms into the quasiequilibrium distribution (2.8), as it should be. In a general case, $u(\boldsymbol{r};t)$ is a functional of the nonequilibrium density of particles number $n(\boldsymbol{r};t)$ and $\beta(\boldsymbol{r};t)$, which is an analogue of the inverse local temperature. Nevertheless, one should handle this analogy with care, as far as definition (2.25) can describe states which are far from local equilibrium. In particular, $f_1(x;t)$ can differ considerably from the local Maxwellian distribution.

The entropy expression (2.19) can be transformed according to the structure of the quasiequilibrium distribution function (2.25)

$$S(t) = \int\mathrm{d}\boldsymbol{r}\ \beta(\boldsymbol{r};t)\langle\hat{\mathcal{E}}_{\mathrm{int}}(\boldsymbol{r})\rangle^t - \int\mathrm{d}x\ f_1(x;t)\ln\frac{f_1(x;t)}{u(\boldsymbol{r};t)}. \tag{2.27}$$

Here the potential and kinetic parts are separated. In the case of low density gases, the influence of the potential energy can be neglected and $u(\boldsymbol{r};t) = \mathrm{e}$. Then, expression (2.27) tends to the usual Boltzmann entropy.

Thus, determining the quasiequilibrium distribution function $\varrho_{\mathrm{q}}\left(x^N;t\right)$ (2.25) and entropy $S(t)$ (2.27) of the system, when the nonequilibrium one-particle distribution function, as well as the average values of densities for the number of particles, momentum and energy are parameters of an abbreviated description of the nonequilibrium state which are locally conserved, the Liouville equation with the source (2.7) can be presented in the form [1,5]:

$$\left(\frac{\partial}{\partial t} + \mathrm{i}L_N\right)\varrho\left(x^N;t\right) = -\varepsilon\left(\varrho\left(x^N;t\right) - \exp\left\{-U_N\left(\boldsymbol{r}^N;t\right)\right\}\prod_{j=1}^{N}\frac{f_1(x_j;t)}{u(\boldsymbol{r}_j;t)}\right). \tag{2.28}$$

Further, on the basis of this equation, one obtains the BBGKY hierarchy for nonequilibrium distribution functions of classical particles with modified boundary conditions which take into account the nonequilibriumnes of the one-particle distribution function, as well as the local conservation laws. To obtain the first equation of the hierarchy, let us integrate with respect to the variables $x^{N-1} = \{x_2,\ldots,x_N\}$ the both parts of (2.28). Taking into account (2.10), (2.11) and (2.17), one obtains:

$$\left(\frac{\partial}{\partial t} + \mathrm{i}L(1)\right)f_1(x_1;t) + \int \mathrm{d}x_2\ \mathrm{i}L(1,2)f_2(x_1,x_2;t) = 0. \tag{2.29}$$

Integrating now (2.28) over the variables $x^{N-2} = \{x_3,\ldots,x_N\}$, after simple transformations one can obtain an equation for the two-particle distribution function $f_2(x_1,x_2;t)$ which differs from the corresponding equation in the Bogolubov hierarchy [1,3] by a source in the right-hand side:

$$\begin{aligned}&\left(\frac{\partial}{\partial t} + \mathrm{i}L_2\right)f_2(x_1,x_2;t) + \int \mathrm{d}x_3\Big(\mathrm{i}L(1,3) + \mathrm{i}L(2,3)\Big)f_3(x_1,x_2,x_3;t) = \\ &-\varepsilon\Big(f_2(x_1,x_2;t) - \mathrm{g}_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)f_1(x_1;t)f_1(x_2;t)\Big).\end{aligned} \tag{2.30}$$

In this equation, $L_2 = L(1) + L(2) + L(1,2)$ is the Liouville two-particle operator and $\mathrm{g}_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)$ denotes the binary coordinate distribution function for the quasiequilibrium state (2.25); $f_2(x_1,x_2;t)$ is the nonequilibrium two-particle distribution function:

$$f_2(x_1,x_2;t) = \langle\hat{n}_2(x_1,x_2;t)\rangle^t = \int \mathrm{d}\varGamma_{N-2}\ \varrho(x_1,x_2,x^{N-2};t).$$

Similarly, integrating equation (2.28) over $x^{N-s} = \{x_{s+1},\ldots,x_N\}$ phase variables, one derives next equations of the chain with the corresponding sources which define boundary conditions for reduced distribution functions. For the s-particle nonequilibrium distribution function

$$f_s(x^s;t) = \langle\hat{n}_s(x^s)\rangle^t = \int \mathrm{d}\varGamma_{N-s}\ \varrho(x_1,\ldots,x_s,x^{N-s};t)$$

one has the following equation:

$$\left(\frac{\partial}{\partial t}+\mathrm{i}L_s\right)f_s(x^s;t)+\int \mathrm{d}x_{s+1}\sum_{j=1}^{s}\mathrm{i}L(j,s+1)f_{s+1}(x^{s+1};t)=$$
$$-\varepsilon\Big(f_s(x^s;t)-\mathrm{g}_s(\boldsymbol{r}_s;t)\prod_{j=1}^{s}f_1(x_j;t)\Big), \tag{2.31}$$

where

$$\begin{aligned} L_s &= \sum_{j=1}^{s}L(j)+\frac{1}{2}\sum_{j\neq k=1}^{s}L(j,k), \qquad \mathrm{g}_s(\boldsymbol{r}^s;t) = f_s(\boldsymbol{r}^s;t)\Big/\prod_{j=1}^{s}n(\boldsymbol{r}_j;t), \\ \hat{n}_s(\boldsymbol{r}^s) &= \sum_{\substack{j_1=1\\ j_1\neq j_2\neq\ldots j_s}}^{s}\ldots\sum_{j_s=1}^{s}\sum_{k=1}^{s}\delta(\boldsymbol{r}_k-\boldsymbol{r}'_{j_k}), \qquad f_s(\boldsymbol{r}^s;t) = \langle\hat{n}_s(\boldsymbol{r}^s)\rangle_{\mathrm{q}}^{t}. \end{aligned} \tag{2.32}$$

As usual, we shall assume that the principle of weakening spatial correlations is valid for the quasiequilibrium state. Then, in the thermodynamic limit, the coordinate distribution functions $f_s(\boldsymbol{r}^s;t)$, $\mathrm{g}_s(\boldsymbol{r}^s;t)$ satisfy the boundary relations:

$$\lim_{(\min|\boldsymbol{r}_j-\boldsymbol{r}_k|)\to\infty}f_s(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_s;t) = \prod_{j=1}^{s}n(\boldsymbol{r}_j;t), \tag{2.33}$$

$$\lim_{(\min|\boldsymbol{r}_j-\boldsymbol{r}_k|)\to\infty}\mathrm{g}_s(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_s;t) = 1. \tag{2.34}$$

Therefore, taking into account "slow" hydrodynamical variables (the density of the interaction energy in the present case) leads to a modification of boundary conditions for the chain of equations (2.29)–(2.31) for nonequilibrium distribution functions. In order to reproduce the usual Bogolubov boundary conditions for the weakening of correlations [22], it is necessary to replace all $\mathrm{g}_s(\boldsymbol{r}^s;t)$ by their limiting values (2.34). Such a replacing is valid in the case of small density, however, new boundary conditions can be more useful for dense gasses, since they automatically take into account spatial correlations connected with the interaction of a pair of particles with the rest of the particles of the system. It is obvious that the influence of such an interaction increases as the density rises.

We note that the chain of equations (2.29)–(2.31) requires us to add equations for coordinate quasiequilibrium distribution functions which are functionals on the nonequilibrium density of the particles number $n(\boldsymbol{r};t)$ and the inverse local temperature $\beta(\boldsymbol{r};t)$. In particular, in [54] it is shown that the binary quasiequilibrium distribution function $\mathrm{g}_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)$ is connected with the pair quasiequilibrium correlation function $h_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)=\mathrm{g}_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)-1$ which satisfies the Ornstein-Zernike equation:

$$h_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)=c_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)+\int \mathrm{d}\boldsymbol{r}_3\; c_2(\boldsymbol{r}_1,\boldsymbol{r}_3;t)h_2(\boldsymbol{r}_2,\boldsymbol{r}_3;t)n(\boldsymbol{r}_3;t), \tag{2.35}$$

where $c_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t)$ is a direct quasiequilibrium correlation function.

Thus, the chain of equations (2.29)–(2.31) is distinguished from the usual Bogolubov hierarchy by the existence of sources in the right-hand sides, beginning from the second equation, and it takes into account both one-particle and collective hydrodynamical effects. It is important to investigate solutions to this chain of equations in the simplest approximations which lead to model kinetic equations for dense gases and simple liquids. We shall consider the most investigated binary collision approximation.

2.3. Binary collision approximation

In this section, we consider as an example of the use of hierarchy (2.31), the simplest approximation of binary collisions which in the case of the Bogolubov ordinary conditions of correlations weakening leads to the Boltzmann equation for $f_1(x_1; t)$. In equation (2.30) for $f_2(x_1, x_2; t)$ we omit the term with the three-particle distribution function, i.e. we take into account the influence of the "medium" on the evolution of the distinguished pair of particles only through the correlation corrections in the boundary condition. Then, we arrive at the equation:

$$\left(\frac{\partial}{\partial t} + \mathrm{i}L_2 + \varepsilon\right) f_2(x_1, x_2; t) = \varepsilon g_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t) f_1(x_1; t) f_1(x_2; t). \tag{2.36}$$

The formal solution of (2.36) has the form:

$$f_2(x_1, x_2; t) = \varepsilon \int\limits_{-\infty}^{0} \mathrm{d}\tau\; \mathrm{e}^{(\varepsilon + \mathrm{i}L_2)\tau} g_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t+\tau) f_1(x_1; t+\tau) f_1(x_2; t+\tau). \tag{2.37}$$

Following the Abel theorem [9,10,51,55,56], this solution can be written in the form:

$$f_2(x_1, x_2; t) = \lim_{\tau \to -\infty} \mathrm{e}^{\mathrm{i}L_2\tau} g_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t+\tau) f_1(x_1; t+\tau) f_1(x_2; t+\tau). \tag{2.38}$$

Substituting expression (2.38) into equation (2.29), one obtains a kinetic equation in the binary collision approximation

$$\left(\frac{\partial}{\partial t} + \mathrm{i}L(1)\right) f_1(x_1; t) = I_{\mathrm{coll}}(x_1; t), \tag{2.39}$$

where

$$I_{\mathrm{coll}}(x_1; t) = \int \mathrm{d}x_2\; \mathrm{i}L(1,2) \lim_{\tau \to -\infty} \mathrm{e}^{\mathrm{i}L_2\tau} g_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t+\tau) f_1(x_1; t+\tau) f_1(x_2; t+\tau) \tag{2.40}$$

is a collision integral. It is necessary to point out that equation (2.35) for the pair quasiequilibrium correlation function must be added to kinetic equation (2.29). Equation (2.35) takes into account an essential part of the many-particles correlations.

2.3.1. Kinetic equation of the revised Enskog theory

In papers [1,3] it was shown how the collision integral (2.40) in the case of a system of hard spheres transforms into the collision integral of the RET kinetic equation [34]. For subsequent manipulations, it is convenient to represent the particle interaction potential in the form:

$$\varPhi^{\rm hs}(|\boldsymbol{r}_{jk}|)=\lim_{a\to+\infty}\varPhi^{a}(|\boldsymbol{r}_{jk}|),\quad \varPhi^{a}(|\boldsymbol{r}_{jk}|)=\begin{cases} a, & |\boldsymbol{r}_{jk}|<\sigma,\\ 0, & |\boldsymbol{r}_{jk}|\geqslant\sigma,\end{cases} \tag{2.41}$$

where σ is a hard sphere diameter. Since the potential $\varPhi^{\rm hs}(|\boldsymbol{r}_{jk}|)$ is strongly singular (in particular, the operator $\mathrm{i}L(1,2)$ is not well defined), we shall operate with the potential $\varPhi^{a}(|\boldsymbol{r}_{jk}|)$, when deriving the kinetic equation, and set $a\to+\infty$ in the final expressions.

The limit $\tau\to-\infty$ in the collision integral (2.40) is mathematically formal. At the physical level of description, this limit assumes that $|\tau|\gg\tau_0$, where $\tau_0>0$ is some characteristic time scale. Depending on the choice of τ_0, we obtain different stages of the evolution of the system (kinetic, hydrodynamic). At the kinetic stage, τ_0 is a characteristic interaction time for which the singular potential (2.41) is an arbitrarily small quantity ($\tau_0\to+0$). Therefore, the limit $\tau-\tau_0\to-\infty$ keeps its form even in the case $\tau\to-0$, provided τ_0 is of the higher order of smallness relatively to τ :

$$\lim_{\tau\to-0}\left\{\lim_{\tau_0\to+0}\frac{\tau}{\tau_0}\right\}=-\infty. \tag{2.42}$$

In this case the collision integral (2.40) [1,3,54] for the hard spheres interparticle potential of interaction transforms into the following final expression:

$$\begin{aligned} I^{\rm hs}_{\rm coll}(x_1;t)=\sigma^2\int \mathrm{d}\hat{r}_{12}\,\mathrm{d}\boldsymbol{v}_2\,\theta(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g})(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g})\,\times \\ \left\{g^{\rm hs}_2(\boldsymbol{r}_1,\boldsymbol{r}_1+\sigma^+\hat{\boldsymbol{r}}_{12};t)f_1(\boldsymbol{r}_1,\boldsymbol{v}_1';t)f_1(\boldsymbol{r}_1+\sigma^+\hat{\boldsymbol{r}}_{12},\boldsymbol{v}_2';t)-\right.\\ \left. g^{\rm hs}_2(\boldsymbol{r}_1,\boldsymbol{r}_1-\sigma^+\hat{\boldsymbol{r}}_{12};t)f_1(\mathbf{r}_1,\boldsymbol{v}_1;t)f_1(\boldsymbol{r}_1-\sigma^+\hat{\boldsymbol{r}}_{12},\boldsymbol{v}_2;t)\right\}, \end{aligned} \tag{2.43}$$

where

$$\begin{aligned} \boldsymbol{v}_1' &= \boldsymbol{v}_1+\hat{\boldsymbol{r}}_{12}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g}),\\ \boldsymbol{v}_2' &= \boldsymbol{v}_2-\hat{\boldsymbol{r}}_{12}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g}). \end{aligned} \tag{2.44}$$

The collision integral in the form (2.43) is identical to the collision integral of the RET theory first introduced by van Beijeren and Ernst [34] on the basis of a diagram method. As Résibois showed [35,36], the kinetic equation of the RET theory satisfies an H-theorem.

We represent the collision integral (2.43) in a more compact form by using the two-particle quasi-Liouvillian (evolution pseudo-operator) $\hat{T}(1,2)$ [26]. Then, the kinetic equation takes the form:

$$\left(\frac{\partial}{\partial t}+\boldsymbol{v}_1\frac{\partial}{\partial\boldsymbol{r}_1}\right)f_1(x_1;t)=\int\mathrm{d}x_2\,\hat{T}(1,2)g^{\rm hs}_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)f_1(x_1;t)f_1(x_2;t), \tag{2.45}$$

$$\hat{T}(1,2) = \sigma^2 \int \mathrm{d}\hat{\boldsymbol{r}}_{12}\, \theta(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g})(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g}) \times \left\{\delta(\boldsymbol{r}_{12}+\sigma^+\hat{\boldsymbol{r}}_{12})\hat{B}(\hat{\boldsymbol{r}}_{12}) - \delta(\boldsymbol{r}_{12}-\sigma^+\hat{\boldsymbol{r}}_{12})\right\}, \tag{2.46}$$

$$\hat{B}(\hat{\boldsymbol{r}}_{12})\Psi(\boldsymbol{v}_1,\boldsymbol{v}_2) = \Psi(\boldsymbol{v}_1',\boldsymbol{v}_2'), \tag{2.47}$$

where Ψ is an arbitrary function of the velocities.

Thus, on the basis of the considered approach we have derived the kinetic equation of the RET theory without additional phenomenological assumptions. We have shown that within the framework of the employed method this equation corresponds to the simplest pair-collision approximation without allowance for retardation in time.

2.3.2. Kinetic equation for a multistep potential of interaction

The collision integral (2.40) is still rather complicated to be written in an explicit form for an arbitrary potential of interaction. As it is now known [30,32], only in two particular cases, namely, for hard spheres and square-well potentials, this integral is reduced to an analytical form. In order to extend these previous results, we consider the interparticle potential in the form of a multistep function

$$\mathit{\Phi}_{jk} \equiv \mathit{\Phi}_{jk}^{\mathrm{ms}} = \lim_{\varepsilon_0\to\infty} \mathit{\Phi}_{jk}^{\varepsilon_0}(r_{jk}), \tag{2.48}$$

where

$$\mathit{\Phi}_{jk}^{\varepsilon_0}(r_{jk}) = \begin{cases} \varepsilon_0, & r_{jk} < \sigma_0; \\ \varepsilon_l, & \sigma_{l-1} < r_{jk} < \sigma_l, \quad l = 1,\ldots,N^*; \\ 0, & \sigma_{N^*} < r_{jk}; \end{cases} \tag{2.49}$$

and N^* is the total number of walls except for a hard sphere wall. Potential of a hard sphere contains strong singularity (operator $L(j,k)$ is hard to define). That is why we shall deal with the potential $\mathit{\Phi}_{jk}^{\varepsilon_0}(r_{jk})$ and only in final expressions we shall put $\varepsilon_0 \to \infty$. Let us for convenience separate the systems of attractive and repulsive walls. Let n^* be the number of repulsive walls at the distance $\sigma_{\mathrm{r}i} \equiv \sigma_i$ and of the height $\triangle\varepsilon_i^{\mathrm{r}} = \varepsilon_i^{\mathrm{r}} - \varepsilon_{i+1}^{\mathrm{r}} > 0$, where $\varepsilon_i^{\mathrm{r}} \equiv \varepsilon_i$, $i = 1,\ldots,n^*$; and m^* – the number of attractive walls with the corresponding parameters $\sigma_{\mathrm{a}j} \equiv \sigma_{j+n^*}$, $\triangle\varepsilon_j^{\mathrm{a}} = \varepsilon_{j+1}^{\mathrm{a}} - \varepsilon_j^{\mathrm{a}} > 0$, where $\varepsilon_j^{\mathrm{a}} \equiv \varepsilon_{j+n^*}$, $j = 1,\ldots,m^*$; $\varepsilon_{n^*+1}^{\mathrm{r}} = \varepsilon_1^{\mathrm{a}}$, $\varepsilon_{m^*+1}^{\mathrm{a}} = 0$, $N^* = n^* + m^*$. Thus, the parameters σ_0 (hard sphere diameter), n^*, $\sigma_{\mathrm{r}i}$, $\triangle\varepsilon_i^{\mathrm{r}}$, m^*, $\sigma_{\mathrm{a}j}$ and $\triangle\varepsilon_j^{\mathrm{a}}$ completely determine the geometry of the multistep potential (see figure 1 where a specific case $n^* = 1$, $m^* = 3$ is shown and the multistep function approximates some real smooth potential).

The limit $\tau \to -\infty$ in the collision integral (2.40) is a formal one in the mathematical sense. On the physical level of description this means that $|\tau| \gg \tau_0$, where $\tau_0 > 0$ is some characteristic interval of time. For different values of τ_0 we can investigate different stages of the evolution of the system. In the Boltzmann theory [57] $|\tau| \gg \tau_{\mathrm{c}}$, where τ_{c} is the time of binary interactions (collision time). On the other hand, $|\tau|$ is far less than the characteristic scale of time τ_{m} for hydrodynamic

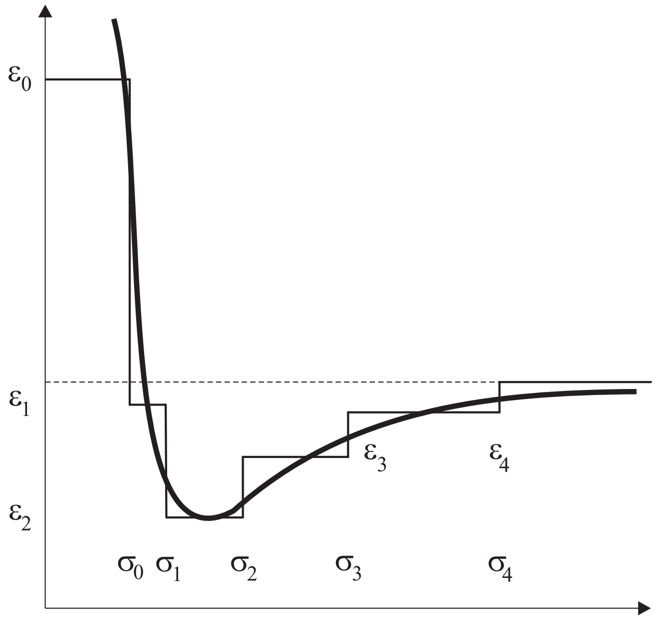

Figure 1. The multistep potential at $n^* = 1$ and $m^* = 3$.

variables. The above situation is possible because of the fact that for dilute gases the kinetic ($\tau \sim \tau_{\mathrm{c}}$) and hydrodynamic ($\tau \sim \tau_{\mathrm{m}}$) stages of the evolution are far from one another in time, i.e. $\tau_{\mathrm{c}} \ll \tau_{\mathrm{f}} \ll \tau_{\mathrm{m}}$, where τ_{f} is the characteristic time of free motion. The pattern is qualitatively different in dense gases and liquids with a realistic smooth potential of interaction. The kinetic and hydrodynamic stages appear to be closely connected. Besides, such quantities as the length of free motion and the time of interaction are not defined in a usual way, because all particles make influence on the dynamics of interaction for some chosen pair of particles.

However, for special types of potentials such a classification of times remains valid even for high densities. For a multi-step potential the region Ω of binary interactions consists of the following subregions $[\sigma_k - \triangle r_0, \sigma_k + \triangle r_0]\,, k = 0, 1, \ldots, N^*$, where $\triangle r_0 \to +0$ due to the singular nature of the potential under consideration. This potential has the finite range $\max\{\sigma_k\} = \sigma_{N^*} > 0$ of action. We can introduce the following set of specific time intervals: $\tau_0 = \triangle r_0/g_0 \to +0$ is the time of pair interactions on the walls, $\triangle\tau = \min\{\sigma_k - \sigma_{k-1}\}/g_0 > 0$ is the time of motion between the two nearest neighbouring walls and $t_{\mathrm{whole}} = \sigma_{N^*}/g_0 > 0$ is the time of motion of the whole systems of walls for some pair of particles, where g_0 is an average relative velocity of two particles. As the potential contains the horizontal parts, where the force of the interparticle interaction is equal to zero, it is also possible to introduce time τ_{f} as an average time of free motion of particles in the system. Changing the geometry of the potential and increasing the density, we can make this time arbitrarily small in order to match the relation

$$\tau_0 \ll \tau_{\mathrm{f}} \ll \triangle\tau < t_{\mathrm{whole}} < \tau_{\mathrm{m}}. \tag{2.50}$$

The kinetic stage of the evolution corresponds to small times of order $\tau \sim \tau_{\mathrm{f}}$. Therefore, as far as the relation (2.50) is satisfied, the formal limit $\tau \to -\infty$ should be considered as $\tau/\tau_0 \to -\infty$ or merely as $|\tau| \gg \tau_0$. For smooth non-singular potentials the region of interaction has some non-zero size and τ_0 is finite. For singular potentials the case $\tau_0 \to +0$ is possible and the limit $\tau/\tau_0 \to -\infty$ remains valid even for $\tau \to -0$ if only τ_0 is a value of the higher infinitesimal order

$$\lim_{\tau\to-0}\left\{\lim_{\tau_0\to+0}\frac{\tau}{\tau_0}\right\} \to -\infty. \tag{2.51}$$

Thus, the formal limit $\tau \to -0$ can be applied to the collision integral (2.40) in the sense of (2.50) and (2.51). As it is now well established, the limit $\tau \to -0$ leads to the kinetic equations of the RET [3,34,38] and RDRS [37] theories.

As it was shown in [2,8], the collision integral (2.40) for potential (2.48) taking into account (2.50), (2.51), can be presented in a compact form:

$$I_{\mathrm{coll}}^{\mathrm{ms}}(x_1;t) = \int \mathrm{d}x_2\, \hat{T}_{12} g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t) f_1(x_1;t) f_1(x_2;t) \tag{2.52}$$

in terms of the two-particle pseudo-Liouville operator

$$\hat{T}_{12} = \hat{T}_{\mathrm{hs}} + \sum_{i=1}^{n^*}\sum_{p=b,c,d} \hat{T}_{\mathrm{r}i}^{p} + \sum_{j=1}^{m^*}\sum_{p=b,c,d} \hat{T}_{\mathrm{a}j}^{p}, \tag{2.53}$$

which consists of the pseudo-Liouville operators for a hard sphere wall, for ith repulsive wall and for jth attractive wall, correspondingly:

$$\hat{T}_{\mathrm{hs}} = \sigma_0^2 \int \mathrm{d}\hat{\boldsymbol{r}}_{12}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g})\theta(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g})\left\{\delta(\boldsymbol{r}_{12}+\sigma_0^+\hat{\boldsymbol{r}}_{12})\hat{B}^a(\hat{\boldsymbol{r}}_{12}) - \delta(\boldsymbol{r}_{12}-\sigma_0^+\hat{\boldsymbol{r}}_{12})\right\}, \tag{2.54}$$

$$\hat{T}_{\mathrm{r}i}^{p} = \sigma_{\mathrm{r}i}^2 \int \mathrm{d}\hat{\boldsymbol{r}}_{12}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g})\theta_{\mathrm{r}i}^{p}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g})\left\{\delta(\boldsymbol{r}_{12}+\sigma_{\mathrm{r}i}^{1,p}\hat{\boldsymbol{r}}_{12})\hat{B}_{\mathrm{r}i}^{p}(\hat{\boldsymbol{r}}_{12}) - \delta(\boldsymbol{r}_{12}-\sigma_{\mathrm{r}i}^{2,p}\hat{\boldsymbol{r}}_{12})\right\}, \tag{2.55}$$

$$\hat{T}_{\mathrm{a}j}^{p} = \sigma_{\mathrm{a}j}^2 \int \mathrm{d}\hat{\boldsymbol{r}}_{12}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g})\theta_{\mathrm{a}j}^{p}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g})\left\{\delta(\boldsymbol{r}_{12}+\sigma_{\mathrm{a}j}^{1,p}\hat{\boldsymbol{r}}_{12})\hat{B}_{\mathrm{a}j}^{p}(\hat{\boldsymbol{r}}_{12}) - \delta(\boldsymbol{r}_{12}-\sigma_{\mathrm{a}j}^{2,p}\hat{\boldsymbol{r}}_{12})\right\}. \tag{2.56}$$

Here,

$$\begin{aligned}
&\theta_{\mathrm{r}i}^{b}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g}) = \theta(-\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g}), && \theta_{\mathrm{a}j}^{b}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g}) = \theta(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g}),\\
&\theta_{\mathrm{r}i}^{c}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g}) = \theta(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g}-\alpha_i^{\mathrm{r}}), && \theta_{\mathrm{a}j}^{c}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g}) = \theta(-\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g}-\alpha_j^{\mathrm{a}}),\\
&\theta_{\mathrm{r}i}^{d}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g}) = \theta(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g})\theta(\alpha_i^{\mathrm{r}}-\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g}), && \theta_{\mathrm{a}j}^{d}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g}) = \theta(-\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g})\theta(\alpha_i^{\mathrm{r}}+\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g})
\end{aligned} \tag{2.57}$$

are the corresponding unit step functions,

$$\begin{aligned}
&\sigma_{\mathrm{r}i}^{1,b} = \sigma_{\mathrm{r}i}^{+}, && \sigma_{\mathrm{r}i}^{2,b} = \sigma_{\mathrm{r}i}^{-}, && \sigma_{\mathrm{a}j}^{1,b} = \sigma_{\mathrm{a}j}^{-}, && \sigma_{\mathrm{a}j}^{2,b} = \sigma_{\mathrm{a}j}^{+},\\
&\sigma_{\mathrm{r}i}^{1,c} = \sigma_{\mathrm{r}i}^{-}, && \sigma_{\mathrm{r}i}^{2,c} = \sigma_{\mathrm{r}i}^{+}, && \sigma_{\mathrm{a}j}^{1,c} = \sigma_{\mathrm{a}j}^{+}, && \sigma_{\mathrm{a}j}^{2,c} = \sigma_{\mathrm{a}j}^{-},\\
&\sigma_{\mathrm{r}i}^{1,d} = \sigma_{\mathrm{r}i}^{+}, && \sigma_{\mathrm{r}i}^{2,d} = \sigma_{\mathrm{r}i}^{+}, && \sigma_{\mathrm{a}j}^{1,d} = \sigma_{\mathrm{a}j}^{-}, && \sigma_{\mathrm{a}j}^{2,d} = \sigma_{\mathrm{a}j}^{-},
\end{aligned} \tag{2.58}$$

and $\hat{B}^p$ are operators of the velocity displacement caused by an interaction of the p-type at each of the walls, namely,

$$
\begin{aligned}
\hat{B}^a(\hat{\boldsymbol{r}}_{12})\Psi(\boldsymbol{v}_1,\boldsymbol{v}_2) \;&=\; \hat{B}^d_{\mathrm{r}i}(\hat{\boldsymbol{r}}_{12})\Psi(\boldsymbol{v}_1,\boldsymbol{v}_2) \;=\; \hat{B}^d_{\mathrm{a}j}(\hat{\boldsymbol{r}}_{12})\Psi(\boldsymbol{v}_1,\boldsymbol{v}_2) \;=\; \Psi(\boldsymbol{v}'_1,\boldsymbol{v}'_2),\\
\hat{B}^b_{\mathrm{r}i}(\hat{\boldsymbol{r}}_{12})\Psi(\boldsymbol{v}_1,\boldsymbol{v}_2) \;&=\; \Psi(\boldsymbol{v}''_{\mathrm{r}1},\boldsymbol{v}''_{\mathrm{r}2}), \qquad \hat{B}^b_{\mathrm{a}j}(\hat{\boldsymbol{r}}_{12})\Psi(\boldsymbol{v}_1,\boldsymbol{v}_2) \;=\; \Psi(\boldsymbol{v}''_{\mathrm{a}1},\boldsymbol{v}''_{\mathrm{a}2}),\\
\hat{B}^c_{\mathrm{r}i}(\hat{\boldsymbol{r}}_{12})\Psi(\boldsymbol{v}_1,\boldsymbol{v}_2) \;&=\; \Psi(\boldsymbol{v}'''_{\mathrm{r}1},\boldsymbol{v}'''_{\mathrm{r}2}), \qquad \hat{B}^c_{\mathrm{a}j}(\hat{\boldsymbol{r}}_{12})\Psi(\boldsymbol{v}_1,\boldsymbol{v}_2) \;=\; \Psi(\boldsymbol{v}'''_{\mathrm{a}1},\boldsymbol{v}'''_{\mathrm{a}2}).
\end{aligned}
\tag{2.59}
$$

Physically, the values $\sigma^{1,p}_{\mathrm{r}i}$ ($\sigma^{1,p}_{\mathrm{a}j}$) (2.58) correspond to the distances between the particles just after the interaction of the p-type on the ith repulsive (jth attractive) wall of the potential, whereas the values $\sigma^{2,p}_{\mathrm{r}i}$ ($\sigma^{2,p}_{\mathrm{a}j}$) correspond to the distances just before the collision.

The H-theorem for the kinetic equation with a multistep potential of interaction (2.52) was proved in [8]. The normal solution by using the boundary conditions method [40,58] was found in [2,45]. On the basis of the solution found, numerical calculations of transport coefficients for liquid Argon along the liquid-vapour curve were performed.

2.3.3. Enskog-Landau kinetic equations for systems of charged hard spheres

We consider kinetic equation (2.39) in the binary collision approximation with collision integral (2.40), when the interaction potentials $\varPhi(|\boldsymbol{r}_{jk}|)$ of classical particles at short distances can be modelled by the hard-sphere potential $\varPhi^{\mathrm{hs}}(|\boldsymbol{r}_{jk}|)$ (2.41), and at large distances by a certain long-range smooth “tail” $\varPhi^{\mathrm{l}}(|\boldsymbol{r}_{jk}|)$, i.e.

$$
\varPhi(|\boldsymbol{r}_{jk}|) = \varPhi^{\mathrm{hs}}(|\boldsymbol{r}_{jk}|) + \varPhi^{\mathrm{l}}(|\boldsymbol{r}_{jk}|), \tag{2.60}
$$

where

$$
\varPhi^{\mathrm{l}}(|\boldsymbol{r}_{jk}|) = \begin{cases} 0, & |\boldsymbol{r}_{jk}| < \sigma, \\ \varPhi^{\mathrm{l}}(|\boldsymbol{r}_{jk}|), & |\boldsymbol{r}_{jk}| \geqslant \sigma. \end{cases} \tag{2.61}
$$

We note that breaking the particle interaction potential (2.60) into short- and long-range parts is not unique and, therefore, there arises the problem of optimal separation.

These problems have often been discussed in equilibrium statistical mechanics [59]. The similar ideas about the separation of the interaction potential at the derivation of kinetic equations were used by Rudyak [60,61].

With allowance for (2.3) and (2.41), the collision integral (2.40) for the interparticle potential (2.60), (2.61) takes the form:

$$
I_{\mathrm{coll}}(x_1;t) \;=\; I_1(x_1;t) + I_2(x_1;t), \tag{2.62}
$$

$$
\begin{aligned}
I_1(x_1;t) \;=\; -\lim_{a\to+\infty}\int_0^{\sigma^+} \mathrm{d}r_{12}\, r_{12}^2 \int \mathrm{d}\hat{r}_{12} \int \mathrm{d}\boldsymbol{v}_2\; \mathrm{i}L^a(1,2) \lim_{\tau\to-0} \mathrm{e}^{\mathrm{i}L_2^a\tau} \times \\
g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t+\tau) f_1(x_1;t+\tau) f_1(x_2;t+\tau),
\end{aligned}
\tag{2.63}
$$

$$I_2(x_2;t) = -\int\limits_{\sigma^+}^{\infty} \mathrm{d}r_{12}\, r_{12}^2 \int \mathrm{d}\hat{r}_{12} \int \mathrm{d}\boldsymbol{v}_2\, \mathrm{i}L^{\mathrm{l}}(1,2) \lim_{\tau\to-\infty} \mathrm{e}^{\mathrm{i}\left(L_2^{(0)}+L^{\mathrm{l}}(1,2)\right)\tau} \times g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t+\tau) f_1(x_1;t+\tau) f_1(x_2;t+\tau), \tag{2.64}$$

where

$$L_2^{(0)} = L(1) + L(2), \quad L^{\mathrm{l}}(1,2) = \mathrm{i}\frac{\hat{\boldsymbol{r}}_{12}}{m^*}\frac{\partial}{\partial r_{12}}\varPhi^{\mathrm{l}}(|\boldsymbol{r}_{12}|)\left(\frac{\partial}{\partial \boldsymbol{v}_1} - \frac{\partial}{\partial \boldsymbol{v}_2}\right) \tag{2.65}$$

and we have used the idea of time separation of the interactions: instant collisions of hard spheres ($\tau \to -0$) and an extended process of interaction with the long-range potential ($\tau \to -\infty$). In accordance with the results obtained in subsection 3.1, the first term in the right-hand side of (2.62) is identical to the collision integral of the hard-sphere system (the right-hand side of (2.45)):

$$I_1(x_1;t) = \int \mathrm{d}x_2\, \hat{T}(1,2) g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t) f_1(x_1;t) f_1(x_2;t) \tag{2.66}$$

with the only difference that here $g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)$ is a quasiequilibrium binary distribution function of the particles in the system with a total interparticle interaction potential (2.60).

In the case when the long-range part of the interaction potential is absent ($\varPhi^{\mathrm{l}}(r)=0$), the first part $I_1(x_1;t)$ of the collision integral is identical to the collision integral of the RET theory; the second part $I_2(x_1;t)$ is then identically equal to zero. In the second limiting case, when the density of the system is low (rarefied gases: $n \to 0$, $g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t) \to 1$ and the hard-sphere part of the potential vanishes ($\sigma \to +0$), the second part $I_2(x_1;t)$ of the collision integral is identical to the collision integral of the Boltzmann equation [30], and at the same time $I_1(x_1;t) \to 0$.

In a special case, when the long-range interaction is weak, we make an expansion for $\exp\{(\mathrm{i}L_2^{(0)} + \mathrm{i}L^{\mathrm{l}}(1,2))\tau\}$, restricting ourselves to the term linear in $\mathrm{i}L^{\mathrm{l}}(1,2)$. Then, the second part $I_2(x_1;t)$ (2.64) reduces to the form:

$$I_2(x_1;t) = I_2^{(0)}(x_1;t) + I_2^{(1)}(x_1;t), \tag{2.67}$$

$$I_2^{(0)}(x_1;t) = -\int\limits_{\sigma^+}^{\infty} \mathrm{d}r_{12}\, r_{12}^2 \int \mathrm{d}\hat{r}_{12} \int \mathrm{d}\boldsymbol{v}_2\, \mathrm{i}L^{\mathrm{l}}(1,2) \lim_{\tau\to-\infty} \mathrm{e}^{\mathrm{i}L_2^{(0)}\tau} \times g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t+\tau) f_1(x_1;t+\tau) f_1(x_2;t+\tau), \tag{2.68}$$

$$I_2^{(1)}(x_1;t) = \int\limits_{\sigma^+}^{\infty} \mathrm{d}r_{12}\, r_{12}^2 \int \mathrm{d}\hat{r}_{12} \int \mathrm{d}\boldsymbol{v}_2\, \mathrm{i}L^{\mathrm{l}}(1,2) \lim_{\tau\to-\infty} \mathrm{e}^{\mathrm{i}L_2^{(0)}\tau} \times \int\limits_0^{\tau} \mathrm{d}\tau'\, \mathrm{e}^{-\mathrm{i}L_2^{(0)}\tau'} \mathrm{i}L^{\mathrm{l}}(1,2) \mathrm{e}^{\mathrm{i}L_2^{(0)}\tau'} g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t+\tau) f_1(x_1;t+\tau) f_1(x_2;t+\tau). \tag{2.69}$$

The first term $I_2^{(0)}(x_1;t)$ is a generalization of the Vlasov mean field in KMFT [38], and the second $I_2^{(1)}(x_1;t)$ is a generalized Landau collision integral with allowance for retardation in time in the approximation of the second order on interaction. Indeed, if we set formally $g_2(\boldsymbol{r};t) \equiv 1$ (rarefied gases) and $\sigma \to 0$, and completely ignore the spatial inhomogeneity of $f_1(x_1;t)$ and the time retardation, then the second term $I_2^{(1)}(x_1;t)$ (2.69) is transformed into the ordinary Landau collision integral [62]. In the case when $\Phi^{\mathrm{l}}(|\boldsymbol{r}_{12}|)$ is the Coulomb interaction potential, the complete collision integral (2.62), (2.66), (2.67) can be called the Enskog-Landau collision integral for a system of charged hard spheres which, in contrast to the ordinary Landau collision integral, does not diverge at short distances. However, at large distances we will observe a divergence, as usual. To eliminate this, it is necessary to take into account the effects of screening [24,63,64]. To avoid this problem *sequentially* we have to consider the kinetic equation with taking into account the dynamical screening effects [63]. But this way is impossible with the Enskog-Landau kinetic equation. The only thing we can do for further calculation is to change the upper integral limit to some finite value which could have a meaning of the statical screening value in our system (see the following subsections). To *solve* this problem we must consider dynamical screening effects.

Using the boundary conditions method, a normal solution to the generalized Enskog-Landau kinetic equation was found in [65,66]. This solution coincides with that obtained in [3,7] for a stationary case. On the basis of normal solutions, numerical calculations of such transport coefficients as viscosity and thermal conductivity were performed for once-ionized Argon [7] and for mixtures of ionized inert gases [46].

2.4. Modified group expansions for the construction of solutions to the BBGKY hierarchy

2.4.1. Modified group expansions

Recently, a kinetic equation of the revised Enskog theory for a dense system of hard spheres (2.45) and an Enskog-Landau kinetic equation for a dense system of charged hard spheres with collision integrals (2.66), (2.67) have been obtained from the BBGKY hierarchy in the binary collisions approximation [3,7]. It should be noted that this approximation does not correspond to the usual two-particle approximation inherent in the Boltzmann theory, because an essential part of the many-particle correlations is implicitly taken into account by the pair quasiequilibrium distribution function $g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)$.

To analyze solutions to the BBGKY hierarchy (2.31) [1,7] in higher approximations on interparticle correlations, it is more convenient to use the concept of group expansions [67–69]. This was applied to the BBGKY hierarchy in previous investigations [26,28,52,67,69] using the boundary conditions which correspond to the weakening correlations principle by Bogolubov [22]. The same conception was envolved in papers by Zubarev and Novikov [52], where a diagram method for obtaining solutions to the BBGKY hierarchy was developed.

To analyze the BBGKY hierarchy (2.31), we turn to the papers by Zubarev and Novikov [52] and earlier ones by Green [67] and Cohen [68,69], and pass from the nonequilibrium distribution functions $f_s(x^s;t)$ to the irreducible distribution ones $G_s(x^s;t)$, which can be introduced by the equalities presented in [52,67]. In our case, with some modifications we obtain:

$$\begin{aligned}
&f_1(x_1;t) = G_1(x_1;t), \\
&f_2(x_1,x_2;t) = G_2(x_1,x_2;t) + g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)G_1(x_1;t)G_1(x_2;t), \\
&f_3(x_1,x_2,x_3;t) = G_3(x_1,x_2,x_3;t) + \sum\nolimits_P G_2(x_1,x_2;t)G_1(x_3;t) + \\
&g_3(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;t)G_1(x_1;t)G_1(x_2;t)G_1(x_3;t), \\
&\vdots\,\vdots
\end{aligned} \tag{2.70}$$

Here, the position-dependent quasiequilibrium distribution functions $g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)$, $g_3(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;t)$, $g_s(\boldsymbol{r}^s;t)$ are defined in [11,12]. The modification of group expansions (2.70) consists in the fact that a considerable part of space time correlations is accumulated in the quasiequilibrium functions $g_s(\boldsymbol{r}^s;t)$. If all $g_s(\boldsymbol{r}^s;t) = 1$ for $s = 2,3,\ldots$, these group expansions coincide with those of papers [52,67–69]. As far as each line in (2.70) brings in new functions $G_s(\boldsymbol{r}^s;t)$, $s = 1,2,3,\ldots$, the corresponding equations can be solved with respect to irreducible distribution functions and we can write the following:

$$\begin{aligned}
&G_1(x_1;t) = f_1(x_1;t), \\
&G_2(x_1,x_2;t) = f_2(x_1,x_2;t) - g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)f_1(x_1;t)f_1(x_2;t), \\
&G_3(x_1,x_2,x_3;t) = f_3(x_1,x_2,x_3;t) - \sum\nolimits_P f_2(x_1,x_2;t)f_1(x_3;t) - \\
&h_3(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;t)f_1(x_1;t)f_1(x_2;t)f_1(x_3;t), \\
&\vdots\,\vdots .
\end{aligned} \tag{2.71}$$

In (2.70) and (2.71), the symbol $\sum_P$ denotes the sum of all different permutations of coordinates for three and more particles

$$\begin{aligned}
h_3(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;t) &= g_3(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;t) - g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t) - g_2(\boldsymbol{r}_1,\boldsymbol{r}_3;t) - g_2(\boldsymbol{r}_2,\boldsymbol{r}_3;t) \\
&\equiv h_3'(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;t) - 2,
\end{aligned} \tag{2.72}$$

where $h_3'(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;t)$ is a three-particle quasiequilibrium correlation function. Now let us write the BBGKY hierarchy (2.31) [1,3,7] for the irreducible distribution functions $G_s(x^s;t)$, namely, the first two equations,

$$\begin{aligned}
&\left(\frac{\partial}{\partial t} + \mathrm{i}L(1)\right) G_1(x_1;t) + \\
&\int \mathrm{d}x_2\; \mathrm{i}L(1,2)g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)G_1(x_1;t)G_1(x_2;t) + \int \mathrm{d}x_2\; \mathrm{i}L(1,2)G_2(x_1,x_2;t) = 0.
\end{aligned} \tag{2.73}$$

Differentiating the relation for $G_2(x_1,x_2;t)$ in (2.71) with respect to time and using the second equation from the BBGKY hierarchy for the function $f_2(x_1,x_2;t)$, we can get for the pair irreducible distribution function $G_2(x_1, x_2; t)$ an equation, which reads:

$$\left(\frac{\partial}{\partial t} + \mathrm{i}L_2 + \varepsilon\right) G_2(x_1, x_2; t) = -\left(\frac{\partial}{\partial t} + \mathrm{i}L_2\right) g_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t) G_1(x_1; t) G_1(x_2; t) -$$
$$\int \mathrm{d}x_3 \left\{\mathrm{i}L(1,3) + \mathrm{i}L(2,3)\right\} \left\{G_3(x_1, x_2, x_3; t) + \sum_P G_2(x_1, x_2; t) G_1(x_3; t) +\right.$$
$$\left. g_3(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3; t) G_1(x_1; t) G_1(x_2; t) G_1(x_3; t)\right\}. \tag{2.74}$$

In a similar way, we can obtain other equations for the three-particle irreducible function $G_3(x_1, x_2, x_3; t)$ and the higher $G_s(x^s; t)$ ones. One remembers now that the appearance of the quasiequilibrium distribution functions $g_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t)$, $g_3(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3; t)$, $g_s(\boldsymbol{r}^s; t)$ in the hierarchy is closely connected with the fact that the boundary conditions for the solutions of the Liouville equation take into consideration both the nonequilibrium character of the one-particle distribution function and the local conservation laws, which corresponds to a consistent description of the kinetics and hydrodynamics of the system [1,3]. Since in the present paper we analyze two first equations, (2.73) and (2.74) only, we will not write the others. It is important to note that, if we put formally $g_s(\boldsymbol{r}^s; t) \equiv 1$ for all $s = 2, 3, \ldots$ in (2.73) and (2.74), we come to the first two equations of the BBGKY hierarchy for the irreducible distribution functions $G_1(x_1; t)$ and $G_2(x_1, x_2; t)$, which were obtained in paper [52] by D.N.Zubarev and M.Yu.Novikov. The first term in the right-hand side of (2.74) is a peculiarity of (2.73) and (2.74) equation system. This is a term with a time derivative of the pair quasiequilibrium distribution function $g_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t)$. As it was shown in [1,3], the binary quasiequilibrium distribution function is a functional of the local values of temperature $\beta(\boldsymbol{r}; t)$ and mean particle density $n(\boldsymbol{r}; t)$. Thus, time derivatives of $g_2(\boldsymbol{r}_1, \boldsymbol{r}_2 | \beta(t), n(t))$ will conform to $\beta(\boldsymbol{r}; t)$ and $n(\boldsymbol{r}; t)$. These quantities, in their turn, according to the self-consistency conditions [1,7], will be expressed via the average energy value $\langle \hat{\mathcal{E}}'(\boldsymbol{r}) \rangle^t$ in an accompanying reference frame and via $\langle \hat{n}(\boldsymbol{r}) \rangle^t$, which constitute a basis of the hydrodynamical description of a nonequilibrium state of the system. Solving equation (2.74) for the irreducible quasiequilibrium two-particle distribution function $G_2(x_1, x_2; t)$ in the generalized polarization approximation and taking into account the first equation of the chain (2.73), lead to [11,12]:

$$\left(\frac{\partial}{\partial t} + \mathrm{i}L(1)\right) G_1(x_1; t) + \int \mathrm{d}x_2\, \mathrm{i}L(1,2) g_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t) G_1(x_1; t) G_1(x_2; t) =$$
$$\int \mathrm{d}x_2 \int\limits_{-\infty}^{t} \mathrm{d}t'\, \mathrm{e}^{\varepsilon(t'-t)} \mathrm{i}L(1,2) U(t,t') \times \tag{2.75}$$
$$\left(\frac{\partial}{\partial t'} + \mathrm{i}L_2 + \mathscr{L}(x_1, x_2; t')\right) g_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t') G_1(x_1; t') G_1(x_2; t'),$$

where

$$
\begin{aligned}
U(t,t') &= \exp_+\left\{-\int_{t'}^{t} \mathrm{d}t''\ [\mathrm{i}L_2 + \mathscr{L}(x_1,x_2;t'')]\right\},\\
\mathscr{L}(x_1,x_2;t) &= \mathscr{L}(x_1;t) + \mathscr{L}(x_2;t).
\end{aligned} \tag{2.76}
$$

The operator $\mathscr{L}(x_1;t)$ can be obtained by a variation of the Vlasov collision integral near the nonequilibrium distribution $G_1(x_1;t)$, namely,

$$
\begin{aligned}
\delta\left(\int \mathrm{d}x_3\ \mathrm{i}L(1,3)G_1(x_3;t)G_1(x_1;t)\right) &= \\
\int \mathrm{d}x_3\ \mathrm{i}L(1,3)G_1(x_3;t)\delta G_1(x_1;t) &= \mathscr{L}(x_1;t)\delta G_1(x_1;t).
\end{aligned} \tag{2.77}
$$

This is a kinetic equation for a nonequilibrium one-particle distribution function with the non-Markovian collision integral in the generalized polarization approximation. It should be noted that the presence of the Vlasov operator $\mathscr{L}(x_1,x_2;t)$ in the collision integral (2.75) indicates taking into consideration collective effects. Analysis of the collision integral (2.75) in a general case is a rather complicated problem. But it is obvious that the collision integral in (2.75) or an expression for $G_2(x_1,x_2;t)$ with (2.74) may be much simplified for every physical model of a particle system or for each nonequilibrium state of the collision integral in (2.75). To show this, we shall consider two particular cases: a hard spheres model and Coulomb plasma.

2.4.2. Hard spheres model in the polarization approximation

In this subsection we shall investigate kinetic processes for a hard spheres model in the approximations which are higher than a binary collisions one. We take into account the character of the model parameters and the results of the previous section of this article and papers [26,49]. This investigation is convenient to carry out on the basis of the equation chain (2.73), (2.74) at the formal substitution of a potential part of the Liouville operator $\mathrm{i}L(1,2)$ by the Enskog collision operator $\hat{T}(1,2)$ [26,70]. In this case, equations (2.73) and (2.74) have the form:

$$
\begin{aligned}
&\left(\frac{\partial}{\partial t} + \mathrm{i}L(1)\right) G_1(x_1;t) + \\
&\int \mathrm{d}x_2\ \hat{T}(1,2) g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)G_1(x_1;t)G_1(x_2;t) + \int \mathrm{d}x_2\ \hat{T}(1,2)G_2(x_1,x_2;t) = 0,
\end{aligned} \tag{2.78}
$$

$$
\begin{aligned}
&\left(\frac{\partial}{\partial t} + \mathrm{i}L_2^0 + \hat{T}(1,2) + \varepsilon\right) G_2(x_1,x_2;t) = \\
&\quad -\left(\frac{\partial}{\partial t} + \mathrm{i}L_2^0 + \hat{T}(1,2)\right) g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t)G_1(x_1;t)G_1(x_2;t) \\
&\quad -\int \mathrm{d}x_3\ \left\{\hat{T}(1,3) + \hat{T}(2,3)\right\}\Big\{G_3(x_1,x_2,x_3;t) + \sum_P G_2(x_1,x_2;t)G_1(x_3;t) \\
&\quad + g_3(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;t)G_1(x_1;t)G_1(x_2;t)G_1(x_3;t)\Big\}.
\end{aligned} \tag{2.79}
$$

Further, we will consider the same approximations concerning equation (2.79), in which $G_3(x_1,x_2,x_3;t)$ and $h_3(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;t)$ are neglected. Then, if we introduce similarly to (2.77) the Boltzmann-Enskog collision operator $C(x_1;t)$ ($\mathscr{L}(x_1;t) \to C(x_1;t)$), equation (2.79) could be rewritten in the next form:

$$\delta \int \mathrm{d}x_3\, \hat{T}(1,3) G_1(x_1;t) G_1(x_3;t) = C(x_1;t)\delta G_1(x_1;t), \tag{2.80}$$

$$\left(\frac{\partial}{\partial t} + \mathrm{i}L_2^0 + \hat{T}(1,2) + C(x_1,x_2;t) + \varepsilon\right) G_2(x_1,x_2;t) = \tag{2.81}$$
$$- \left(\frac{\partial}{\partial t} + \mathrm{i}L_2^0 + \hat{T}(1,2) + C(x_1,x_2;t)\right) g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t) G_1(x_1;t) G_1(x_2;t),$$

Hence it appears that the formal solution to $G_2(x_1,x_2;t)$ reads:

$$G_2(x_1,x_2;t) = -\int\limits_{-\infty}^{0} \mathrm{d}t'\, \mathrm{e}^{\varepsilon(t'-t)} U_{\mathrm{hs}}(t,t') \times \tag{2.82}$$
$$\left\{\frac{\partial}{\partial t'} + \mathrm{i}L_2^0 + \hat{T}(1,2) + C(x_1,x_2;t')\right\} g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t') G_1(x_1;t') G_1(x_2;t'),$$

where $U_{\mathrm{hs}}(t,t')$ is an evolution operator for the system of hard spheres:

$$\begin{aligned} U_{\mathrm{hs}}(t,t') &= \exp_+\left\{-\int_{t'}^{t} \mathrm{d}t''\, \left[\mathrm{i}L_2^0 + \hat{T}(1,2) + C(x_1,x_2;t'')\right]\right\}, \\ C(x_1,x_2;t) &= C(x_1;t) + C(x_2;t). \end{aligned} \tag{2.83}$$

Now let us put (2.82) into the first equation (2.78). Then, the resulting equation takes the form [11,12]:

$$\left(\frac{\partial}{\partial t} + \mathrm{i}L(1)\right) G_1(x_1;t) = \int \mathrm{d}x_2\, \hat{T}(1,2) g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t) G_1(x_1;t) G_1(x_2;t) - \tag{2.84}$$
$$\int \mathrm{d}x_2\, \hat{T}(1,2) \int\limits_{-\infty}^{0} \mathrm{d}t'\, \mathrm{e}^{\varepsilon(t'-t)} U_{\mathrm{hs}}(t,t') \left\{\frac{\partial}{\partial t'} + \mathrm{i}L_2^0 + \hat{T}(1,2) + C(x_1,x_2;t')\right\} \times$$
$$g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t') G_1(x_1;t') G_1(x_2;t').$$

This equation can be called a generalized kinetic equation for the nonequilibrium one-particle distribution function of hard spheres with a non-Markovian collision integral in the generalized polarization approximation. The first term in the right-hand side of this equation is the collision integral from the revised Enskog theory [1,7,34,54]. Neglecting time retardation effects and assuming that the operator $C(x_1,x_2;t)$ does not depend on time when

$$G_1(x_1;t) = f_0(\boldsymbol{p}) = n\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left\{-\frac{p^2}{2mkT}\right\}$$

is a local equilibrium Maxwell distribution function, the next term can be rewritten in a simplified form:

$$I_{\mathrm{R}}(x_1;t) = -\int\limits_{-\infty}^{t} \mathrm{d}t'\ \mathrm{e}^{\varepsilon(t'-t)} R_0(x_1;t,t')G_1(x_1;t') - \int\limits_{-\infty}^{t} \mathrm{d}t'\ \mathrm{e}^{\varepsilon(t'-t)} R_1(x_1;t,t')G_1(x_1;t'), \tag{2.85}$$

where

$$R_0(x_1;t,t') = \int \mathrm{d}x_2\, \hat{T}(1,2) \exp\Big\{(t'-t)\Big[\mathrm{i}L_2^0 + \hat{T}(1,2) + C(x_1,x_2)\Big]\Big\} \times \\ \big[\mathrm{i}L_2^0 + C(x_1,x_2)\big]\, g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t')G_1(x_2;t'), \tag{2.86}$$

$$R_1(x_1;t,t') = \int \mathrm{d}x_2\, \hat{T}(1,2) \exp\Big\{(t'-t)\Big[\mathrm{i}L_2^0 + \hat{T}(1,2) + C(x_1,x_2)\Big]\Big\} \times \\ \hat{T}(1,2) g_2(\boldsymbol{r}_1,\boldsymbol{r}_2;t')G_1(x_2;t'), \tag{2.87}$$

$R_1(x_1;t,t')$ is a generalized ring operator. The kinetic equation (2.84) together with (2.86) and (2.87) is the generalization of the kinetic equation for a system of hard spheres which was obtained by Bogolubov in [49,70]. It coincides with the case, when the quasiequilibrium pair distribution function of hard spheres is set formally to be a unity.

2.4.3. Coulomb plasma in the polarization approximation

Here we shall study an electron gas, which is contained in a homogeneous positively charged equilibrating background. This background can be created, for example, by hard motionless ions. Then, electrons interact according to the Coulomb law:

$$\varPhi(|\boldsymbol{r}_{12}|) = \frac{e^2}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|} = \frac{e^2}{|\boldsymbol{r}_{12}|},$$

the Fourier transform of which exists in the form of a real function $\varPhi(|\boldsymbol{k}|)$:

$$\frac{e^2}{r_{12}} = \int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3} \varPhi(|\boldsymbol{k}|) \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_{12}}, \quad \varPhi(|\boldsymbol{k}|) = \varPhi(k) = \frac{4\pi e^2}{k^2}, \tag{2.88}$$

here $\boldsymbol{k}$ is a wavevector, e is an electron charge. Let us consider equation chain (2.73), (2.74), when $G_3(x_1,x_2,x_3;t)=0$, $g_3(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;t)=0$ in the homogeneous case, when $G_1(x_1;t)=G_1(\boldsymbol{p}_1;t)$ and pair distribution functions depend on $|\boldsymbol{r}_{12}|$. Following the Bogolubov method [22], we shall assume that the one-particle distribution function $G_1(\boldsymbol{p}_1;t)$ is calculated in the "zeroth" order on the interaction constant q, pair distribution functions $G_2(\boldsymbol{r}_{12},\boldsymbol{p}_1,\boldsymbol{p}_2;t)$ and $g_2(\boldsymbol{r}_{12};t)$ in the first order on q, and $G_3(x_1,x_2,x_3;t)$, $g_3(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;t) \sim q^2$, where $q = \frac{e^2}{r_{\mathrm{d}}}\varTheta$, $r_{\mathrm{d}} = \sqrt{\varTheta/4\pi e^2 n}$ is the Debye radius, $n = N/V$, $\varTheta = k_{\mathrm{B}}T$, k_{B} is the Boltzmann constant, T is thermodynamic temperature. Therefore, to obtain an equation for $G_2(\boldsymbol{r}_{12},\boldsymbol{p}_1,\boldsymbol{p}_2;t)$ in the first approximation on the interaction constant q without time retardment effects, it is necessary to retain all integral terms, but omit all the others. In this case, using the Fourier transform with respect to spatial coordinates for a homogeneous Coulomb electron gas, the set of equations (2.73), (2.74) yields:

$$\begin{aligned}\frac{\partial}{\partial t}G_1(\boldsymbol{p}_1;t) \;=\; & -\frac{\partial}{\partial \boldsymbol{p}_1}\int \mathrm{d}\boldsymbol{k}\;\mathrm{d}\boldsymbol{p}_2\;\mathrm{i}\varPhi(|\boldsymbol{k}|)g_2(\boldsymbol{k};t)G_1(\boldsymbol{p}_1;t)G_1(\boldsymbol{p}_2;t)\\ & -\frac{\partial}{\partial \boldsymbol{p}_1}\int \mathrm{d}\boldsymbol{k}\;\mathrm{d}\boldsymbol{p}_2\;\mathrm{i}\varPhi(|\boldsymbol{k}|)G_2(\boldsymbol{k},\boldsymbol{p}_1,\boldsymbol{p}_2;t),\end{aligned}$$

or

$$\begin{aligned}\frac{\partial}{\partial t}G_1(\boldsymbol{p}_1;t) \;=\; & \frac{\partial}{\partial \boldsymbol{p}_1}G_1(\boldsymbol{p}_1;t)\int \mathrm{d}\boldsymbol{k}\;\boldsymbol{k}\varPhi(|\boldsymbol{k}|)\,\Im\mathfrak{m}\,g_2(\boldsymbol{k};t)\\ & +\frac{\partial}{\partial \boldsymbol{p}_1}\int \mathrm{d}\boldsymbol{k}\;\boldsymbol{k}\varPhi(|\boldsymbol{k}|)\,\Im\mathfrak{m}\,G_2(\boldsymbol{k},\boldsymbol{p}_1;t)\end{aligned} \tag{2.89}$$

and an equation for $G_2(\boldsymbol{k},\boldsymbol{p}_1,\boldsymbol{p}_2;t)$:

$$\begin{aligned}&\left(\frac{\partial}{\partial t}+\mathrm{i}\boldsymbol{k}\frac{\boldsymbol{p}_{12}}{m}+\varepsilon\right)G_2(\boldsymbol{k},\boldsymbol{p}_1,\boldsymbol{p}_2;t)= \\ &\mathrm{i}\boldsymbol{k}\varPhi(|\boldsymbol{k}|)\left\{\frac{\partial}{\partial \boldsymbol{p}_1}G_1(\boldsymbol{p}_1;t)\int \mathrm{d}\boldsymbol{p}_3\;G_2(\boldsymbol{k},\boldsymbol{p}_2,\boldsymbol{p}_3;t)-\frac{\partial}{\partial \boldsymbol{p}_2}G_1(\boldsymbol{p}_2;t)\int \mathrm{d}\boldsymbol{p}_3\;G_2(\boldsymbol{k},\boldsymbol{p}_1,\boldsymbol{p}_3;t)\right\}+\\ &\mathrm{i}\boldsymbol{k}\varPhi(|\boldsymbol{k}|)\left\{\frac{\partial}{\partial \boldsymbol{p}_1}G_1(\boldsymbol{p}_1;t)g_2(-\boldsymbol{k};t)G_1(\boldsymbol{p}_2;t)-\frac{\partial}{\partial \boldsymbol{p}_2}G_1(\boldsymbol{p}_2;t)g_2(\boldsymbol{k};t)G_1(\boldsymbol{p}_1;t)\right\},\end{aligned} \tag{2.90}$$

$\varepsilon\to+0$, and $G_2(\boldsymbol{k},\boldsymbol{p}_1;t)=\int \mathrm{d}\boldsymbol{p}_2\;\;G_2(\boldsymbol{k},\boldsymbol{p}_1,\boldsymbol{p}_2;t)$; $\Im\mathfrak{m}\,g_2(\boldsymbol{k};t)$, $\Im\mathfrak{m}\,G_2(x_1,x_2;t)$ are imaginary parts of the corresponding distribution functions. The following properties should be noted:

$$\begin{aligned}G_2(-\boldsymbol{k},\boldsymbol{p}_1,\boldsymbol{p}_2;t) \;&=\; G_2^*(\boldsymbol{k},\boldsymbol{p}_1,\boldsymbol{p}_2;t),\\ g_2(-\boldsymbol{k};t) \;&=\; g_2^*(\boldsymbol{k};t),\end{aligned}$$

where $*$ denotes a complex conjugation. The solution to (2.90), neglecting time retardment effects, reads:

$$\begin{aligned}&G_2(\boldsymbol{k},\boldsymbol{p}_1,\boldsymbol{p}_2;t)= \\ &\frac{\boldsymbol{k}\varPhi(|\boldsymbol{k}|)}{\boldsymbol{k}\cdot\dfrac{\boldsymbol{p}_{12}}{m}-\mathrm{i}0}\left\{\frac{\partial}{\partial \boldsymbol{p}_1}G_1(\boldsymbol{p}_1;t)G_2(-\boldsymbol{k},\boldsymbol{p}_2;t)-\frac{\partial}{\partial \boldsymbol{p}_2}G_1(\boldsymbol{p}_2;t)G_2(\boldsymbol{k},\boldsymbol{p}_1;t)\right\}+\\ &\frac{\boldsymbol{k}\varPhi(|\boldsymbol{k}|)}{\boldsymbol{k}\cdot\dfrac{\boldsymbol{p}_{12}}{m}-\mathrm{i}0}\left\{\frac{\partial}{\partial \boldsymbol{p}_1}G_1(\boldsymbol{p}_1;t)g_2(-\boldsymbol{k};t)G_1(\boldsymbol{p}_2;t)-\frac{\partial}{\partial \boldsymbol{p}_2}G_1(\boldsymbol{p}_2;t)g_2(\boldsymbol{k};t)G_1(\boldsymbol{p}_1;t)\right\}.\end{aligned} \tag{2.91}$$

It should also be noted that equation (2.89) contains an imaginary part of the irreducible pair nonequilibrium distribution function, to be integrated with respect to momentum of the second particle. Now one integrates equation (2.91) over all the values of momentum $\boldsymbol{p}_2$ and defines in such a way some function $G_2(\boldsymbol{k},\boldsymbol{p}_1;t)$:

$$\left[1+\int \mathrm{d}\boldsymbol{p}_2\;\frac{\boldsymbol{k}\varPhi(|\boldsymbol{k}|)}{\boldsymbol{k}\cdot\dfrac{\boldsymbol{p}_{12}}{m}-\mathrm{i}0}\frac{\partial}{\partial \boldsymbol{p}_2}G_1(\boldsymbol{p}_2;t)\right]G_2(\boldsymbol{k},\boldsymbol{p}_1;t)= \tag{2.92}$$

$$\frac{\partial}{\partial \boldsymbol{p}_1} G_1(\boldsymbol{p}_1;t) \int \mathrm{d}\boldsymbol{p}_2 \, \frac{\boldsymbol{k}\Phi(|\boldsymbol{k}|)}{\boldsymbol{k}\cdot\frac{\boldsymbol{p}_{12}}{m} - \mathrm{i}0} G_2(-\boldsymbol{k},\boldsymbol{p}_2;t) +$$
$$\int \mathrm{d}\boldsymbol{p}_2 \, \frac{\boldsymbol{k}\Phi(|\boldsymbol{k}|)}{\boldsymbol{k}\cdot\frac{\boldsymbol{p}_{12}}{m} - \mathrm{i}0} \left\{ \frac{\partial}{\partial \boldsymbol{p}_1} G_1(\boldsymbol{p}_1;t) g_2(-\boldsymbol{k};t) G_1(\boldsymbol{p}_2;t) - \frac{\partial}{\partial \boldsymbol{p}_2} G_1(\boldsymbol{p}_2;t) g_2(\boldsymbol{k};t) G_1(\boldsymbol{p}_1;t) \right\}.$$

Further, we should exclude from (2.92) the term with $G_2(-\boldsymbol{k},\boldsymbol{p}_2;t)$. To do this, we follow Lenard [47,49] and integrate equation (2.92) over the momentum component $\boldsymbol{p}_{1\perp}$, which is perpendicular to wavevector $\boldsymbol{k}$. The resulting expression then reads:

$$[1+\Phi(|\boldsymbol{k}|)\chi(\boldsymbol{k},p_1;t)]\, G_2(\boldsymbol{k},p_1;t) = \frac{\partial}{\partial \boldsymbol{p}_1} G_1(p_1;t) \int \mathrm{d}\boldsymbol{p}_2 \, \frac{\boldsymbol{k}\Phi(|\boldsymbol{k}|)}{k\cdot\frac{p_{12}}{m} - \mathrm{i}0} G_2(-\boldsymbol{k},\boldsymbol{p}_2;t) +$$
$$\int \mathrm{d}\boldsymbol{p}_2 \, \frac{\boldsymbol{k}\Phi(|\boldsymbol{k}|)}{k\cdot\frac{p_{12}}{m} - \mathrm{i}0} \left\{ \frac{\partial}{\partial \boldsymbol{p}_1} G_1(p_1;t) g_2(-\boldsymbol{k};t) G_1(\boldsymbol{p}_2;t) - \frac{\partial}{\partial \boldsymbol{p}_2} G_1(\boldsymbol{p}_2;t) g_2(\boldsymbol{k};t) G_1(p_1;t) \right\},. \tag{2.93}$$

Here the following conventional designations have been introduced:

$$\begin{aligned}
\chi(\boldsymbol{k},p_1;t) &= \int \mathrm{d}\boldsymbol{p}_2 \, \frac{\boldsymbol{k}}{k\cdot\frac{p_{12}}{m} - \mathrm{i}0} \frac{\partial}{\partial \boldsymbol{p}_2} G_1(\boldsymbol{p}_2;t), & p_1 &= \frac{\boldsymbol{p}_1\cdot\boldsymbol{k}}{k}, \\
G_1(p_1;t) &= \int \mathrm{d}\boldsymbol{p}_{1\perp} \, G_1(\boldsymbol{p}_1;t), & p_2 &= \frac{\boldsymbol{p}_2\cdot\boldsymbol{k}}{k}, \\
G_2(\boldsymbol{k},p_1;t) &= \int \mathrm{d}\boldsymbol{p}_{1\perp} \, G_2(\boldsymbol{k},\boldsymbol{p}_1;t), & k &= |\boldsymbol{k}|.
\end{aligned} \tag{2.94}$$

Now we multiply the both equations, (2.92) and (2.93), by $\frac{\partial}{\partial p_1} G_1(p_1;t)$ and by $\frac{\partial}{\partial p_1} G_1(\boldsymbol{p}_1;t)$, respectively, and subtract them:

$$\Big(1+\Phi(|\boldsymbol{k}|)\chi(\boldsymbol{k},p_1;t)\Big) \left[G_2(\boldsymbol{k},\boldsymbol{p}_1;t)\frac{\partial}{\partial p_1} G_1(p_1;t) - G_2(\boldsymbol{k},p_1;t)\frac{\partial}{\partial p_1} G_1(\boldsymbol{p}_1;t) \right] =$$
$$\Phi(|\boldsymbol{k}|)\chi(\boldsymbol{k},p_1;t) g_2(\boldsymbol{k};t) \left[G_1(p_1;t)\frac{\partial}{\partial p_1} G_1(\boldsymbol{p}_1;t) - G_1(\boldsymbol{p}_1;t)\frac{\partial}{\partial p_1} G_1(p_1;t) \right]. \tag{2.95}$$

If we extract the imaginary part of this equation, one can find the unknown quantity $\Im\mathfrak{m}\, G_2(\boldsymbol{k},\boldsymbol{p}_1;t)$, provided $\Im\mathfrak{m}\, G_2(\boldsymbol{k},p_1;t) = 0$ [11,12]:

$$\begin{aligned}
\frac{\partial}{\partial p_1} G_1(p_1;t)\, \Im\mathfrak{m}\, G_2(\boldsymbol{k},\boldsymbol{p}_1;t) &= \frac{\Phi(|\boldsymbol{k}|)\, \Im\mathfrak{m}\, [\chi(\boldsymbol{k},p_1;t) g_2(\boldsymbol{k};t)]}{|1+\Phi(|\boldsymbol{k}|)\chi(\boldsymbol{k},p_1;t)|^2} \\
&\times \left[G_1(p_1;t)\frac{\partial}{\partial p_1} G_1(\boldsymbol{p}_1;t) - G_1(\boldsymbol{p}_1;t)\frac{\partial}{\partial p_1} G_1(p_1;t) \right].
\end{aligned} \tag{2.96}$$

Since $\Im\mathfrak{m}\, \chi(\boldsymbol{k},p_1;t)$=-$\pi\frac{\partial}{\partial p_1} G_1(p_1;t)$ [64], putting an expression for $\Im\mathfrak{m}\, G_2(\boldsymbol{k},\boldsymbol{p}_1;t)$ into equation (2.89) gives the generalized Bogolubov-Lenard-Balescu kinetic equation for an electron gas in an equilibrating background:

$$\begin{aligned}
\frac{\partial}{\partial t} G_1(\boldsymbol{p}_1;t) &= \frac{\partial}{\partial \boldsymbol{p}_1} G_1(\boldsymbol{p}_1;t) \int \mathrm{d}\boldsymbol{k}\; \boldsymbol{k}\Phi(|\boldsymbol{k}|)\, \Im\mathfrak{m}\, g_2(\boldsymbol{k};t) + \\
&\frac{\partial}{\partial \boldsymbol{p}_1} \int \mathrm{d}\boldsymbol{p}_2 \; Q(\boldsymbol{p}_1,\boldsymbol{p}_2;t) \left[\frac{\partial}{\partial \boldsymbol{p}_1} - \frac{\partial}{\partial \boldsymbol{p}_2} \right] G_1(\boldsymbol{p}_1;t) G_1(\boldsymbol{p}_2;t),
\end{aligned} \tag{2.97}$$

where $Q(\boldsymbol{p}_1,\boldsymbol{p}_2;t)$ is a second rank tensor

$$Q(\boldsymbol{p}_1,\boldsymbol{p}_2;t) = -\pi \int \mathrm{d}\boldsymbol{k}\ \frac{|\varPhi(|\boldsymbol{k}|)|^2\,\boldsymbol{k}\cdot\boldsymbol{k}}{|1+\varPhi(|\boldsymbol{k}|)\chi(\boldsymbol{k},p_1;t)|^2}\,\Im\mathfrak{m}\,g_2(\boldsymbol{k};t)\delta(\boldsymbol{k}\cdot(\boldsymbol{p}_1-\boldsymbol{p}_2)), \quad (2.98)$$

which coincides with $Q(\boldsymbol{p}_1,\boldsymbol{p}_2)$ [64] at $\Im\mathfrak{m}\,g_2(\boldsymbol{k};t) = 1$. In this case, the kinetic equation (2.97) transforms into the well-known Lenard-Balescu equation [47–49,64]. Evidently, the generalized Bogolubov-Lenard-Balescu kinetic equation (2.97) claims the description of a dense electron gas, since in both the generalized mean field and the generalized Bogolubov-Lenard-Balescu collision integrals, many-particle correlations are treated by the imaginary part of $g_2(\boldsymbol{k};t)$. Nevertheless, the problem of divergence in the collision integral of equation (2.97) at small distances ($k\to\infty$) still remains. There are papers where the divergence of collision integrals is avoided with the help of a special choice of the differential cross section (quantum systems [71]), or via a combination of simpler collision integrals (classical systems [24]). These generalizations for collision integrals are attractive by their simplicity and helpful for ideal plasma. But, contrary to the obtained by us Bogolubov-Lenard-Balescu kinetic equation, they do not work for nonideal plasma. In accordance with the proposed structure of the collision integral

$$I_{\text{total}} = I_{\text{Boltzmann}} - I_{\text{Landau}} + I_{\text{Lenard-Balescu}}$$

the influence of particles interaction on plasma energy will be defined by the correlation function $g_2(r)$. Its asymptotic is

$$\lim_{r\to\infty}\left[\underbrace{\exp\left\{-\frac{e_a e_b}{r k_{\mathrm{B}}T}\right\}-1}_{\text{Boltzmann}}+\underbrace{\frac{e_a e_b}{r k_{\mathrm{B}}T}}_{\text{Landau}}-\underbrace{\frac{e_a e_b}{r k_{\mathrm{B}}T}\mathrm{e}^{-r/r_{\mathrm{D}}}}_{\text{Lenard-Balescu}}\right]=\frac{1}{r^2} \qquad (!),$$

where r_{D} denotes the Debye radius. In other combinations one arrives at false expressions for thermodynamic functions [24]. Dynamical screening, which appears in the generalized Bogolubov-Lenard-Balescu collision integral obtained by us, is free of these discrepancies. Generally speaking, the problem of divergency could be solved within the framework of a charged hard spheres model, combining the results of this section and the preciding one. But this step is an intricate and complicated problem and needs a separate consideration.

Evidently, an investigation of the obtained kinetic equation is important in view of its solutions and studying transport coefficients and time correlation functions for model systems.

In view of the dense systems study, where the consideration of spatial interparticle correlations is important, the BBGKY hierarchy (2.73), (2.74) with the modified boundary conditions and group expansions has quite a good perspective. The kinetic equation (2.84)–(2.87) is a generalization of the Bogolubov one [49,70] for a system of hard spheres. M.Ernst and J.Dorfman [72] investigated collective modes in an inhomogeneous gas and showed that the solution of a dispersion equation for hydrodynamic modes leads to a nonanalytic frequency dependence on a wavevector. This is connected with the fact that the ring operator for inhomogeneous systems at small wavenumbers has a term proportional to $\sqrt{k}$. Similar investigations of collective modes and time correlation functions in the hydrodynamic region were carried out by Bogolubov [70]. Nevertheless, it is necessary to carry out analogous investigations of hydrodynamic collective modes and time correlation functions on the basis of kinetic equation (2.84), taking into account (2.85)–(2.87), where some part of space correlations is considered in the pair quasiequilibrium function $g_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t)$. Obviously, these results may appear to be good for very dense gases, which could be described by a hard spheres model. An important factor is that in the kinetic equation (2.84)–(2.87), as well as in the generalized Bogolubov-Lenard-Balescu one, collective effects are taken into account both via the Vlasov mean field and the binary quasiequilibrium correlation function which is a functional of nonequilibrium values of temperature and a chemical potential.

Transferring the obtained results to quantum systems is not obvious. Such a procedure is rather complicated and needs additional investigations. Nevertheless, some steps in this way have been already done by Morozov and Röpke [9,10,16].

3. Generalized transport equations and time correlation functions

3.1. Overview

One of the main problems of the nonequilibrium statistical mechanics of liquids is an investigation of collective excitations, time correlation functions and transport coefficients because, using these quantities, we can compare the corresponding theoretical results with the data on light and neutron scattering as well as with the results obtained by the method of molecular dynamics (MD) [73–79]. At present, one can distinguish three regions in these investigations.

The first one is the usual hydrodynamics in a linear approximation when the time and spatial evolution of small excitations in liquids can be described in terms of hydrodynamic modes (in terms of eigenvalues and eigenvectors of the linear hydrodynamics equations) [26,31,77,80–82]. Such hydrodynamic modes are the heat mode $z_{\mathrm{H}}(\boldsymbol{k}) = -D_{\mathrm{T}}k^2$, two sound modes $z_{\pm}(\boldsymbol{k}) = \pm\mathrm{i}ck - \Gamma k^2$ and two viscosity modes $z_{\nu_{1,2}} = -\nu k^2$ (see, for example, [81]). In our notations $\boldsymbol{k}$ denotes a wavenumber which describes spatial dispersion of excitations, $D_{\mathrm{T}} = \lambda/nc_p$ is a coefficient of thermal diffusion, $\nu = \eta/mn$ is the kinematical viscosity, $\Gamma = \frac{2}{3}\nu + \frac{1}{3}\kappa/mn + \frac{1}{2}(c_p/c_V - 1)D_{\mathrm{T}}$ is the constant of sound decrement. λ, κ and η are the thermal conductivity, bulk and shear viscosity coefficients defined by the Green-Kubo formulas, n is the average density, m is the mass of a separate particle, c_p and c_V are specific heats at constant pressure and volume, c is the sound velocity. In this region, where $|\boldsymbol{k}|^{-1}$ is much larger than atomic sizes of liquid σ, and $\omega\tau \ll 1$, where ω is frequency and τ is the characteristic correlation time, the dynamical structure factor $S(\boldsymbol{k};\omega)$ is well described by the Landau-Plachek formula [26,81,84] in terms of Brillouin lines which corresponds to heat and sound modes. With increasing wavevector values $\boldsymbol{k}$, the precision of the hydrodynamic description decreases, since correlations at short times and small spatial distances which are inherent in neutron scattering in liquids are not described by linear hydrodynamic equations.

Extensions of the usual hydrodynamics were performed on the basis of modern methods of nonequilibrium statistical mechanics by Zwanzig and Mori [85–87] with the use of the method of projection operators, by Sergeev and Tishchenko [88,89] using the NSO method [50,51] and by Tserkovnikov with the use of the method of Green functions [90–92]. Transport equations were obtained for the average values of the densities of mass, momentum and energy. These equations generalize the hydrodynamic ones. In these equations, thermodynamic quantities c_p, c_V depend on wavevector $\boldsymbol{k}$, whereas the transport coefficients λ, κ and η depend on $\boldsymbol{k}$, as well as on frequency ω. Moreover, new generalized transport coefficients appear in the generalized hydrodynamic equations. These coefficients describe dynamical correlations between the thermal and viscous motions which vanish in the limit $k \to 0$ and $\omega \to 0$. This is the second region, namely, the region of the generalized hydrodynamics [26,77,83,87,59,89–94]. Papers by Götze, Lücke, Bosse and others [95–101] are of special interest. In these papers, an investigation of spectra for fluctuations of densities for the number of particles, their longitudinal and transverse fluxes for small as well as for intermediate values of $\boldsymbol{k}$ and ω is performed. For Argon [73,76] and Rubidium [74–76], a good coincidence with the experimental data was obtained. It is necessary to point out papers by Yulmetyev and Shurygin [102–106] where the dynamical structure factor $S(\boldsymbol{k};\omega)$ for Rubidium [103–105] and Argon [102,106] was investigated by the projection operator method taking into account non-Markovian effects. The results obtained in these papers are valid in the low-frequency approximation.

The third region of investigations on collective modes and time correlation functions in dense gases and liquids is connected with the kinetic theory [77,107–109] on the basis of the method of projection operators by Mori and its generalizations [77,79,87]. In this approach, the nonequilibrium one-particle distribution function in phase space of coordinates and momenta is a variable for an abbreviated description of a nonequilibrium state of the system. At the same time, collective effects arise explicitly in memory functions. Approximate calculations of memory functions (expansions on density, weak interaction) in the hydrodynamic limit were performed in papers by Mazenko [109–114], Forster and Martin [108], Forster [115] and others [100,116,117]. John and Forster unified paper [118] and proposed a formalism similar to that of the generalized hydrodynamics [107–109,114,115]. This formalism is distinguished by the fact that the nonequilibrium one-particle distribution function in phase space of coordinates and momenta, together with the density of total energy are included into a set of variables of an abbreviated description. The results of this theory for the dynamical structure factor $S(\boldsymbol{k};\omega)$ of Argon agree well with the experimental data and MD calculations in the regions of small and intermediate values of $\boldsymbol{k}$ and ω. The same formalism was used later in papers by Sjodin and Sjolander [119] in which the dynamical structure factor $S(\boldsymbol{k};\omega)$ for Rubidium was investigated. The authors of this paper touched a number of interesting questions about the influence of one-particle motion and temperature fluctuations on $S(\boldsymbol{k};\omega)$ in the regions of intermediate and great values of wavenumber $\boldsymbol{k}$, as well as about the existence of short-length collective modes [75]. Such questions and a number of others arise in discussing the results of neutron scattering with $\ell \sim \sigma$, ($|\boldsymbol{k}| = 2\pi/\ell$), where the kinetic theory of liquids is developed insufficiently. From the experimental results of neutron scattering in Rubidium [75] it follows that there are sharp side peaks for $S(\boldsymbol{k};\omega)$ at points $\omega = \pm ck$, where c is an adiabatic velocity of sound. Such side peaks could be identified as Brillouin ones. However, the Landau-Plachek formula is not valid in this region, where $\ell \sim \sigma$.

During the last period, the question of the existence of short-length collective modes, beginning from the papers by de Schepper and Cohen [120,121], is intensively discussed [122–135]. The proposed in [120,121] approach is based on the kinetic equation of the revised Enskog theory (RET) [1,3,34], which was derived by us sequentially in subsection 2.3.1. The linearized kinetic equation of RET for hard spheres, which was used in [120,121], was obtained in papers by Mazenko and others as well [112,113,117]. Although the kinetic theory considered by de Schepper and Cohen for short-length collective modes is related to a model system of hard spheres, it has explained many questions which are significant for understanding collective modes in the region of large $\boldsymbol{k}$. First of all, it proposed a concept of extending hydrodynamic modes into the region of large wavevectors [121,122]. Solving the problem on eigenvalues for the generalized Enskog operator (taking into account a mean field) for hard spheres has shown [122,132] the existence of five generalized (extended) hydrodynamic and kinetic modes. The generalized hydrodynamic modes are a generalization of hydrodynamic modes to the region of large $\boldsymbol{k}$ and tend to zero at $k \to 0$. The kinetic modes at $k \to 0$ take positive nonzero values. The dynamical structure factor $S(\boldsymbol{k};\omega)$ is presented in this approach in the form of a sum of spectral terms which correspond to the Lorentzian form of lines. Secondly, to confirm the theoretical predictions of papers [120–135], experiments on neutron scattering in liquid Argon [123], Neon [125], as well as MD calculations for a system of hard spheres [126] and for a liquid with the Lennard-Jones-like interparticle potential of interaction [128] were made. In [123] short-length sound modes and their decrement in the region $\ell \sim \sigma$ for a liquid Argon were investigated. The obtained results [131] for a nonanalytical dependence of the sound dispersion on $\boldsymbol{k}$ were compared with the experimental data and the results of the theory of coupled modes [136]. Eigenvalues for the heat mode $z_{\mathrm{H}}(\boldsymbol{k})$, obtained on the basis of the generalized Enskog equation, were investigated in [130]. The calculations performed agree quantitatively with the MD data for hard spheres [125] and with the data on neutron scattering for Argon [123,124,137] and Crypton [138].

Interesting results of the investigation of the dynamical structure factor $S(\boldsymbol{k};\omega)$ and time correlation functions of flow particle and enthalpy densities, as well as collective modes for a liquid with the Lennard-Jones potential of interaction were obtained in [139]. There a system of equations for time correlation functions of densities for the number of particles, momentum and enthalpy, the generalized stress tensor and the flow of energy obtained by the Mori projection operator method was used. The basis of this system are equations of a generalized hydrodynamic description obtained for the first time in [93] for simple liquid and ionic systems with the use of the NSO method [140]. Solving the system of equations for time correlation functions in the Markovian approximation on eigenvalues in the hydrodynamic limit $k \to 0$, $\omega \to 0$ showed [139] that apart from the eigenvalues which correspond to pure hydrodynamic modes, in particular, to heat $z_{\mathrm{H}}(\boldsymbol{k})$ and two sound modes $z_{\pm}(\boldsymbol{k})$, there exist two eigenvalues that correspond to kinetic modes. They differ from zero in the limit $k \to 0$. The analogous results were obtained in our paper [141]. Further, such an approach was developed for a Lennard-Jones fluid in a number of works [142–154]. The concept of generalized collective modes was developed [142,143] on the basis of equations of extended hydrodynamics and the relevant equilibrium static quantities which normally are obtained within the method of molecular dynamics (MD). This approach gives the possibility to calculate generalized-mode spectra of a Lennard-Jones fluid using nine- and four-mode descriptions for longitudinal [142,147] and transverse [142,146] fluctuations, respectively. The investigation of generalized transport coefficients dependent on wavevector and frequency was carried out for the first time in [148]. Further, this approach was developed for mixtures [154–156] and polar liquids, in particular, for Stockmayer models and TIP4P water [157–165].

Problems of the description of kinetic and hydrodynamic fluctuations, taking into account correlations in a simple fluid, were discussed in papers by Peletminskii, Sokolovskii and Slusarenko [166,167]. They used a functional hypothesis and the method of an abbreviated description [168].

On the basis of general equations of fluctuational hydrodynamics obtained in [166], we carried out investigations of liquid hydrodynamics near the equilibrium without taking into account far correlations and spatial dispersion of transport coefficients [169,170].

It is necessary to point out a series of papers by Balabanyan [171–175] devoted to the investigation of classical time correlation and Poisson Green functions and spectra of collective modes on the basis of the Boltzmann kinetic equation by the method of moments for a system of hard spheres and Maxwellian molecules.

The problem of the construction of interpolation formulas for density-density and flow-flow correlation functions which are valid for a wide range of frequencies and wavevectors was considered in papers by Tserkovnikov [91,92,176,177]. Here a method of Green functions was applied to the molecular hydrodynamics of quantum Bose-systems. These questions were presented in detail in [176,177] for a weakly nonideal Bose-gas. The obtained interpolation formulas for the Green functions of transverse components of flow density [176], as well as of fluctuations of the number of particles and energy [177], appear to be valid in the hydrodynamic region and in the region of great frequencies and small wavelengths. These results are obviously valid in the classical case as well.

From the presented survey, one can draw the following conclusions: firstly, for a more detailed study of collective modes, generalized transport coefficients and time correlation functions in liquids and dense gases, it is necessary to have a theory in which kinetic and hydrodynamic processes are considered simultaneously; secondly, at the present time, a consistent theory for the description of light and neutron scattering in the whole range of wavevector $\boldsymbol{k}$ and frequency ω has not been formulated yet.

In our papers [1,3,5,7] we proposed an approach in which the kinetics and hydrodynamics of transport processes in dense gases and liquids are considered as coupled. As a result, we obtained a system of coupled generalized equations for the nonequilibrium one-particle distribution function and the mean density of the total energy. It can be applied to the description of nonequilibrium states of the system of particles which are as close as they are far from the equilibrium state. In the next subsections of this section we shall present this approach in comparison with other theories and investigate time correlation functions, generalized transport coefficients and collective modes in comparison with the molecular hydrodynamics [77] and its extended versions [139,141–152].

3.2. The generalized kinetic equation for the nonequilibrium one-particle distribution function with taking into account transport equations for the mean density of energy

According to the formulation of a modified boundary condition for the Liouville equation (2.4) with choosing parameters of an abbreviated description $\langle\hat{n}_1(x)\rangle^t = f_1(x;t)$ and $\langle\hat{\mathcal{E}}(\boldsymbol{r})\rangle^t$ (the definition of the quasiequilibrium distribution function is presented by equation (2.12)), the dependence of the the quasiequilibrium distribution function on time is defined by variation in time of the average values $\langle\hat{n}_1(x)\rangle^t$ and $\langle\hat{\mathcal{E}}(\boldsymbol{r})\rangle^t$:

$$\varrho\left(x^N;t\right) = \varrho\Big(\ldots,\{\langle\hat{n}_1(x)\rangle^t,\langle\hat{\mathcal{E}}(\boldsymbol{r})\rangle^t\}\ldots\Big). \tag{3.1}$$

Time evolution of the parameters of an abbreviated description $f_1(x;t)$, $\langle\hat{\mathcal{E}}(\boldsymbol{r})\rangle^t$ is described by transport equations. For obtaining these equations it is more convenient to use the method of projection operators, which was widely used by Robertson [178,179] and modified by Kawasaki and Gunton [180]. We shall reformulate the NSO method in order to take into account projection operators.

Let us write the Liouville equation with source (2.4) in the form:

$$\left(\frac{\partial}{\partial t} + \mathrm{i}L_N + \varepsilon\right)\triangle\varrho\left(x^N;t\right) = -\left(\frac{\partial}{\partial t} + \mathrm{i}L_N\right)\varrho_{\mathrm{q}}\left(x^N;t\right), \tag{3.2}$$

introducing $\triangle\varrho\left(x^N;t\right) = \varrho\left(x^N;t\right) - \varrho_{\mathrm{q}}\left(x^N;t\right)$. We shall use the quasiequilibrium distribution function $\varrho_{\mathrm{q}}\left(x^N;t\right)$ defined as follows:

$$\varrho_{\mathrm{q}}\left(x^N;t\right) = \exp\left\{-\Phi(t) - \int \mathrm{d}\boldsymbol{r}\;\beta(\boldsymbol{r};t)\hat{\mathcal{E}}(\boldsymbol{r}) - \int \mathrm{d}x\;b(x;t)\hat{n}_1(x)\right\}, \tag{3.3}$$

which is found from (2.12), taking into account (2.13), by redefinition of the parameter conjugated to $\langle\hat{n}_1(x)\rangle^t$:

$$b(x;t) = a(x;t) - \beta(\boldsymbol{r};t)\frac{p^2}{2m}. \tag{3.4}$$

Parameters $\beta(\boldsymbol{r};t)$ and $b(x;t)$ in (3.3) are defined from the corresponding self-consistency conditions:

$$\begin{aligned} \langle\hat{\mathcal{E}}(\boldsymbol{r})\rangle^t &= \langle\hat{\mathcal{E}}(\boldsymbol{r})\rangle^t_{\mathrm{q}}, \\ \langle\hat{n}_1(x)\rangle^t &= \langle\hat{n}_1(x)\rangle^t_{\mathrm{q}}. \end{aligned} \tag{3.5}$$

Taking into account the structure of the quasiequilibrium distribution function (3.3), the time derivative $\frac{\partial}{\partial t}$ of this function in the right-hand side of equation (3.2) can be presented as

$$\frac{\partial}{\partial t}\varrho_{\mathrm{q}}\left(x^N;t\right) = -\mathscr{P}_{\mathrm{q}}(t)\mathrm{i}L_N\varrho\left(x^N;t\right), \tag{3.6}$$

where $\mathscr{P}_{\mathrm{q}}(t)$ is the Kawasaki-Gunton projection operator, defined as follows:

$$\begin{aligned} &\mathscr{P}_{\mathrm{q}}(t)\varrho' = \\ &\left\{\varrho_{\mathrm{q}}\left(x^N;t\right) - \int \mathrm{d}\boldsymbol{r}\, \frac{\partial\varrho_{\mathrm{q}}\left(x^N;t\right)}{\partial\langle\hat{\mathcal{E}}(\boldsymbol{r})\rangle^t}\langle\hat{\mathcal{E}}(\boldsymbol{r})\rangle^t - \int \mathrm{d}x\, \frac{\partial\varrho_{\mathrm{q}}\left(x^N;t\right)}{\partial\langle\hat{n}_1(x)\rangle^t}\langle\hat{n}_1(x)\rangle^t\right\}\int \mathrm{d}\Gamma_N\, \varrho' + \\ &\int \mathrm{d}\boldsymbol{r}\, \frac{\partial\varrho_{\mathrm{q}}\left(x^N;t\right)}{\partial\langle\hat{\mathcal{E}}(\boldsymbol{r})\rangle^t}\int \mathrm{d}\Gamma_N\, \hat{\mathcal{E}}(\boldsymbol{r})\varrho' + \int \mathrm{d}x\, \frac{\partial\varrho_{\mathrm{q}}\left(x^N;t\right)}{\partial\langle\hat{n}_1(x)\rangle^t}\int \mathrm{d}\Gamma_N\, \hat{n}_1(x)\varrho'. \end{aligned} \tag{3.7}$$

This operator acts only on distribution functions and has the properties:

$$\begin{aligned} \mathscr{P}_{\mathrm{q}}(t)\varrho(x^N;t') &= \varrho_{\mathrm{q}}(x^N;t), \\ \mathscr{P}_{\mathrm{q}}(t)\varrho_{\mathrm{q}}(x^N;t') &= \varrho_{\mathrm{q}}(x^N;t), \\ \mathscr{P}_{\mathrm{q}}(t)\mathscr{P}_{\mathrm{q}}(t') &= \mathscr{P}_{\mathrm{q}}(t). \end{aligned}$$

Taking into account (3.6), the Liouville equation (2.4) transforms into the form

$$\left(\frac{\partial}{\partial t} - \Big(1-\mathscr{P}_{\mathrm{q}}(t)\Big)\mathrm{i}L_N + \varepsilon\right)\triangle\varrho\left(x^N;t\right) = -\Big(1-\mathscr{P}_{\mathrm{q}}(t)\Big)\mathrm{i}L_N\varrho_{\mathrm{q}}\left(x^N;t\right), \tag{3.8}$$

the formal solution to which is

$$\triangle\varrho\left(x^N;t\right) = -\int_{-\infty}^{t} \mathrm{d}t'\, \mathrm{e}^{\varepsilon(t'-t)}T(t,t')\Big(1-\mathscr{P}_{\mathrm{q}}(t')\Big)\mathrm{i}L_N\varrho_{\mathrm{q}}\left(x^N;t'\right), \tag{3.9}$$

and one finds an expression for the nonequilibrium distribution function

$$\varrho\left(x^N;t\right) = \varrho_{\mathrm{q}}\left(x^N;t\right) - \int_{-\infty}^{t} \mathrm{d}t'\, \mathrm{e}^{\varepsilon(t'-t)}T(t,t')\Big(1-\mathscr{P}_{\mathrm{q}}(t')\Big)\mathrm{i}L_N\varrho_{\mathrm{q}}\left(x^N;t'\right), \tag{3.10}$$

where

$$T(t,t') = \exp_+ \left\{ -\int_{t'}^{t} \mathrm{d}t'' \left(1 - \mathscr{P}_\mathrm{q}(t'')\right) \mathrm{i}L_N \right\} \tag{3.11}$$

is a generalized evolution operator with taking account the Kawasaki-Gunton projection operator, $\exp_+$ is an ordering exponent. Solution (3.10) is exact. It corresponds to the idea of an abbreviated description of a nonequilibrium state of the system (3.1). Acting by the operators $(1 - \mathscr{P}_\mathrm{q}(t'))$ and $\mathrm{i}L_N$ on $\varrho_\mathrm{q}(x^N;t')$ in the right-hand side of expression (3.10), we write an expression for the nonequilibrium distribution function in the explicit form:

$$\begin{aligned} \varrho\left(x^N;t\right) = \varrho_\mathrm{q}\left(x^N;t\right) \, + \int \mathrm{d}\boldsymbol{r} \int\limits_{-\infty}^{t} \mathrm{d}t' \, \mathrm{e}^{\varepsilon(t'-t)} T(t,t') I_\mathcal{E}(\boldsymbol{r},t') \beta(\boldsymbol{r};t') \varrho_\mathrm{q}\left(x^N;t'\right) \\ - \int \mathrm{d}x \int\limits_{-\infty}^{t} \mathrm{d}t' \, \mathrm{e}^{\varepsilon(t'-t)} T(t,t') I_n(x,t') b(x;t') \varrho_\mathrm{q}\left(x^N;t'\right), \end{aligned} \tag{3.12}$$

where

$$I_\mathcal{E}(\boldsymbol{r},t') \, = \, \left(1 - \mathscr{P}(t')\right) \dot{\hat{\mathcal{E}}}(\boldsymbol{r}), \tag{3.13}$$

$$I_n(x,t') \, = \, \left(1 - \mathscr{P}(t')\right) \dot{\hat{n}}_1(x) \tag{3.14}$$

are generalized flows, $\dot{\hat{\mathcal{E}}}(\boldsymbol{r}) = \mathrm{i}L_N \hat{\mathcal{E}}(\boldsymbol{r})$, $\dot{\hat{n}}_1(x) = \mathrm{i}L_N \hat{n}_1(x)$. $\mathscr{P}(t')$ is a generalized Mori projection operator which acts on dynamical variables $\hat{A}(\boldsymbol{r})$ and has the following structure:

$$\begin{gathered} \mathscr{P}(t)\hat{A}(\boldsymbol{r}') = \langle \hat{A}(\boldsymbol{r}')\rangle_\mathrm{q}^t + \\ \int \mathrm{d}\boldsymbol{r} \, \frac{\partial \langle \hat{A}(\boldsymbol{r}')\rangle_\mathrm{q}^t}{\partial \langle \hat{\mathcal{E}}(\boldsymbol{r})\rangle^t} \left(\hat{\mathcal{E}}(\boldsymbol{r}) - \langle \hat{\mathcal{E}}(\boldsymbol{r})\rangle^t\right) + \int \mathrm{d}x \, \frac{\partial \langle \hat{A}(\boldsymbol{r}')\rangle_\mathrm{q}^t}{\partial \langle \hat{n}_1(x)\rangle^t} \left(\hat{n}_1(x) - \langle \hat{n}_1(x)\rangle^t\right). \end{gathered} \tag{3.15}$$

The projection operator $\mathscr{P}(t)$ has the following properties:

$$\begin{array}{ll} \mathscr{P}(t)\hat{\mathcal{E}}(\boldsymbol{r}) \, = \, \hat{\mathcal{E}}(\boldsymbol{r}), & \mathscr{P}(t)(1 - \mathscr{P}(t)) \, = \, 0, \\ \mathscr{P}(t)\hat{n}_1(x) \, = \, \hat{n}_1(x), & \mathscr{P}(t)\mathscr{P}(t') \, = \, \mathscr{P}(t). \end{array} \tag{3.16}$$

With the help of the solution to the Liouville equation (3.12), one obtains a system of coupled equations for the nonequilibrium one-particle distribution function $f_1(x;t)$ and the average density of energy $\langle \hat{\mathcal{E}}(\boldsymbol{r})\rangle^t$. To do it explicitly, it is necessary to calculate

$$\begin{aligned} \frac{\partial}{\partial t}\langle \hat{n}_1(x)\rangle^t \, &= \, \frac{\partial}{\partial t} f_1(x;t) = \langle \dot{\hat{n}}_1(x)\rangle^t, \\ \frac{\partial}{\partial t}\langle \hat{\mathcal{E}}(\boldsymbol{r})\rangle^t \, &= \, \langle \dot{\hat{\mathcal{E}}}(\boldsymbol{r})\rangle^t. \end{aligned}$$

To this end let us use the equalities

$$\Big\langle\Big(1-\mathscr{P}(t)\Big)\dot{\hat{n}}_1(x)\Big\rangle^t = \langle\dot{\hat{n}}_1(x)\rangle^t - \langle\dot{\hat{n}}_1(x)\rangle^t_{\mathrm{q}},$$
$$\Big\langle\Big(1-\mathscr{P}(t)\Big)\dot{\hat{\mathcal{E}}}(\boldsymbol{r})\Big\rangle^t = \langle\dot{\hat{\mathcal{E}}}(\boldsymbol{r})\rangle^t - \langle\dot{\hat{\mathcal{E}}}(\boldsymbol{r})\rangle^t_{\mathrm{q}}$$

and perform averaging in the right-hand side with the nonequilibrium distribution function (3.12). Then we obtain, calculating $\langle\dot{\hat{n}}_1(x)\rangle^t_{\mathrm{q}}$, $\langle\dot{\hat{\mathcal{E}}}(\boldsymbol{r})\rangle^t_{\mathrm{q}}$ [5]:

$$\begin{aligned}\frac{\partial}{\partial t}f_1(x;t)+\frac{\boldsymbol{p}}{m}\frac{\partial}{\partial\boldsymbol{r}}f_1(x;t) &= \int \mathrm{d}x'\ \frac{\partial}{\partial\boldsymbol{r}}\varPhi(|\boldsymbol{r}-\boldsymbol{r}'|)\frac{\partial}{\partial\boldsymbol{p}}\mathrm{g}_2(\boldsymbol{r},\boldsymbol{r}';t)f_1(x;t)f_1(x';t)+\\ &\int \mathrm{d}x' \int\limits_{-\infty}^{t} \mathrm{d}t'\ \mathrm{e}^{\varepsilon(t'-t)}\varphi_{nn}(x,x';t,t')b(x';t')+\\ &\int \mathrm{d}\boldsymbol{r}' \int\limits_{-\infty}^{t} \mathrm{d}t'\ \mathrm{e}^{\varepsilon(t'-t)}\varphi_{n\mathcal{E}}(x,\boldsymbol{r}';t,t')\beta(\boldsymbol{r}';t'),\end{aligned} \tag{3.17}$$

$$\begin{aligned}-\frac{\partial}{\partial t}\langle\hat{\mathcal{E}}(\boldsymbol{r})\rangle^t &= \frac{1}{2}\int \mathrm{d}\boldsymbol{r}'\ \mathrm{d}\boldsymbol{p}\ \mathrm{d}\boldsymbol{p}'\ \frac{\boldsymbol{p}}{m}\frac{\partial}{\partial\boldsymbol{r}}\varPhi(|\boldsymbol{r}-\boldsymbol{r}'|)\mathrm{g}_2(\boldsymbol{r},\boldsymbol{r}';t)f_1(x;t)f_1(x';t) \qquad (3.18)\\ &+\int \mathrm{d}\boldsymbol{r}'\ \mathrm{d}\boldsymbol{p}\ \mathrm{d}\boldsymbol{p}'\ \frac{\boldsymbol{p}}{m}\frac{\partial}{\partial\boldsymbol{r}}\left[\frac{p^2}{2m}+\frac{1}{2}\varPhi(|\boldsymbol{r}-\boldsymbol{r}'|)\right]\mathrm{g}_2(\boldsymbol{r},\boldsymbol{r}';t)f_1(x;t)f_1(x';t)\\ &-\int \mathrm{d}x' \int\limits_{-\infty}^{t} \mathrm{d}t'\ \mathrm{e}^{\varepsilon(t'-t)}\varphi_{\mathcal{E}n}(\boldsymbol{r},x';t,t')b(x';t')\\ &-\int \mathrm{d}\boldsymbol{r}' \int\limits_{-\infty}^{t} \mathrm{d}t'\ \mathrm{e}^{\varepsilon(t'-t)}\varphi_{\mathcal{E}\mathcal{E}}(\boldsymbol{r},\boldsymbol{r}';t,t')\beta(\boldsymbol{r}';t'),\end{aligned}$$

where $\mathrm{g}_2(\boldsymbol{r},\boldsymbol{r}';t)$ is a binary quasiequilibrium distribution function (2.32), and

$$\varphi_{nn}(x,x';t,t') = \int \mathrm{d}\varGamma_N\ I_n(x;t)T(t,t')I_n(x';t')\varrho_{\mathrm{q}}(x^N;t'), \tag{3.19}$$

$$\varphi_{n\mathcal{E}}(x,\boldsymbol{r}';t,t') = \int \mathrm{d}\varGamma_N\ I_n(x;t)T(t,t')I_{\mathcal{E}}(\boldsymbol{r}';t')\varrho_{\mathrm{q}}(x^N;t'), \tag{3.20}$$

$$\varphi_{\mathcal{E}n}(\boldsymbol{r},x';t,t') = \int \mathrm{d}\varGamma_N\ I_{\mathcal{E}}(\boldsymbol{r};t)T(t,t')I_n(x';t')\varrho_{\mathrm{q}}(x^N;t'), \tag{3.21}$$

$$\varphi_{\mathcal{E}\mathcal{E}}(\boldsymbol{r},\boldsymbol{r}';t,t') = \int \mathrm{d}\varGamma_N\ I_{\mathcal{E}}(\boldsymbol{r};t)T(t,t')I_{\mathcal{E}}(\boldsymbol{r}';t')\varrho_{\mathrm{q}}(x^N;t') \tag{3.22}$$

are generalized transport kernels which describe the kinetic and hydrodynamic dissipative processes in a system. We obtained generalized transport equations for a nonequilibrium one-particle distribution function and the average energy density, which describe both strong and weak nonequilibrium processes. From these equations one can derive interesting limiting cases. First of all, if two last terms in the right-hand side of equation (3.17) are not taken into account (these terms describe non-Markovian dissipative processes), then one obtains a generalized kinetic equation in the mean field approximation [5,38]:

$$\frac{\partial}{\partial t} f_1(x;t) + \frac{\boldsymbol{p}}{m}\frac{\partial}{\partial \boldsymbol{r}} f_1(x;t) = \int \mathrm{d}x' \, \frac{\partial}{\partial \boldsymbol{r}} \varPhi(|\boldsymbol{r}-\boldsymbol{r}'|) \frac{\partial}{\partial \boldsymbol{p}} \mathrm{g}_2(\boldsymbol{r},\boldsymbol{r}';t) f_1(x;t) f_1(x';t). \tag{3.23}$$

From equation (3.23) at $\mathrm{g}_2(\boldsymbol{r},\boldsymbol{r}';t)=1$ we have a usual Vlasov kinetic equation for a nonequilibrium one-particle distribution function. The kinetic equation (3.23) for $f_1(x;t)$ must be complemented by an equation for the average energy density (3.18), without taking into account two last terms which describe non-Markovian dissipative processes. It is important to remember here that the binary quasiequilibrium distribution function $\mathrm{g}_2(\boldsymbol{r},\boldsymbol{r}';t)$ is a functional of the local values of inversed temperature $\beta(\boldsymbol{r};t)$ and average density $n(\boldsymbol{r};t)$: $\mathrm{g}_2(\boldsymbol{r},\boldsymbol{r}';t)=\mathrm{g}_2(\boldsymbol{r},\boldsymbol{r}'|\beta(t),n(t))$. If spatial correlations connected with an interaction between a separated group of particles and the media are neglected (that is valid in the case of small densities), i.e. formally putting for $\varrho_{\mathrm{q}}\left(x^N;t\right)$ in (2.22) the density of interaction energy $\hat{\mathcal{E}}^{\mathrm{int}}(\boldsymbol{r})$ to be equal to zero ($U_N(\boldsymbol{r}^N;t)=0$), then (2.22) transforms into a quasiequilibrium distribution (2.5) that corresponds to the usual boundary conditions of the weakening of correlations by Bogolubov [22] for the solution of the Liouville equation. In this case, from the system of equations (3.17), (3.18), we have a generalized kinetic equation for the nonequilibrium one-particle distribution function which was obtained earlier by Zubarev and Novikov [52]

$$\begin{aligned} \frac{\partial}{\partial t} f_1(x;t) + \frac{\boldsymbol{p}}{m}\frac{\partial}{\partial \boldsymbol{r}} f_1(x;t) \;&=\; \int \mathrm{d}x' \, \frac{\partial}{\partial \boldsymbol{r}} \varPhi(|\boldsymbol{r}-\boldsymbol{r}'|) \frac{\partial}{\partial \boldsymbol{p}} f_1(x;t) f_1(x';t) + \\ &\int \mathrm{d}x' \int\limits_{-\infty}^{t} \mathrm{d}t' \; \mathrm{e}^{\varepsilon(t'-t)} \varphi_{nn}(x,x';t,t') a(x';t'). \end{aligned} \tag{3.24}$$

However, in this equation, contrary to the kinetic equation for $f_1(x;t)$ of paper [52], we used a projection procedure for the exclusion of time derivatives for parameters $a(x;t)$ which are determined from the self-consistency condition (3.5) and in our case, according to (2.12), (2.22), (2.23) and $u=\mathrm{e}$, they are equal to $\mathrm{e}^{-a(x;t)} = f_1(x;t)/\mathrm{e}$. In the kinetic equation (3.24), the transport kernel has the following structure:

$$\varphi_{nn}(x,x';t,t') = \int \mathrm{d}\varGamma_N \, \Big(1-\mathscr{P}'(t)\Big)\dot{\hat{n}}_1(x) T(t,t') \Big(1-\mathscr{P}'(t')\Big)\dot{\hat{n}}_1(x') \prod_{j=1}^{N} \frac{f_1(x_j;t)}{\mathrm{e}}, \tag{3.25}$$

where the projection operator $\mathscr{P}'(t)$ is defined as

$$\mathscr{P}'(t)\hat{A}(x') = \langle \hat{A}(x')\rangle_{\mathrm{q}}^t + \int \mathrm{d}x \, \frac{\partial \langle \hat{A}(x')\rangle_{\mathrm{q}}^t}{\partial \langle \hat{n}_1(x)\rangle^t} \left(\hat{n}_1(x) - \langle \hat{n}_1(x)\rangle^t\right), \tag{3.26}$$

and the average values $\langle\hat{A}(x')\rangle^t_{\mathrm{q}}$ are calculated with the help of the quasiequilibrium distribution function (2.5). The kernel $\varphi_{nn}(x,x';t,t')$ describes kinetic processes in the system.

A coupled system of equations for the nonequilibrium one-particle distribution function and the density of the total energy (3.17), (3.18) is strongly nonlinear. It takes into account complicated kinetic and hydrodynamic processes and such a system can be used for describing both strongly and weakly nonequilibrium states.

In the next subsection we apply equations (3.17), (3.18) to the investigation of nonequilibrium states of the system, which are close to equilibrium. In this case the equations are simplified significantly.

3.3. Kinetics and hydrodynamics of nonequilibrium state near equilibrium

Let us assume that the average energy density $\langle\hat{\mathcal{E}}(\boldsymbol{r})\rangle^t$, the nonequilibrium one-particle distribution function $f_1(x;t)$ and parameters $\beta(\boldsymbol{r};t)$, $b(x;t)$ deviate slightly from their equilibrium values. Then, being restricted to the linear approximation [5], the quasiequilibrium distribution function $\varrho_{\mathrm{q}}\left(x^N;t\right)$ (3.3) can be expanded over deviations of the parameters $\beta(\boldsymbol{r};t)$, $b(x;t)$ from their equilibrium values:

$$\varrho_{\mathrm{q}}\left(x^N;t\right)=\varrho_0(x^N)\left[1-\int\mathrm{d}\boldsymbol{r}\;\delta\beta(\boldsymbol{r};t)\hat{\mathcal{E}}(\boldsymbol{r})-\int\mathrm{d}x\;\delta b(x;t)\hat{n}_1(x)\right],\tag{3.27}$$

where $\varrho_0(x^N)=Z^{-1}\mathrm{e}^{-\beta(H-\mu N)}$ is an equilibrium distribution of particles, $Z=\int\mathrm{d}\Gamma_N\;\mathrm{e}^{-\beta(H-\mu N)}$ is a grand partition function, $\beta=1/k_{\mathrm{B}}T$ is an equilibrium value for inverse temperature, k_{B} is the Boltzmann constant, $\delta\beta(\boldsymbol{r};t)=\beta(\boldsymbol{r};t)-\beta$, $\delta b(x;t)=b(x;t)+\beta\mu$, μ is an equilibrium value of the chemical potential. In formula (3.27) it is convenient to transform the dynamical variables $\hat{\mathcal{E}}(\boldsymbol{r})$, $\hat{n}_1(x)=\hat{n}_1(\boldsymbol{r},\boldsymbol{p})$ and parameters $\delta\beta(\boldsymbol{r};t)$, $\delta b(x;t)$ into Fourier-components. Then we obtain:

$$\varrho_{\mathrm{q}}\left(x^N;t\right)=\varrho_0(x^N)\left[1-{\sum_{\boldsymbol{k}}}'\delta\beta_{-\boldsymbol{k}}(t)\hat{\mathcal{E}}_{\boldsymbol{k}}-{\sum_{\boldsymbol{k}}}'\int\mathrm{d}\boldsymbol{p}\;\delta b_{-\boldsymbol{k}}(\boldsymbol{p};t)\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})\right],\tag{3.28}$$

where $\sum_{\boldsymbol{k}}'=\sum_{\boldsymbol{k}(\boldsymbol{k}\neq0)}$ and

$$\begin{aligned}\hat{\mathcal{E}}_{\boldsymbol{k}}&=\int\mathrm{d}\boldsymbol{r}\;\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\hat{\mathcal{E}}(\boldsymbol{r}), &\delta\beta_{-\boldsymbol{k}}(t)&=\int\mathrm{d}\boldsymbol{r}\;\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\delta\beta(\boldsymbol{r};t),\\ \hat{n}_{\boldsymbol{k}}(\boldsymbol{p})&=\int\mathrm{d}\boldsymbol{r}\;\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\hat{n}_1(\boldsymbol{r},\boldsymbol{p}), &\delta b_{-\boldsymbol{k}}(\boldsymbol{p};t)&=\int\mathrm{d}\boldsymbol{r}\;\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\delta b(x;t).\end{aligned}$$

Using the self-consistency conditions (3.5), let us consequently exclude parameters $\delta\beta_{-\boldsymbol{k}}(t)$, $\delta b_{-\boldsymbol{k}}(\boldsymbol{p};t)$ in equation (3.28). From $\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})\rangle^t=\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})\rangle^t_{\mathrm{q}}$ one finds:

$$\int\mathrm{d}\boldsymbol{p}'\;\Phi_{\boldsymbol{k}}(\boldsymbol{p},\boldsymbol{p}')\delta b_{-\boldsymbol{k}}(\boldsymbol{p}';t)=-\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})\rangle^t-\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})\hat{\mathcal{E}}_{-\boldsymbol{k}}\rangle_0\delta\beta_{\boldsymbol{k}}(t),\tag{3.29}$$

where $\langle\ldots\rangle_0$ denotes averaging over the equilibrium distribution, namely $\langle\ldots\rangle_0=\int\mathrm{d}\Gamma_N\;\ldots\varrho_0(x^N)$, $\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})\rangle_0=0$, $(\boldsymbol{k}\neq0)$;

$$\Phi_{\boldsymbol{k}}(\boldsymbol{p},\boldsymbol{p}')=\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})\hat{n}_{-\boldsymbol{k}}(\boldsymbol{p}')\rangle_0=n\delta(\boldsymbol{p}-\boldsymbol{p}')f_0(p')+n^2f_0(p)f_0(p')h_2(\boldsymbol{k})\tag{3.30}$$

is an equilibrium correlation function, $n = N/V$, $f_0(p) = (\beta/2\pi m)^{3/2}\mathrm{e}^{-\beta p^2/2m}$ is a Maxwell distribution, $h_2(\boldsymbol{k})$ is a Fourier-image of the function $\mathrm{g}_2(\boldsymbol{R}) - 1$:

$$h_2(\boldsymbol{k}) = \int \mathrm{d}\boldsymbol{R}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{R}}\Big(\mathrm{g}_2(\boldsymbol{R}) - 1\Big), \tag{3.31}$$

$\mathrm{g}_2(\boldsymbol{R}) = \mathrm{g}_2(|\boldsymbol{r} - \boldsymbol{r}'|)$ is a pair equilibrium distribution function. Now let us define the function $\Phi_{\boldsymbol{k}}^{-1}(\boldsymbol{p},\boldsymbol{p}')$ inversed to the function $\Phi_{\boldsymbol{k}}(\boldsymbol{p},\boldsymbol{p}')$ by means of the relation

$$\int \mathrm{d}\boldsymbol{p}'\;\Phi_{\boldsymbol{k}}^{-1}(\boldsymbol{p}'',\boldsymbol{p}')\Phi_{\boldsymbol{k}}(\boldsymbol{p}',\boldsymbol{p}) = \delta(\boldsymbol{p}'' - \boldsymbol{p}). \tag{3.32}$$

Taking into account (3.30), from (3.32) we find $\Phi_{\boldsymbol{k}}^{-1}(\boldsymbol{p}'',\boldsymbol{p}')$ in an explicit form

$$\Phi_{\boldsymbol{k}}^{-1}(\boldsymbol{p}'',\boldsymbol{p}') = \frac{\delta(\boldsymbol{p}'' - \boldsymbol{p}')}{n f_0(p'')} - c_2(\boldsymbol{k}), \tag{3.33}$$

where $c_2(\boldsymbol{k})$ denotes a direct correlation function which is connected with the correlation function $h_2(\boldsymbol{k})$ as: $h_2(\boldsymbol{k}) = c_2(\boldsymbol{k})[1 - nc_2(\boldsymbol{k})]^{-1}$. Let us multiply the left-hand side of (3.29) by the function $\Phi_{\boldsymbol{k}}^{-1}(\boldsymbol{p}'',\boldsymbol{p}')$ and integrate it with respect to $\boldsymbol{p}'$. Taking into account (3.32), we find:

$$\delta b_{-\boldsymbol{k}}(\boldsymbol{p};t) = -\int \mathrm{d}\boldsymbol{p}'\;\Phi_{\boldsymbol{k}}^{-1}(\boldsymbol{p},\boldsymbol{p}')\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p}')\rangle^t - \delta\beta_{\boldsymbol{k}}(t)\int \mathrm{d}\boldsymbol{p}'\;\Phi_{\boldsymbol{k}}^{-1}(\boldsymbol{p},\boldsymbol{p}')\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p}')\hat{\mathcal{E}}_{-\boldsymbol{k}}\rangle_0.$$

Substituting the obtained value of the parameter $\delta b_{-\boldsymbol{k}}(\boldsymbol{p};t)$ in (3.28) we find after some transformations:

$$\varrho_{\mathrm{q}}\left(x^N;t\right) = \varrho_0(x^N)\left[1 - {\sum_{\boldsymbol{k}}}'\delta\beta_{-\boldsymbol{k}}(t)\hat{h}_{\boldsymbol{k}}^{\mathrm{int}} + {\sum_{\boldsymbol{k}}}'\int \mathrm{d}\boldsymbol{p}\,\mathrm{d}\boldsymbol{p}'\;\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p}')\rangle^t\Phi_{\boldsymbol{k}}^{-1}(\boldsymbol{p}',\boldsymbol{p})\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})\right], \tag{3.34}$$

where

$$\hat{h}_{\boldsymbol{k}}^{\mathrm{int}} = \hat{\mathcal{E}}_{\boldsymbol{k}} - \int \mathrm{d}\boldsymbol{p}\,\mathrm{d}\boldsymbol{p}'\;\langle\hat{\mathcal{E}}_{\boldsymbol{k}}\hat{n}_{-\boldsymbol{k}}(\boldsymbol{p}')\rangle_0\Phi_{\boldsymbol{k}}^{-1}(\boldsymbol{p}',\boldsymbol{p})\hat{n}_{\boldsymbol{k}}(\boldsymbol{p}) = \hat{\mathcal{E}}_{\boldsymbol{k}}^{\mathrm{int}} - \langle\hat{\mathcal{E}}_{\boldsymbol{k}}^{\mathrm{int}}\hat{n}_{-\boldsymbol{k}}\rangle_0 S_2^{-1}(\boldsymbol{k})\hat{n}_{\boldsymbol{k}}, \tag{3.35}$$

$$\hat{\mathcal{E}}_{\boldsymbol{k}}^{\mathrm{int}} = \frac{1}{2}\sum_{l\neq j=1}^{N}\varPhi(|\boldsymbol{r}_{lj}|)\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_l}, \qquad \hat{n}_{\boldsymbol{k}} = \sum_{l=1}^{N}\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_l} \tag{3.36}$$

are the Fourier-components of densities for the interaction energy and the number of particles, respectively. Further, it is more convenient, instead of the dynamical variable of energy $\hat{\mathcal{E}}_{\boldsymbol{k}}$, to use the variable $\hat{h}_{\boldsymbol{k}}^{\mathrm{int}}$ (3.35) which is orthogonal to $\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})$ by means of the equality:

$$\langle\hat{h}_{\boldsymbol{k}}^{\mathrm{int}}\hat{n}_{-\boldsymbol{k}}\rangle_0 = 0. \tag{3.37}$$

From the structure of the dynamical variable $\hat{h}_{\boldsymbol{k}}^{\mathrm{int}}$ (3.35) it can be seen that it corresponds to a potential part of the Fourier-component of the generalized enthalpy $\hat{h}_{\boldsymbol{k}}$, which is introduced in molecular hydrodynamics [5,77,79]:

$$\hat{h}_{\boldsymbol{k}} = \hat{\mathcal{E}}_{\boldsymbol{k}} - \langle\hat{\mathcal{E}}_{\boldsymbol{k}}\hat{n}_{-\boldsymbol{k}}\rangle_0\hat{n}_{\boldsymbol{k}} = \hat{h}_{\boldsymbol{k}}^{\mathrm{kin}} + \hat{h}_{\boldsymbol{k}}^{\mathrm{int}}, \tag{3.38}$$

where

$$\hat{h}^{\mathrm{kin}}_{\boldsymbol{k}} = \hat{\mathcal{E}}^{\mathrm{kin}}_{\boldsymbol{k}} - \langle \hat{\mathcal{E}}^{\mathrm{kin}}_{\boldsymbol{k}} \hat{n}_{-\boldsymbol{k}} \rangle_0 S_2^{-1}(\boldsymbol{k}) \hat{n}_{\boldsymbol{k}} \tag{3.39}$$

is a kinetic part of the generalized enthalpy,

$$\hat{\mathcal{E}}^{\mathrm{kin}}_{\boldsymbol{k}} = \sum_{l=1}^{N} \frac{p_l^2}{2m} \mathrm{e}^{-\boldsymbol{k}\cdot\boldsymbol{r}_l}$$

is the Fourier-component of the kinetic energy density. $S_2 = \langle \hat{n}_{\boldsymbol{k}} \hat{n}_{-\boldsymbol{k}} \rangle_0$ is a static structure factor of the system. Taking into account the orthogonality of the dynamical variables $\hat{h}^{\mathrm{int}}_{\boldsymbol{k}}$ and $\hat{n}_{\boldsymbol{k}}$ (3.37), from the self-consistency condition $\langle \hat{h}^{\mathrm{int}}_{\boldsymbol{k}} \rangle^t = \langle \hat{h}^{\mathrm{int}}_{\boldsymbol{k}} \rangle^t_{\mathrm{q}}$ that is equivalent to $\langle \hat{\mathcal{E}}_{\boldsymbol{k}} \rangle^t = \langle \hat{\mathcal{E}}_{\boldsymbol{k}} \rangle^t_{\mathrm{q}}$ one defines the parameter $\delta\beta_{\boldsymbol{k}}(t)$:

$$\delta\beta_{\boldsymbol{k}}(t) = -\langle \hat{h}^{\mathrm{int}}_{\boldsymbol{k}} \rangle^t \Phi^{-1}_{hh}(\boldsymbol{k}), \tag{3.40}$$

where $\Phi_{hh}(\boldsymbol{k}) = \langle \hat{h}^{\mathrm{int}}_{\boldsymbol{k}} \hat{h}^{\mathrm{int}}_{-\boldsymbol{k}} \rangle_0$. Finally, let us substitute (3.40) into (3.34). As a result, we obtain a quasiequilibrium distribution function in the linear approximation:

$$\varrho_{\mathrm{q}}\left(x^N;t\right) = \varrho_0(x^N) \left[1 + \sum_{\boldsymbol{k}}{}' \langle \hat{h}^{\mathrm{int}}_{\boldsymbol{k}} \rangle^t \Phi^{-1}_{hh}(\boldsymbol{k}) \hat{h}^{\mathrm{int}}_{\boldsymbol{k}} + \sum_{\boldsymbol{k}}{}' \int \mathrm{d}\boldsymbol{p}\, \mathrm{d}\boldsymbol{p}'\ \langle \hat{n}_{\boldsymbol{k}}(\boldsymbol{p}') \rangle^t \Phi^{-1}_{\boldsymbol{k}}(\boldsymbol{p}',\boldsymbol{p}) \hat{n}_{\boldsymbol{k}}(\boldsymbol{p}) \right]. \tag{3.41}$$

Taking into account (3.41), the nonequilibrium distribution function $\varrho(x^N;t)$ (3.10) or (3.12) has the following form in this approximation [5]:

$$\begin{aligned}
&\varrho\left(x^N;t\right) = \varrho_0(x^N) \times \\
&\Big[1 + \sum_{\boldsymbol{k}}{}' \langle \hat{h}^{\mathrm{int}}_{\boldsymbol{k}} \rangle^t \Phi^{-1}_{hh}(\boldsymbol{k}) \hat{h}^{\mathrm{int}}_{\boldsymbol{k}} + \\
&\quad \sum_{\boldsymbol{k}}{}' \int \mathrm{d}\boldsymbol{p}\, \mathrm{d}\boldsymbol{p}'\ \langle \hat{n}_{\boldsymbol{k}}(\boldsymbol{p}') \rangle^t \Phi^{-1}_{\boldsymbol{k}}(\boldsymbol{p}',\boldsymbol{p}) \hat{n}_{\boldsymbol{k}}(\boldsymbol{p}) - \\
&\quad \sum_{\boldsymbol{k}}{}' \int_{-\infty}^{t} \mathrm{d}t'\ \mathrm{e}^{\varepsilon(t'-t)} \langle \hat{h}^{\mathrm{int}}_{\boldsymbol{k}} \rangle^{t'} \Phi^{-1}_{hh}(\boldsymbol{k}) T_0(t,t') I^{\mathrm{int}}_h(-\boldsymbol{k}) - \\
&\quad \sum_{\boldsymbol{k}}{}' \int \mathrm{d}\boldsymbol{p}\, \mathrm{d}\boldsymbol{p}' \int_{-\infty}^{t} \mathrm{d}t'\ \mathrm{e}^{\varepsilon(t'-t)} \langle \hat{n}_{\boldsymbol{k}}(\boldsymbol{p}') \rangle^{t'} \Phi^{-1}_{\boldsymbol{k}}(\boldsymbol{p}',\boldsymbol{p}) T_0(t,t') I_n(-\boldsymbol{k},\boldsymbol{p}) \Big],
\end{aligned} \tag{3.42}$$

where

$$I_n(\boldsymbol{k},\boldsymbol{p}) = (1-\mathscr{P}_0)\dot{\hat{n}}_{\boldsymbol{k}}(\boldsymbol{p}), \tag{3.43}$$

$$I^{\mathrm{int}}_h(\boldsymbol{k}) = (1-\mathscr{P}_0)\dot{\hat{h}}^{\mathrm{int}}_{\boldsymbol{k}} \tag{3.44}$$

are generalized fluxes in the linear approximation, $\dot{\hat{n}}_{\boldsymbol{k}}(\boldsymbol{p}) = \mathrm{i}L_N \hat{n}_{\boldsymbol{k}}(\boldsymbol{p})$, $\dot{\hat{h}}^{\mathrm{int}}_{\boldsymbol{k}} = \mathrm{i}L_N \hat{h}^{\mathrm{int}}_{\boldsymbol{k}}$, $T_0(t,t') = \mathrm{e}^{(t-t')(1-\mathscr{P}_0)\mathrm{i}L_N}$ is a time evolution operator with the projection operator $\mathscr{P}_0$ which is a linear approximation of the generalized Mori projection operator $\mathscr{P}(t)$ (3.15), with taking into account orthogonalization of variables

$\{\hat{\mathcal{E}}_{\boldsymbol{k}}, \hat{n}_{\boldsymbol{k}}(\boldsymbol{p})\} \rightarrow \{\hat{h}^{\rm int}_{\boldsymbol{k}}, \hat{n}_{\boldsymbol{k}}(\boldsymbol{p})\}$. According to the structure (3.41), $\mathscr{P}_0$ acts on the dynamical variables $\hat{A}_{\boldsymbol{k}}$

$$\mathscr{P}_0 \hat{A}_{\boldsymbol{k}'} = \sum_{\boldsymbol{k}} \langle \hat{A}_{\boldsymbol{k}'} \hat{h}^{\rm int}_{-\boldsymbol{k}} \rangle_0 \Phi^{-1}_{hh}(\boldsymbol{k}) \hat{h}^{\rm int}_{\boldsymbol{k}} + \sum_{\boldsymbol{k}} \int {\rm d}\boldsymbol{p}\ {\rm d}\boldsymbol{p}'\ \langle \hat{A}_{\boldsymbol{k}'} \hat{n}_{-\boldsymbol{k}}(\boldsymbol{p}') \rangle_0 \Phi^{-1}_{\boldsymbol{k}}(\boldsymbol{p}', \boldsymbol{p}) \hat{n}_{\boldsymbol{k}}(\boldsymbol{p}). \tag{3.45}$$

For kinetics and hydrodynamics of nonequilibrium processes which are close to an equilibrium state, the generalized transport equations (3.17), (3.18) in the linear approximation for the nonequilibrium distribution function $\varrho\left(x^N; t\right)$ (3.42) transform into a transport equation for $f_{\boldsymbol{k}}(\boldsymbol{p}; t) = \langle \hat{n}_{\boldsymbol{k}}(\boldsymbol{p}) \rangle^t$, $h^{\rm int}_{\boldsymbol{k}}(t) = \langle \hat{h}^{\rm int}_{\boldsymbol{k}} \rangle^t$

$$\begin{aligned} \frac{\partial}{\partial t} f_{\boldsymbol{k}}(\boldsymbol{p}; t) + \frac{{\rm i}\boldsymbol{k}\cdot\boldsymbol{p}}{m} f_{\boldsymbol{k}}(\boldsymbol{p}; t) = -\frac{{\rm i}\boldsymbol{k}\cdot\boldsymbol{p}}{m} n f_0(p) c_2(\boldsymbol{k}) \int {\rm d}\boldsymbol{p}'\ f_{\boldsymbol{k}}(\boldsymbol{p}'; t) + {\rm i}\Omega_{nh}(\boldsymbol{k}, \boldsymbol{p}) h^{\rm int}_{\boldsymbol{k}}(t) - \\ \int {\rm d}\boldsymbol{p}' \int\limits_{-\infty}^{t} {\rm d}t'\ {\rm e}^{\varepsilon(t'-t)} \varphi_{nn}(\boldsymbol{k}, \boldsymbol{p}, \boldsymbol{p}'; t, t') f_{\boldsymbol{k}}(\boldsymbol{p}'; t') - \int\limits_{-\infty}^{t} {\rm d}t'\ {\rm e}^{\varepsilon(t'-t)} \varphi_{nh}(\boldsymbol{k}, \boldsymbol{p}; t, t') h^{\rm int}_{\boldsymbol{k}}(t'), \end{aligned} \tag{3.46}$$

$$\begin{aligned} \frac{\partial}{\partial t} h^{\rm int}_{\boldsymbol{k}}(t) = \int {\rm d}\boldsymbol{p}\ {\rm i}\Omega_{hn}(\boldsymbol{k}, \boldsymbol{p}) f_{\boldsymbol{k}}(\boldsymbol{p}; t) - \\ \int {\rm d}\boldsymbol{p} \int\limits_{-\infty}^{t} {\rm d}t'\ {\rm e}^{\varepsilon(t'-t)} \varphi_{hn}(\boldsymbol{k}, \boldsymbol{p}; t, t') f_{\boldsymbol{k}}(\boldsymbol{p}; t') - \int\limits_{-\infty}^{t} {\rm d}t'\ {\rm e}^{\varepsilon(t'-t)} \varphi_{hh}(\boldsymbol{k}; t, t') h^{\rm int}_{\boldsymbol{k}}(t'), \end{aligned} \tag{3.47}$$

where ${\rm i}\Omega_{nh}(\boldsymbol{k}, \boldsymbol{p})$, ${\rm i}\Omega_{hn}(\boldsymbol{k}, \boldsymbol{p})$ are normalized static correlation functions:

$${\rm i}\Omega_{nh}(\boldsymbol{k}, \boldsymbol{p}\,) = \langle \dot{\hat{n}}_{\boldsymbol{k}}(\boldsymbol{p}) \hat{h}^{\rm int}_{-\boldsymbol{k}} \rangle_0 \Phi^{-1}_{hh}(\boldsymbol{k}), \tag{3.48}$$

$${\rm i}\Omega_{hn}(\boldsymbol{k}, \boldsymbol{p}) = \int {\rm d}\boldsymbol{p}'\ \langle \dot{\hat{h}}^{\rm int}_{\boldsymbol{k}}\ \hat{n}_{-\boldsymbol{k}}(\boldsymbol{p}') \rangle_0 \Phi^{-1}_{\boldsymbol{k}}(\boldsymbol{p}', \boldsymbol{p}) \tag{3.49}$$

and

$$\varphi_{nn}(\boldsymbol{k}, \boldsymbol{p}, \boldsymbol{p}'; t, t') = \int {\rm d}\boldsymbol{p}''\ \langle I_n(\boldsymbol{k}, \boldsymbol{p}) T_0(t, t') I_n(-\boldsymbol{k}, \boldsymbol{p}'') \rangle_0 \Phi^{-1}_{\boldsymbol{k}}(\boldsymbol{p}'', \boldsymbol{p}'), \tag{3.50}$$

$$\varphi_{hn}(\boldsymbol{k}, \boldsymbol{p}; t, t') = \int {\rm d}\boldsymbol{p}'\ \langle I^{\rm int}_h(\boldsymbol{k}) T_0(t, t') I_n(-\boldsymbol{k}, \boldsymbol{p}') \rangle_0 \Phi^{-1}_{\boldsymbol{k}}(\boldsymbol{p}', \boldsymbol{p}), \tag{3.51}$$

$$\varphi_{nh}(\boldsymbol{k}, \boldsymbol{p}; t, t') = \langle I_n(\boldsymbol{k}, \boldsymbol{p}) T_0(t, t') I^{\rm int}_h(-\boldsymbol{k}) \rangle_0 \Phi^{-1}_{hh}(\boldsymbol{k}), \tag{3.52}$$

$$\varphi_{hh}(\boldsymbol{k}; t, t') = \langle I^{\rm int}_h(\boldsymbol{k}) T_0(t, t') I^{\rm int}_h(-\boldsymbol{k}) \rangle_0 \Phi^{-1}_{hh}(\boldsymbol{k}) \tag{3.53}$$

are generalized transport kernels (memory functions) which describe kinetic and hydrodynamic processes. The system of transport equations (3.46), (3.47) is closed. Eliminating $h^{\rm int}_{\boldsymbol{k}}(t)$, it is possible to obtain a closed kinetic equation for a Fourier component of the nonequilibrium one-particle distribution function. For this purpose one uses a Laplace transform with respect to time, assuming that at $t > 0$ the quantities $f_{\boldsymbol{k}}(\boldsymbol{p}; t = 0)$, $h^{\rm int}_{\boldsymbol{k}}(t = 0)$ are known

$$A(z) = {\rm i} \int_0^{\infty} {\rm d}t\ {\rm e}^{{\rm i}zt} A(t), \qquad z = \omega + {\rm i}\varepsilon, \qquad \varepsilon \rightarrow +0. \tag{3.54}$$

Then, equations (6.46) and (6.47) are presented in the form:

$$z f_{\boldsymbol{k}}(\boldsymbol{p};z) + \frac{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{p}}{m} f_{\boldsymbol{k}}(\boldsymbol{p};z) = -\frac{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{p}}{m} n f_0(p) c_2(\boldsymbol{k}) \int \mathrm{d}\boldsymbol{p}'\; f_{\boldsymbol{k}}(\boldsymbol{p}';z) + \tag{3.55}$$
$$\Sigma_{nh}(\boldsymbol{k},\boldsymbol{p};z) h^{\mathrm{int}}_{\boldsymbol{k}}(z) - \int \mathrm{d}\boldsymbol{p}'\; \varphi_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';z) f_{\boldsymbol{k}}(\boldsymbol{p}';z) + f_{\boldsymbol{k}}(\boldsymbol{p};t=0),$$

$$z h^{\mathrm{int}}_{\boldsymbol{k}}(z) = \int \mathrm{d}\boldsymbol{p}'\; \Sigma_{hn}(\boldsymbol{k},\boldsymbol{p}';z) f_{\boldsymbol{k}}(\boldsymbol{p}';z) - \varphi_{hh}(\boldsymbol{k};z) h^{\mathrm{int}}_{\boldsymbol{k}}(z) + h^{\mathrm{int}}_{\boldsymbol{k}}(t=0), \tag{3.56}$$

where

$$\Sigma_{nh}(\boldsymbol{k},\boldsymbol{p};z) = \mathrm{i}\Omega_{nh}(\boldsymbol{k},\boldsymbol{p}) - \varphi_{nh}(\boldsymbol{k},\boldsymbol{p};z), \tag{3.57}$$
$$\Sigma_{hn}(\boldsymbol{k},\boldsymbol{p};z) = \mathrm{i}\Omega_{hn}(\boldsymbol{k},\boldsymbol{p}) - \varphi_{hn}(\boldsymbol{k},\boldsymbol{p};z). \tag{3.58}$$

Let us solve equation (3.56) with respect to $h^{\mathrm{int}}_{\boldsymbol{k}}(z)$ and substitute the result into (3.55). Then one obtains a closed kinetic equation for $f_{\boldsymbol{k}}(\boldsymbol{p};z)$ (at $h^{\mathrm{int}}_{\boldsymbol{k}}(t=0)=0$):

$$z f_{\boldsymbol{k}}(\boldsymbol{p};z) + \frac{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{p}}{m} f_{\boldsymbol{k}}(\boldsymbol{p};z) = -\frac{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{p}}{m} n f_0(p) c_2(\boldsymbol{k}) \int \mathrm{d}\boldsymbol{p}'\; f_{\boldsymbol{k}}(\boldsymbol{p}';z) -$$
$$\int \mathrm{d}\boldsymbol{p}'\; D_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';z) f_{\boldsymbol{k}}(\boldsymbol{p}';z) + f_{\boldsymbol{k}}(\boldsymbol{p};t=0), \tag{3.59}$$

where

$$D_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';z) = \varphi_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';z) - \Sigma_{nh}(\boldsymbol{k},\boldsymbol{p};z)\frac{1}{z+\varphi_{hh}(\boldsymbol{k};z)}\Sigma_{hn}(\boldsymbol{k},\boldsymbol{p}';z) \tag{3.60}$$

is a generalized transport kernel of kinetic processes, which is renormalized taking into account the processes of transport of the potential energy of interaction between the particles. If we put formally in $D_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';z)$ that $\hat{h}^{\mathrm{int}}_{-\boldsymbol{k}} = 0$ (such equality is valid when the contribution of the average potential energy of interaction is much smaller than that of the average kinetic energy), then from (3.59) one obtains a kinetic equation for $f_{\boldsymbol{k}}(\boldsymbol{p};z)$,

$$z f_{\boldsymbol{k}}(\boldsymbol{p};z) + \frac{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{p}}{m} f_{\boldsymbol{k}}(\boldsymbol{p};z) = -\frac{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{p}}{m} n f_0(p) c_2(\boldsymbol{k}) \int \mathrm{d}\boldsymbol{p}'\; f_{\boldsymbol{k}}(\boldsymbol{p}';z) -$$
$$\int \mathrm{d}\boldsymbol{p}'\; \varphi'_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';z) f_{\boldsymbol{k}}(\boldsymbol{p}';z) + f_{\boldsymbol{k}}(\boldsymbol{p};t=0). \tag{3.61}$$

Equation (3.61) was derived for the first time by using the Mori projection operators method in [107–109], when the microscopic phase density $\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})$ was a parameter of an abbreviated description. In this case the memory function $\varphi'_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';z)$ has the following structure:

$$\varphi'_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';t,t') = \int \mathrm{d}\boldsymbol{p}''\; \langle I^0_n(\boldsymbol{k},\boldsymbol{p}) T'_0(t,t') I^0_n(-\boldsymbol{k},\boldsymbol{p}'')\rangle_0 \Phi^{-1}_{\boldsymbol{k}}(\boldsymbol{p}'',\boldsymbol{p}'), \tag{3.62}$$

where

$$I^0_n(\boldsymbol{k},\boldsymbol{p}) = (1-\mathscr{P}'_0)\dot{\hat{n}}_{\boldsymbol{k}}(\boldsymbol{p}) \tag{3.63}$$

is a generalized flow, $\mathscr{P}'_0$ is the Mori projection operator introduced in [107–109]:

$$\mathscr{P}'_0 \hat{A}_{\boldsymbol{k}'} = \sum_{\boldsymbol{k}} \int \mathrm{d}\boldsymbol{p}\, \mathrm{d}\boldsymbol{p}'\, \langle \hat{A}_{\boldsymbol{k}'} \hat{n}_{-\boldsymbol{k}}(\boldsymbol{p}') \rangle_0 \Phi^{-1}_{\boldsymbol{k}}(\boldsymbol{p}', \boldsymbol{p}) \hat{n}_{\boldsymbol{k}}(\boldsymbol{p}), \tag{3.64}$$

$T'_0(t,t') = \mathrm{e}^{(t-t')(1-\mathscr{P}'_0)\mathrm{i}L_N}$ is the corresponding time evolution operator. The kinetic equation (3.59) for the Fourier component of the nonequilibrium one-particle distribution function stimulates investigations of the dynamical structure factor $S(\boldsymbol{k};\omega)$, time correlation functions for transverse and longitudinal flows of the particles, diffusion coefficients, viscosities for dense gases and liquids [77,79,107–119]. In papers by Mazenko, there was performed a derivation of the linearized Boltzmann-Enskog equation by expanding memory functions $\varphi'_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';t,t')$ on density. In the case of weak interactions a Fokker-Planck equation was obtained. Mazenko [114] and Forster [115,136] calculated the generalized viscosity coefficient with the help of kinetic equation (3.59). Self-diffusion of a one-component plasma [181], as well as spectra of mass and charge fluctuations in ionic solutions were investigated on the basis of this equation. In actual calculations, certain approximations for the memory functions $\varphi'_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';t,t')$ were used. However, the main drawback of the kinetic equation (3.59) lies in its non-self-consistency with the conservation law of the total energy for dense gases and liquids when the contribution of the interaction energy between the particles to thermodynamic quantities and transport coefficients becomes important. This fact was also pointed out earlier in the paper by John and Forster [118], where they performed an investigation of the dynamical structure factor $S(\boldsymbol{k};\omega)$ in the intermediate region of wavevector $\boldsymbol{k}$ and frequency values ω for a simple liquid on the basis of the set of dynamical variables of an abbreviated description $\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})$ and $\hat{\mathcal{E}}_{\boldsymbol{k}}$.

In the next subsection, on the basis of a system of transport equations for Fourier components of the nonequilibrium one-particle distribution function and the potential part of enthalpy (3.46), (3.47), we shall obtain equations for time correlation functions. We shall also investigate the spectrum of collective excitations and the structure of generalized transport coefficients.

3.4. Time correlation functions, collective modes and generalized transport coefficients

With the help of combined equations (3.46), (3.47) one obtains a system for time correlation functions:

$$\Phi_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';t) = \int \mathrm{d}\boldsymbol{p}''\, \langle \hat{n}_{\boldsymbol{k}}(\boldsymbol{p};t) \hat{n}_{-\boldsymbol{k}}(\boldsymbol{p}'';0) \rangle_0 \Phi^{-1}_{\boldsymbol{k}}(\boldsymbol{p}'',\boldsymbol{p}'), \tag{3.65}$$

$$\Phi_{hn}(\boldsymbol{k},\boldsymbol{p};t) = \int \mathrm{d}\boldsymbol{p}'\, \langle \hat{h}^{\mathrm{int}}_{\boldsymbol{k}}(t) \hat{n}_{-\boldsymbol{k}}(\boldsymbol{p}';0) \rangle_0 \Phi^{-1}_{\boldsymbol{k}}(\boldsymbol{p}',\boldsymbol{p}), \tag{3.66}$$

$$\Phi_{nh}(\boldsymbol{k},\boldsymbol{p};t) = \langle \hat{n}_{\boldsymbol{k}}(\boldsymbol{p};t) \hat{h}^{\mathrm{int}}_{-\boldsymbol{k}}(0) \rangle_0 \Phi^{-1}_{hh}(\boldsymbol{k}), \tag{3.67}$$

$$\Phi_{hh}(\boldsymbol{k};t) = \langle \hat{h}^{\mathrm{int}}_{\boldsymbol{k}}(t) \hat{h}^{\mathrm{int}}_{-\boldsymbol{k}}(0) \rangle_0 \Phi^{-1}_{hh}(\boldsymbol{k}), \tag{3.68}$$

where $\hat{n}_{\boldsymbol{k}}(\boldsymbol{p};t) = \mathrm{e}^{-\mathrm{i}L_N t} \hat{n}_{\boldsymbol{k}}(\boldsymbol{p};0)$, $\hat{h}^{\mathrm{int}}_{\boldsymbol{k}}(t) = \mathrm{e}^{-\mathrm{i}L_N t} \hat{h}^{\mathrm{int}}_{\boldsymbol{k}}(0)$.

One uses the Fourier transform with respect to time

$$\langle a\rangle_{\omega}=\int_{-\infty}^{\infty}\mathrm{d}t\ \mathrm{e}^{\mathrm{i}\omega t}\langle a\rangle^{t}.$$

Then we write the system of equations (3.46), (3.47) in the form:

$$-\mathrm{i}\omega\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})\rangle_{\omega}=\int\mathrm{d}\boldsymbol{p}'\ \Sigma_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';\omega+\mathrm{i}\varepsilon)\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p}')\rangle_{\omega}+\Sigma_{nh}(\boldsymbol{k},\boldsymbol{p};\omega+\mathrm{i}\varepsilon)\langle\hat{h}^{\mathrm{int}}_{\boldsymbol{k}}\rangle_{\omega},\tag{3.69}$$

$$-\mathrm{i}\omega\langle\hat{h}^{\mathrm{int}}_{\boldsymbol{k}}\rangle_{\omega}=\int\mathrm{d}\boldsymbol{p}'\ \Sigma_{hn}(\boldsymbol{k},\boldsymbol{p}';\omega+\mathrm{i}\varepsilon)\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p}')\rangle_{\omega}-\varphi_{hh}(\boldsymbol{k};\omega+\mathrm{i}\varepsilon)\langle\hat{h}^{\mathrm{int}}_{\boldsymbol{k}}\rangle_{\omega},\tag{3.70}$$

where

$$\Sigma_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';\omega+\mathrm{i}\varepsilon)=\mathrm{i}\Omega_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}')-\varphi_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';\omega+\mathrm{i}\varepsilon),\tag{3.71}$$

$$\Sigma_{nh}(\boldsymbol{k},\boldsymbol{p};\omega+\mathrm{i}\varepsilon)=\mathrm{i}\Omega_{nh}(\boldsymbol{k},\boldsymbol{p})-\varphi_{nh}(\boldsymbol{k},\boldsymbol{p};\omega+\mathrm{i}\varepsilon),\tag{3.72}$$

$$\Sigma_{hn}(\boldsymbol{k},\boldsymbol{p};\omega+\mathrm{i}\varepsilon)=\mathrm{i}\Omega_{hn}(\boldsymbol{k},\boldsymbol{p})-\varphi_{hn}(\boldsymbol{k},\boldsymbol{p};\omega+\mathrm{i}\varepsilon).\tag{3.73}$$

It is more convenient to present the system of equations (3.69), (3.70) in a matrix form:

$$-\mathrm{i}\omega\langle\tilde{a}_{\boldsymbol{k}}\rangle_{\omega}=\tilde{\Sigma}(\boldsymbol{k};\omega+\mathrm{i}\varepsilon)\langle\tilde{a}_{\boldsymbol{k}}\rangle_{\omega},\tag{3.74}$$

where $\tilde{a}_{\boldsymbol{k}}=\mathrm{col}(\hat{n}_{\boldsymbol{k}}(\boldsymbol{p}),\hat{h}^{\mathrm{int}}_{\boldsymbol{k}})$ is a vector-column and

$$\tilde{\Sigma}(\boldsymbol{k};\omega+\mathrm{i}\varepsilon)=\begin{bmatrix}\int\mathrm{d}\boldsymbol{p}'\ \Sigma_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';\omega+\mathrm{i}\varepsilon) & \Sigma_{nh}(\boldsymbol{k},\boldsymbol{p};\omega+\mathrm{i}\varepsilon)\\ \int\mathrm{d}\boldsymbol{p}'\ \Sigma_{hn}(\boldsymbol{k},\boldsymbol{p}';\omega+\mathrm{i}\varepsilon) & -\varphi_{hh}(\boldsymbol{k};\omega+\mathrm{i}\varepsilon)\end{bmatrix},\tag{3.75}$$

$$\tilde{\Sigma}(\boldsymbol{k};\omega+\mathrm{i}\varepsilon)=\int_{0}^{\infty}\mathrm{d}t\ \mathrm{e}^{\mathrm{i}(\omega+\mathrm{i}\varepsilon)t}\tilde{\Sigma}(\boldsymbol{k};t).$$

Now, one uses the solution to the Liouville equation in approximation (3.41) without introducing the projection operator $\mathscr{P}_{\mathrm{q}}(t)$:

$$\varrho\left(x^{N};t\right)=\varrho_{\mathrm{q}}\left(x^{N};t\right)-\int\limits_{-\infty}^{t}\mathrm{d}t'\ \mathrm{e}^{\varepsilon(t'-t)}\mathrm{e}^{\mathrm{i}L_{N}(t'-t)}\left(\frac{\partial}{\partial t}'+\mathrm{i}L_{N}\right)\varrho_{\mathrm{q}}\left(x^{N};t'\right).$$

Then, from the self-consistency conditions $\langle\tilde{a}_{\boldsymbol{k}}\rangle^{t}=\langle\tilde{a}_{\boldsymbol{k}}\rangle^{t}_{\mathrm{q}}$ one obtains a system of equations which connects the average values $\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})\rangle_{\omega}$ and $\langle\hat{h}^{\mathrm{int}}_{\boldsymbol{k}}\rangle_{\omega}$ with spectral functions of time correlation functions:

$$\mathrm{i}\omega\begin{bmatrix}\int\mathrm{d}\boldsymbol{p}'\ \Phi_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';\omega+\mathrm{i}\varepsilon) & \Phi^{\mathrm{int}}_{nh}(\boldsymbol{k},\boldsymbol{p};\omega+\mathrm{i}\varepsilon)\\ \int\mathrm{d}\boldsymbol{p}'\ \Phi^{\mathrm{int}}_{hn}(\boldsymbol{k},\boldsymbol{p}';\omega+\mathrm{i}\varepsilon) & -\Phi^{\mathrm{int}}_{hh}(\boldsymbol{k};\omega+\mathrm{i}\varepsilon)\end{bmatrix}\times\begin{bmatrix}\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})\rangle_{\omega}\\ \langle\hat{h}^{\mathrm{int}}_{\boldsymbol{k}}\rangle_{\omega}\end{bmatrix}=$$

$$\begin{bmatrix}\int\mathrm{d}\boldsymbol{p}'\ \Phi_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}') & 0\\ 0 & -\Phi_{hh}(\boldsymbol{k})\end{bmatrix}-$$

$$\mathrm{i}(\omega+\mathrm{i}\varepsilon)\begin{bmatrix}\int\mathrm{d}\boldsymbol{p}'\ \Phi_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';\omega+\mathrm{i}\varepsilon) & \Phi^{\mathrm{int}}_{nh}(\boldsymbol{k},\boldsymbol{p};\omega+\mathrm{i}\varepsilon)\\ \int\mathrm{d}\boldsymbol{p}'\ \Phi^{\mathrm{int}}_{hn}(\boldsymbol{k},\boldsymbol{p}';\omega+\mathrm{i}\varepsilon) & -\Phi^{\mathrm{int}}_{hh}(\boldsymbol{k};\omega+\mathrm{i}\varepsilon)\end{bmatrix}\times\begin{bmatrix}\langle\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})\rangle_{\omega}\\ \langle\hat{h}^{int}_{\boldsymbol{k}}\rangle_{\omega}\end{bmatrix}.$$

Such a system can be presented in the next compact form:

$$\mathrm{i}\omega\tilde{\Phi}(\boldsymbol{k};\omega+\mathrm{i}\varepsilon)\langle\tilde{a}_{\boldsymbol{k}}\rangle_\omega = \left[\tilde{\Phi}(\boldsymbol{k}) - \mathrm{i}(\omega+\mathrm{i}\varepsilon)\tilde{\Phi}(\boldsymbol{k};\omega+\mathrm{i}\varepsilon)\right]\langle\tilde{a}_{\boldsymbol{k}}\rangle_\omega. \tag{3.76}$$

Let us multiply equation (3.74) by the matrix $\tilde{\Phi}(\boldsymbol{k};\omega+\mathrm{i}\varepsilon)$ and compare the result with equation (3.76). So we find

$$\begin{aligned} z\tilde{\Phi}(\boldsymbol{k};z) &= \tilde{\Sigma}(\boldsymbol{k};z)\tilde{\Phi}(\boldsymbol{k};z) - \tilde{\Phi}(\boldsymbol{k}), \qquad z=\omega+\mathrm{i}\varepsilon, \\ \tilde{\Phi}(\boldsymbol{k};z) &= \mathrm{i}\int_0^\infty \mathrm{d}t\ \mathrm{e}^{\mathrm{i}zt}\tilde{\Phi}(\boldsymbol{k};t), \qquad \varepsilon\to 0 \end{aligned} \tag{3.77}$$

or in an explicit form:

$$\begin{aligned} z\Phi_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';z) &= \int \mathrm{d}\boldsymbol{p}''\ \Sigma_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}'';z)\Phi_{nn}(\boldsymbol{k},\boldsymbol{p}'',\boldsymbol{p}';z) + \\ &\quad \Sigma_{nh}(\boldsymbol{k},\boldsymbol{p};z)\Phi_{hn}^{\mathrm{int}}(\boldsymbol{k},\boldsymbol{p}';z) - \Phi_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}'), \end{aligned} \tag{3.78}$$

$$z\Phi_{nh}^{\mathrm{int}}(\boldsymbol{k},\boldsymbol{p};z) = \int \mathrm{d}\boldsymbol{p}''\ \Sigma_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}'';z)\Phi_{nh}^{\mathrm{int}}(\boldsymbol{k},\boldsymbol{p}'';z) + \Sigma_{nh}(\boldsymbol{k},\boldsymbol{p};z)\Phi_{hh}^{\mathrm{int}}(\boldsymbol{k};z), \tag{3.79}$$

$$z\Phi_{hn}^{\mathrm{int}}(\boldsymbol{k},\boldsymbol{p}';z) = \int \mathrm{d}\boldsymbol{p}''\ \Sigma_{hn}(\boldsymbol{k},\boldsymbol{p}'';z)\Phi_{nn}^{\mathrm{int}}(\boldsymbol{k},\boldsymbol{p}'',\boldsymbol{p}';z) - \varphi_{hh}(\boldsymbol{k};z)\Phi_{hn}^{\mathrm{int}}(\boldsymbol{k},\boldsymbol{p}';z), \tag{3.80}$$

$$z\Phi_{hh}^{\mathrm{int}}(\boldsymbol{k};z) = \int \mathrm{d}\boldsymbol{p}''\ \Sigma_{hn}(\boldsymbol{k},\boldsymbol{p}'';z)\Phi_{nh}^{\mathrm{int}}(\boldsymbol{k},\boldsymbol{p}'';z) - \varphi_{hh}(\boldsymbol{k};z)\Phi_{hh}^{\mathrm{int}}(\boldsymbol{k};z) - \Phi_{hh}(\boldsymbol{k}), \tag{3.81}$$

where the condition $\Phi_{hn}(\boldsymbol{k},\boldsymbol{p}') = \Phi_{nh}(\boldsymbol{k},\boldsymbol{p}) = 0$ is taken into account.

Therefore, we obtained a system of equations for normalized correlation functions (3.65)–(3.68) for the description of nonequilibrium states of the system when kinetic and hydrodynamic processes are considered simultaneously. A similar system of equations for time correlation functions in the method of projection operators was obtained in the paper by John and Forster [118]. In this paper the parameters of an abbreviated description were orthogonal dynamical variables $\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})$ and $\hat{\mathcal{E}}_{\boldsymbol{k}}$. Solutions of such an equation system in the hydrodynamic limit were found using the projection procedure for the first five moments of the nonequilibrium one-particle distribution function. In view of this, the dynamical structure factor $S(\boldsymbol{k};\omega)$ for intermediate values of $\boldsymbol{k}$ and ω was investigated using a parametric approximation of memory functions for liquid Argon. However, in this paper the question concerning collective modes and generalized transport coefficients, when kinetics and hydrodynamics are connected between themselves, is not discussed.

In order to solve the system of equations (3.78)–(3.81) we also apply the projection procedure [108]. Let us introduce the dimensionless momentum $\boldsymbol{\xi} = \frac{\boldsymbol{k}}{mv_0}$, $v_0^2 = (m\beta)^{-1}$. Then the system of equations (3.77) can be rewritten in the matrix form:

$$z\tilde{\Phi}(\boldsymbol{k};\boldsymbol{\xi},\boldsymbol{\xi}';z) - \tilde{\Sigma}(\boldsymbol{k};\boldsymbol{\xi},\boldsymbol{\xi}'';z)\tilde{\Phi}(\boldsymbol{k};\boldsymbol{\xi}'',\boldsymbol{\xi}';z) = -\tilde{\Phi}(\boldsymbol{k};\boldsymbol{\xi},\boldsymbol{\xi}'), \tag{3.82}$$

where it is clear that the integration must be performed with respect to the repeating indices $\boldsymbol{\xi}''$. Further, let us introduce the scalar product of two functions,

$\phi(\boldsymbol{\xi})$ and $\psi(\boldsymbol{\xi})$, as

$$\langle\phi|\psi\rangle = \int \mathrm{d}\boldsymbol{\xi}\ \phi^*(\boldsymbol{\xi}) f_0(\xi)\psi(\boldsymbol{\xi}). \tag{3.83}$$

Then, the matrix element for some "operator" M can be determined as

$$\langle\phi|M|\psi\rangle = \int \mathrm{d}\boldsymbol{\xi}\ \mathrm{d}\boldsymbol{\xi}'\ \phi^*(\boldsymbol{\xi}) M(\boldsymbol{\xi},\boldsymbol{\xi}') f_0(\xi')\psi(\boldsymbol{\xi}'). \tag{3.84}$$

Let $\phi(\boldsymbol{\xi}) = \{\phi_\mu(\boldsymbol{\xi})\}$ be the orthogonalized basis of functions with the weight $f_0(\xi)$, so that the following condition is satisfied:

$$\langle\phi_\nu|\phi_\mu\rangle = \delta_{\nu\mu}, \qquad \sum_\nu |\phi_\nu\rangle\langle\phi_\nu| = 1, \tag{3.85}$$

where

$$\phi_\mu(\boldsymbol{\xi}) = \phi_{lmn}(\boldsymbol{\xi}) = (l!m!n!)^{-1/2}\bar{H}_l(\xi_x)\bar{H}_m(\xi_y)\bar{H}_n(\xi_z), \tag{3.86}$$

$\bar{H}_l(\xi) = 2^{-l/2}H_l(\xi/2)$, $H_l(\xi)$ is a Hermite polynomial. Then, each function in the system of equations (3.82), which depends on momentum variables $\boldsymbol{\xi}$, $\boldsymbol{\xi}'$, can be expanded over functions $\phi_\mu(\boldsymbol{\xi})$ in the series:

$$\tilde{\Phi}(\boldsymbol{k};\boldsymbol{\xi},\boldsymbol{\xi}';z) = \sum_{\nu,\mu} \phi^*_\nu(\boldsymbol{\xi})\tilde{\Phi}_{\nu\mu}(\boldsymbol{k};z)\phi_\mu(\boldsymbol{\xi}')f_0(\xi'), \tag{3.87}$$

$$\tilde{\Sigma}(\boldsymbol{k};\boldsymbol{\xi},\boldsymbol{\xi}';z) = \sum_{\nu,\mu} \phi^*_\nu(\boldsymbol{\xi})\tilde{\Sigma}_{\nu\mu}(\boldsymbol{k};z)\phi_\mu(\boldsymbol{\xi}')f_0(\xi'), \tag{3.88}$$

where

$$\tilde{\Phi}_{\nu\mu}(\boldsymbol{k};z) = \langle\phi_\nu|\tilde{\Phi}(\boldsymbol{k};\boldsymbol{\xi},\boldsymbol{\xi}';z)|\phi_\mu\rangle = \int \mathrm{d}\boldsymbol{\xi}\ \mathrm{d}\boldsymbol{\xi}'\ \phi^*_\nu(\boldsymbol{\xi})f_0(\xi)\tilde{\Phi}(\boldsymbol{k};\boldsymbol{\xi},\boldsymbol{\xi}';z)\phi_\mu(\boldsymbol{\xi}'), \tag{3.89}$$

$$\tilde{\Sigma}_{\nu\mu}(\boldsymbol{k};z) = \langle\phi_\nu|\tilde{\Sigma}(\boldsymbol{k};\boldsymbol{\xi},\boldsymbol{\xi}';z)|\phi_\mu\rangle = \int \mathrm{d}\boldsymbol{\xi}\ \mathrm{d}\boldsymbol{\xi}'\ \phi^*_\nu(\boldsymbol{\xi})f_0(\xi)\tilde{\Sigma}(\boldsymbol{k};\boldsymbol{\xi},\boldsymbol{\xi}';z)\phi_\mu(\boldsymbol{\xi}'). \tag{3.90}$$

Let us substitute expansions (3.87)–(3.90) into equation (3.82). As a result, one obtains:

$$z\tilde{\Phi}_{\nu\mu}(\boldsymbol{k};z) - \sum_\gamma \tilde{\Sigma}_{\nu\gamma}(\boldsymbol{k};z)\tilde{\Phi}_{\gamma\mu}(\boldsymbol{k};z) = -\tilde{\Phi}_{\nu\mu}(\boldsymbol{k}). \tag{3.91}$$

In actual calculations, a finite number of functions from the set $\phi_\nu(\boldsymbol{\xi})$ is used. Taking into account this fact, let us introduce the projection operator P which projects arbitrary functions $\psi(\boldsymbol{\xi})$ onto a finite set of functions $\phi_\mu(\boldsymbol{\xi})$:

$$P = \sum_{\nu=1}^{n} |\phi_\nu\rangle\langle\phi_\nu| = 1 - Q, \qquad P\langle\psi| = \sum_{\nu=1}^{n}\langle\psi|\phi_\nu\rangle\langle\phi_\nu|. \tag{3.92}$$

Here n denotes a finite number of functions. Then, from (3.91) we obtain a system of equations for a finite set of functions $\phi_\mu(\boldsymbol{\xi})$,

$$\sum_{\gamma=1}^{n}\left[z\bar{\delta}_{\nu\gamma} - \mathrm{i}\tilde{\Omega}_{\nu\gamma}(\boldsymbol{k}) + \tilde{D}_{\nu\gamma}(\boldsymbol{k};z)\right]\tilde{\Phi}_{\gamma\mu}(\boldsymbol{k};z) = -\tilde{\Phi}_{\nu\mu}(\boldsymbol{k}), \tag{3.93}$$

where

$$\tilde{D}_{\nu\mu}(\boldsymbol{k};z) = \langle\phi_\nu|\tilde{\varphi}(\boldsymbol{k};z) + \tilde{\Sigma}(\boldsymbol{k};z)Q\left[z - Q\tilde{\Sigma}(\boldsymbol{k};z)Q\right]^{-1}Q\tilde{\Sigma}(\boldsymbol{k};z)|\phi_\mu\rangle \qquad (3.94)$$

are generalized hydrodynamic transport kernels and

$$\mathrm{i}\tilde{\Omega}_{\nu\mu}(\boldsymbol{k}) = \langle\phi_\nu|\mathrm{i}\tilde{\Omega}(\boldsymbol{k})|\phi_\mu\rangle \qquad (3.95)$$

is a frequency matrix. Note that matrices $\mathrm{i}\tilde{\Omega}(\boldsymbol{k})$ and $\tilde{\varphi}(\boldsymbol{k};z)$ are defined according to (3.48), (3.49) and (3.50)–(3.53).

Let us find solutions to the system of equations (3.93) in the hydrodynamic region when a set of functions $\phi_\mu(\boldsymbol{\xi})$ present five moments of a one-particle distribution function:

$$\phi_1(\boldsymbol{\xi}) = 1, \quad \phi_2(\boldsymbol{\xi}) = \xi_z, \quad \phi_3(\boldsymbol{\xi}) = \frac{1}{\sqrt{6}}(\xi^2-3), \quad \phi_4(\boldsymbol{\xi}) = \xi_x, \quad \phi_5(\boldsymbol{\xi}) = \xi_y. \quad (3.96)$$

Then, the following relations are fulfilled:

$$\begin{aligned}\langle 1|\hat{n}_{\boldsymbol{k}}(\boldsymbol{\xi}) &= \int \mathrm{d}\boldsymbol{\xi}\ \hat{n}_{\boldsymbol{k}}(\boldsymbol{\xi}) = \hat{n}_{\boldsymbol{k}},\\ \langle\xi_\gamma|\hat{n}_{\boldsymbol{k}}(\boldsymbol{\xi}) &= \int \mathrm{d}\boldsymbol{\xi}\ \hat{n}_{\boldsymbol{k}}(\boldsymbol{\xi})\ \xi_\gamma = \hat{p}^\gamma_{\boldsymbol{k}},\\ \langle 6^{-1/2}(\xi^2-3)|\hat{n}_{\boldsymbol{k}}(\boldsymbol{\xi}) &= \int \mathrm{d}\boldsymbol{\xi}\ \hat{n}_{\boldsymbol{k}}(\boldsymbol{\xi})\ 6^{-1/2}(\xi^2-3) = \hat{\mathcal{E}}^{\mathrm{kin}}_{\boldsymbol{k}} - 3\hat{n}_{\boldsymbol{k}}\beta^{-1} = \hat{h}^{\mathrm{kin}}_{\boldsymbol{k}},\end{aligned} \qquad (3.97)$$

for the Fourier components of densities for the number of particles, momentum and the kinetic part of generalized enthalpy. Besides that, the microscopic conservation laws for densities of the number of particles and momentum can be written in the form:

$$\begin{aligned}\langle 1|\dot{\hat{n}}_{\boldsymbol{k}}(\boldsymbol{\xi}) &= -\mathrm{i}k_\gamma\hat{p}^\gamma_{\boldsymbol{k}}m^{-1},\\ \langle\xi_\alpha|\dot{\hat{n}}_{\boldsymbol{k}}(\boldsymbol{\xi}) &= -\mathrm{i}k_\gamma\hat{\overset{\leftrightarrow}{T}}{}^{\gamma\alpha}_{\boldsymbol{k}},\end{aligned} \qquad (3.98)$$

where $\hat{\overset{\leftrightarrow}{T}}{}^{\gamma\alpha}_{\boldsymbol{k}}$ is a Fourier component of the stress-tensor.

If we choose the direction of wavevector $\boldsymbol{k}$ along oz-axis, then $\phi_\nu(\boldsymbol{\xi})$, $\nu = 1,2,3$ will correspond to longitudinal modes, whereas $\phi_\nu(\boldsymbol{\xi})$ at $\nu = 4,5$ will be related to transverse modes.

From the system of equations (3.93), at $\nu = 4,5$, $\phi_4(\boldsymbol{\xi}) = \xi_x$, $\phi_5(\boldsymbol{\xi}) = \xi_y$, one obtains an equation for the Fourier component of the time correlation function connected with the transverse component of the momentum density $\Phi_{44}(\boldsymbol{k};z)$. From this equation one finds:

$$\Phi_{44}(\boldsymbol{k};z) = \Phi^\perp_{pp}(\boldsymbol{k};z) = -\frac{1}{z + D^\perp_{pp}(\boldsymbol{k};z)}, \qquad (3.99)$$

where

$$\Phi^{\perp}_{pp}(\boldsymbol{k};z) \;=\; \langle\xi_x|\Phi_{nn}(\boldsymbol{k},\boldsymbol{\xi},\boldsymbol{\xi}';z)|\xi'_x\rangle, \tag{3.100}$$

$$D^{\perp}_{pp}(\boldsymbol{k};z) \;=\; D^{\perp(\mathrm{kin})}_{pp}(\boldsymbol{k};z)+D^{\perp(\mathrm{int})}_{pp}(\boldsymbol{k};z), \tag{3.101}$$

$$D^{\perp(\mathrm{kin})}_{pp}(\boldsymbol{k};z) \;=\; \langle\xi_x|\varphi_{nn}(\boldsymbol{k},\boldsymbol{\xi},\boldsymbol{\xi}';z)|\xi'_x\rangle, \tag{3.102}$$

$$D^{\perp(\mathrm{int})}_{pp}(\boldsymbol{k};z) \;=\; \langle\xi_x|\left[\tilde{\Sigma}(\boldsymbol{k},\boldsymbol{\xi},\boldsymbol{\xi}';z)Q\left[z-Q\tilde{\Sigma}(\boldsymbol{k},\boldsymbol{\xi},\boldsymbol{\xi}';z)Q\right]^{-1}Q\tilde{\Sigma}(\boldsymbol{k},\boldsymbol{\xi},\boldsymbol{\xi}';z)\right]_{nn}|\xi'_x\rangle, \tag{3.103}$$

$$D^{\perp}_{pp}(\boldsymbol{k};z) \;=\; \mathrm{i}k^2\eta(\boldsymbol{k};z)(mn)^{-1}, \tag{3.104}$$

where $\eta(\boldsymbol{k};z)$ denotes a generalized coefficient of shear viscosity. Such a coefficient consists of two main contributions. The first one is $D^{\perp(\mathrm{kin})}_{pp}(\boldsymbol{k};z)$, whereas the second contribution $D^{\perp(\mathrm{int})}_{pp}(\boldsymbol{k};z)$ describes a relation of kinetic and hydrodynamic processes. If the last term is neglected, which formally corresponds to $\hat{h}^{\mathrm{int}}_{\boldsymbol{k}}=0$, then one obtains an expression for the generalized coefficient of shear viscosity $\eta(\boldsymbol{k};z)$, obtained earlier by the authors of [114,115] when solving the equation

$$\begin{aligned} z\Phi_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';z)-\int\mathrm{d}\boldsymbol{p}''\;\mathrm{i}\Omega_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}'')\Phi_{nn}(\boldsymbol{k},\boldsymbol{p}'',\boldsymbol{p}';z)\;+&\\ \int\mathrm{d}\boldsymbol{p}''\;\varphi'_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}'';z)\Phi_{nn}(\boldsymbol{k},\boldsymbol{p}'',\boldsymbol{p}';z)\;&=\;-\Phi_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}'), \end{aligned} \tag{3.105}$$

by using the method of projections [108] in the hydrodynamic region. Equation (3.105) corresponds to kinetic equation (3.61) for the Fourier component of the nonequilibrium one-particle distribution function. In $\varphi'_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';z)$ the projection is performed on a space of dynamics of the "slow" variable $\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})$ connected with the microscopic conservation laws for the particle-number density and momentum (3.98). In our approach in $D^{\perp}_{pp}(\boldsymbol{k};z)$, the projection is carried out on an extended space of dynamics of the "slow" variables $\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})$ and $\hat{h}^{\mathrm{int}}_{\boldsymbol{k}}$. Moreover, $\hat{h}^{\mathrm{int}}_{\boldsymbol{k}}$ is a hydrodynamic variable, whereas $\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})$ is a kinetic one and only its average value with five moments (3.96) can be related to hydrodynamic quantities.

If we put $\nu=1,2,3$, $\phi_1(\boldsymbol{\xi})=1$, $\phi_2(\boldsymbol{\xi})=\xi_z$, $\phi_3(\boldsymbol{\xi})-6^{-1/2}(\xi^2-3)$ in the system of equation (3.93), then we obtain:

$$z\Phi_{na}(\boldsymbol{k};z)-\mathrm{i}\Omega_{np}(\boldsymbol{k})\Phi_{pa}(\boldsymbol{k};z) \;=\; -\Phi_{na}(\boldsymbol{k}), \tag{3.106}$$

$$\begin{aligned} z\Phi_{pa}(\boldsymbol{k};z)-\mathrm{i}\Omega_{pn}(\boldsymbol{k})\Phi_{na}(\boldsymbol{k};z)+D^{||}_{pp}(\boldsymbol{k};z)\Phi_{pa}(\boldsymbol{k};z)\;-&\\ \Sigma_{ph^{\mathrm{kin}}}(\boldsymbol{k};z)\Phi_{h^{\mathrm{kin}}a}(\boldsymbol{k};z)-\Sigma_{ph^{\mathrm{int}}}(\boldsymbol{k};z)\Phi_{h^{\mathrm{int}}a}(\boldsymbol{k};z)\;&=\;-\Phi_{pa}(\boldsymbol{k}), \end{aligned} \tag{3.107}$$

$$\begin{aligned} z\Phi_{h^{\mathrm{kin}}a}(\boldsymbol{k};z)-\Sigma_{h^{\mathrm{kin}}p}(\boldsymbol{k};z)\Phi_{pa}(\boldsymbol{k};z)\;+&\\ D_{h^{\mathrm{kin}}h^{\mathrm{kin}}}(\boldsymbol{k};z)\Phi_{h^{\mathrm{kin}}a}(\boldsymbol{k};z)+D_{h^{\mathrm{kin}}h^{\mathrm{int}}}(\boldsymbol{k};z)\Phi_{h^{\mathrm{int}}a}(\boldsymbol{k};z)\;&=\;-\Phi_{h^{\mathrm{kin}}a}(\boldsymbol{k}), \end{aligned} \tag{3.108}$$

$$\begin{aligned} z\Phi_{h^{\mathrm{int}}a}(\boldsymbol{k};z)-\Sigma_{h^{\mathrm{int}}p}(\boldsymbol{k};z)\Phi_{pa}(\boldsymbol{k};z)\;+&\\ D_{h^{\mathrm{int}}h^{\mathrm{kin}}}(\boldsymbol{k};z)\Phi_{h^{\mathrm{kin}}a}(\boldsymbol{k};z)+D_{h^{\mathrm{int}}h^{\mathrm{int}}}(\boldsymbol{k};z)\Phi_{h^{\mathrm{int}}a}(\boldsymbol{k};z)\;&=\;-\Phi_{h^{\mathrm{int}}a}(\boldsymbol{k}), \end{aligned} \tag{3.109}$$

where $a = \{\hat{n}_{\boldsymbol{k}}, \hat{\boldsymbol{p}}_{\boldsymbol{k}}, \hat{h}^{\text{kin}}_{\boldsymbol{k}}, \hat{h}^{\text{int}}_{\boldsymbol{k}}\}$ and

$$\begin{aligned}
\Sigma_{ph^{\text{kin}}}(\boldsymbol{k};z) &= \mathrm{i}\Omega_{ph^{\text{kin}}}(\boldsymbol{k}) - D_{ph^{\text{kin}}}(\boldsymbol{k};z),\\
\Sigma_{ph^{\text{int}}}(\boldsymbol{k};z) &= \mathrm{i}\Omega_{ph^{\text{int}}}(\boldsymbol{k}) - D_{ph^{\text{int}}}(\boldsymbol{k};z),\\
\Sigma_{h^{\text{kin}}p}(\boldsymbol{k};z) &= \mathrm{i}\Omega_{h^{\text{kin}}p}(\boldsymbol{k}) - D_{h^{\text{kin}}p}(\boldsymbol{k};z),\\
\Sigma_{h^{\text{int}}p}(\boldsymbol{k};z) &= \mathrm{i}\Omega_{h^{\text{int}}p}(\boldsymbol{k}) - D_{h^{\text{int}}p}(\boldsymbol{k};z),
\end{aligned} \tag{3.110}$$

$\mathrm{i}\Omega_{ab}(\boldsymbol{k})$ and $D_{ab}(\boldsymbol{k};z)$ are determined according to (3.95), (3.94). From the system of equations (3.106)–(3.109) one can define the Fourier components of the particle number density correlation functions

$$\Phi_{nn}(\boldsymbol{k};z) = \Phi_{11}(\boldsymbol{k};z) = \langle 1|\Phi_{nn}(\boldsymbol{k},\boldsymbol{\xi},\boldsymbol{\xi}';z)|1'\rangle,, \tag{3.111}$$

as well as of the longitudinal component of the momentum density

$$\Phi^{||}_{pp}(\boldsymbol{k};z) = \Phi_{22}(\boldsymbol{k};z) = \langle \xi_z|\Phi_{nn}(\boldsymbol{k},\boldsymbol{\xi},\boldsymbol{\xi}';z)|\xi'_z\rangle, \tag{3.112}$$

for the kinetic part of generalized enthalpy

$$\Phi_{h^{\text{kin}}h^{\text{kin}}}(\boldsymbol{k};z) = \Phi_{33}(\boldsymbol{k};z) = \langle 6^{-1/2}(\xi^2-3)|\Phi_{nn}(\boldsymbol{k},\boldsymbol{\xi},\boldsymbol{\xi}';z)|6^{-1/2}((\xi')^2-3)\rangle \tag{3.113}$$

as well as for the potential part of generalized enthalpy $\Phi_{h^{\text{int}}h^{\text{int}}}(\boldsymbol{k};z)$ and cross correlation functions, especially $\Phi_{h^{\text{int}}h^{\text{kin}}}(\boldsymbol{k};z)$, $\Phi_{nh^{\text{kin}}}(\boldsymbol{k};z)$, $\Phi_{nh^{\text{int}}}(\boldsymbol{k};z)$, $\Phi_{ph^{\text{kin}}}(\boldsymbol{k};z)$, $\Phi_{ph^{\text{int}}}(\boldsymbol{k};z)$. It is important to point out that the system of equations (3.106)–(3.109) corresponds to the system of equations for Fourier components of the average values of densities for the number of particles $\langle\hat{n}_{\boldsymbol{k}}\rangle_z$, longitudinal momentum $\langle\hat{\boldsymbol{p}}_{\boldsymbol{k}}\rangle_z$, kinetic $\langle\hat{h}^{\text{kin}}_{\boldsymbol{k}}\rangle_z$ and potential $\langle\hat{h}^{\text{int}}_{\boldsymbol{k}}\rangle_z$ parts of generalized enthalpy:

$$z\langle\hat{n}_{\boldsymbol{k}}\rangle_z - \mathrm{i}\Omega_{np}(\boldsymbol{k})\langle\hat{\boldsymbol{p}}_{\boldsymbol{k}}\rangle_z = -\langle\hat{n}_{\boldsymbol{k}}(t=0)\rangle, \tag{3.114}$$

$$\begin{aligned}
&z\langle\hat{\boldsymbol{p}}_{\boldsymbol{k}}\rangle_z - \mathrm{i}\Omega_{pn}(\boldsymbol{k})\langle\hat{n}_{\boldsymbol{k}}\rangle_z \;+\\
&D^{||}_{pp}(\boldsymbol{k};z)\langle\hat{\boldsymbol{p}}_{\boldsymbol{k}}\rangle_z - \Sigma_{ph^{\text{kin}}}(\boldsymbol{k};z)\langle\hat{h}^{\text{kin}}_{\boldsymbol{k}}\rangle_z - \Sigma_{ph^{\text{int}}}(\boldsymbol{k};z)\langle\hat{h}^{\text{int}}_{\boldsymbol{k}}\rangle_z = -\langle\hat{\boldsymbol{p}}_{\boldsymbol{k}}(t=0)\rangle,
\end{aligned} \tag{3.115}$$

$$\begin{aligned}
&z\langle\hat{h}^{\text{kin}}_{\boldsymbol{k}}\rangle_z - \Sigma_{h^{\text{kin}}p}(\boldsymbol{k};z)\langle\hat{\boldsymbol{p}}_{\boldsymbol{k}}\rangle_z \;+\\
&D_{h^{\text{kin}}h^{\text{kin}}}(\boldsymbol{k};z)\langle\hat{h}^{\text{kin}}_{\boldsymbol{k}}\rangle_z + D_{h^{\text{kin}}h^{\text{int}}}(\boldsymbol{k};z)\langle\hat{h}^{\text{int}}_{\boldsymbol{k}}\rangle_z = -\langle\hat{h}^{\text{kin}}_{\boldsymbol{k}}(t=0)\rangle,
\end{aligned} \tag{3.116}$$

$$\begin{aligned}
&z\langle\hat{h}^{\text{int}}_{\boldsymbol{k}}\rangle_z - \Sigma_{h^{\text{int}}p}(\boldsymbol{k};z)\langle\hat{\boldsymbol{p}}_{\boldsymbol{k}}\rangle_z \;+\\
&D_{h^{\text{int}}h^{\text{kin}}}(\boldsymbol{k};z)\langle\hat{h}^{\text{kin}}_{\boldsymbol{k}}\rangle_z + D_{h^{\text{int}}h^{\text{int}}}(\boldsymbol{k};z)\langle\hat{h}^{\text{int}}_{\boldsymbol{k}}\rangle_z = -\langle\hat{h}^{\text{int}}_{\boldsymbol{k}}(t=0)\rangle.
\end{aligned} \tag{3.117}$$

This system of equations is similar in construction to the equations of molecular hydrodynamics [77]. The difference consists in the fact that instead of the equations for the Fourier component of the mean enthalpy density $\langle\hat{h}_{\boldsymbol{k}}\rangle^t$ which is introduced in molecular hydrodynamics, there are two connected equations for the Fourier components of mean values of the kinetic and potential parts of enthalpy density. Moreover, instead of the generalized thermal conductivity which appears in molecular hydrodynamics, the dissipation of energy flows is described in equations (3.114)–(3.117) by a set of generalized transport kernels $D_{h^{\text{kin}}h^{\text{kin}}}(\boldsymbol{k};z)$,

$D_{h^{\rm kin}h^{\rm int}}(\boldsymbol{k};z)$, $D_{h^{\rm int}h^{\rm kin}}(\boldsymbol{k};z)$, $D_{h^{\rm int}h^{\rm int}}(\boldsymbol{k};z)$. Obviously, transport kernels give more detailed information on the dissipation of energy flows in the system because they describe the time evolution of dynamical correlations between the kinetic and potential flows of enthalpy density.

Solving the system of equation (3.106)–(3.109) at $a=n$, one obtains an expression for the correlation function “density-density” $\Phi_{nn}(\boldsymbol{k};z)$

$$\Phi_{nn}(\boldsymbol{k};z)=-S_2(\boldsymbol{k})\left[z-\frac{\mathrm{i}\Omega_{np}(\boldsymbol{k})\mathrm{i}\Omega_{pn}(\boldsymbol{k})}{z+\bar{D}^{||}_{pp}(\boldsymbol{k};z)}\right]^{-1}, \tag{3.118}$$

where

$$\begin{aligned}\bar{D}^{||}_{pp}(\boldsymbol{k};z)=D^{||}_{pp}(\boldsymbol{k};z)-\bar{\Sigma}_{ph^{\rm kin}}(\boldsymbol{k};z)\left[z+\bar{D}_{h^{\rm kin}h^{\rm kin}}(\boldsymbol{k};z)\right]^{-1}\bar{\Sigma}_{h^{\rm kin}p}(\boldsymbol{k};z)\ -\\ \Sigma_{ph^{\rm int}}(\boldsymbol{k};z)\ \left[z\ +\ D_{h^{\rm int}h^{\rm int}}(\boldsymbol{k};z)\right]^{-1}\Sigma_{h^{\rm int}p}(\boldsymbol{k};z),\end{aligned} \tag{3.119}$$

$$\begin{aligned}\bar{\Sigma}_{ph^{\rm kin}}(\boldsymbol{k};z)\ &=\ \Sigma_{ph^{\rm kin}}(\boldsymbol{k};z)-\\ &\quad\ \Sigma_{ph^{\rm int}}(\boldsymbol{k};z)\left[z+D_{h^{\rm int}h^{\rm int}}(\boldsymbol{k};z)\right]^{-1}D_{h^{\rm int}h^{\rm kin}}(\boldsymbol{k};z),\end{aligned} \tag{3.120}$$

$$\begin{aligned}\bar{\Sigma}_{h^{\rm kin}p}(\boldsymbol{k};z)\ &=\ \Sigma_{h^{\rm kin}p}(\boldsymbol{k};z)-\\ &\quad\ D_{h^{\rm kin}h^{\rm int}}(\boldsymbol{k};z)\left[z+D_{h^{\rm int}h^{\rm int}}(\boldsymbol{k};z)\right]^{-1}\Sigma_{h^{\rm int}p}(\boldsymbol{k};z),\end{aligned} \tag{3.121}$$

$$\begin{aligned}\bar{D}_{h^{\rm kin}h^{\rm kin}}(\boldsymbol{k};z)\ &=\ D_{h^{\rm kin}h^{\rm kin}}(\boldsymbol{k};z)-\\ &\quad\ D_{h^{\rm kin}h^{\rm int}}(\boldsymbol{k};z)\left[z+D_{h^{\rm int}h^{\rm int}}(\boldsymbol{k};z)\right]^{-1}D_{h^{\rm int}h^{\rm kin}}(\boldsymbol{k};z).\end{aligned} \tag{3.122}$$

In expressions (3.119)–(3.122) we can observe an interesting renormalization of the functions Σ_{ab} and D_{ab} via the generalized transport kernels for fluctuations of flows of the potential part of enthalpy density. It is important to point out that in the mode-coupling theory developed by Götze [95,96], an expression for $\Phi_{nn}(\boldsymbol{k};z)$ has the same form as in (3.118). However, $\bar{D}^{||}_{pp}(\boldsymbol{k};z)$ is connected only with the generalized shear viscosity $\eta^{||}(\boldsymbol{k};z)$, since the densities of the number of particles $\hat{n}_{\boldsymbol{k}}$ and momentum $\hat{\boldsymbol{p}}_{\boldsymbol{k}}$ are included in the set of variables of an abbreviated description. In our case $\bar{D}^{||}_{pp}(\boldsymbol{k};z)$ takes into account both thermal and viscous dynamical correlation processes. Excluding from (3.118) the imaginary part $\Phi^{||}_{nn}(\boldsymbol{k};\omega)$ of the correlation function $\Phi_{nn}(\boldsymbol{k};z)$, one obtains an expression for the dynamical structure factor $S(\boldsymbol{k};\omega)$ in which contributions of transport kernels corresponding to the kinetic and potential parts of the enthalpy density $\hat{h}_{\boldsymbol{k}}$ are separated. It is evident that the main contribution of the transport kernel $D_{h^{\rm int}h^{\rm int}}(\boldsymbol{k};z)$ to the $S(\boldsymbol{k};\omega)$ for liquids was in the hydrodynamical region (the region of small values of wavevector $\boldsymbol{k}$ and frequency ω), whereas $D_{h^{\rm kin}h^{\rm kin}}(\boldsymbol{k};z)$ will contribute to the kinetic region (orders of interatomic distances, small correlation times). In the region of intermediate values of wavevector $\boldsymbol{k}$ and frequency ω, it is necessary to take into account all the transport kernels $\Sigma_{ph^{\rm kin}}(\boldsymbol{k};z)$, $\Sigma_{ph^{\rm int}}(\boldsymbol{k};z)$, $D_{h^{\rm kin}h^{\rm kin}}(\boldsymbol{k};z)$, $D_{h^{\rm int}h^{\rm kin}}(\boldsymbol{k};z)$, $D_{h^{\rm int}h^{\rm int}}(\boldsymbol{k};z)$. Since it is impossible to perform exact calculations of the described above functions, it is necessary in each separate region to accept models corresponding to the physical processes. Obviously, it is necessary to provide modelling on the level of generalized transport kernels $\varphi_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';t,t')$, $\varphi_{nh}(\boldsymbol{k},\boldsymbol{p};t,t')$, $\varphi_{hn}(\boldsymbol{k},\boldsymbol{p}';t,t')$, $\varphi_{hh}(\boldsymbol{k};t,t')$ (3.50)–(3.53). In particular, we can accept for $\varphi_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';t,t')$ the Boltzmann-Enskog model for hard spheres or the Fokker-Planck model for the case of weak correlation, as in the papers by Mazenko [111,114,135]. Such approximations can be applicable to kinetic processes at interatomic distances (region of large values of wavevector $\boldsymbol{k}$) and small correlation times (region of large frequencies ω) of interatomic collisions. Besides, it is necessary to take into account information about interactions of particles at short distances. In the transport kernels $\varphi_{nn}(\boldsymbol{k};\boldsymbol{\xi},\boldsymbol{\xi}';t,t')$, $\varphi_{nh}(\boldsymbol{k};\boldsymbol{\xi};t,t')$, $\varphi_{hn}(\boldsymbol{k};\boldsymbol{\xi}';t,t')\varphi_{hh}(\boldsymbol{k};t,t')$ it is important to distinguish processes connected with a type of particles interaction at short and long distances, analogously to the case of Enskog-Landau kinetic equation for a model of charged hard spheres. The problems of modelling for these kernels are complicated by the absence of a small parameter in terms of which the perturbation theory might be developed.

The transport kernel $\varphi'_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';t,t')$ (3.62) and its moments on space$[1,p_l,\frac{p^2}{2m}]$ were analyzed in the limit $z\to\infty$ and the hydrodynamic limit $k\to 0$, $z\to 0$ in the paper by Forster [115]. The modelling problems of transport kernels for intermediate values of $\boldsymbol{k}$ and ω are reflected in the details of the description of spectra for collective excitations, as well as in the dynamical structure factor. In the limit $k\to 0$, $\omega\to 0$, the cross correlations between the kinetic and potential flows of energy and shear flows become not so important and the system of equations (3.118)–(3.122) gives a spectrum of the collective modes of molecular hydrodynamics [77,79]. For intermediate values of $\boldsymbol{k}$ and ω, the spectrum of collective modes can be found from the condition

$$\begin{bmatrix} z & \mathrm{i}\Omega_{np}(\boldsymbol{k}) & 0 & 0 \\ \mathrm{i}\Omega_{pn}(\boldsymbol{k}) & z+D^{||}_{pp}(\boldsymbol{k};z) & \Sigma_{ph^{\rm kin}}(\boldsymbol{k};z) & \Sigma_{ph^{\rm int}}(\boldsymbol{k};z) \\ 0 & \Sigma_{h^{\rm kin}p}(\boldsymbol{k};z) & z+D_{h^{\rm kin}h^{\rm kin}}(\boldsymbol{k};z) & D_{h^{\rm kin}h^{\rm int}}(\boldsymbol{k};z) \\ 0 & \Sigma_{h^{\rm int}p}(\boldsymbol{k};z) & D_{h^{\rm int}h^{\rm kin}}(\boldsymbol{k};z) & z+D_{h^{\rm int}h^{\rm int}}(\boldsymbol{k};z) \end{bmatrix} = 0, \qquad (3.123)$$

in which contributions of the kinetic and potential parts of generalized enthalpy are separated. This will be reflected in the structure of a heat mode at concrete model calculations of the wavevector- and frequency-dependent transport kernels $D_{h^{\rm kin}h^{\rm kin}}(\boldsymbol{k};z)$, $D_{h^{\rm kin}h^{\rm int}}(\boldsymbol{k};z)$, $D_{h^{\rm int}h^{\rm kin}}(\boldsymbol{k};z)$, $D_{h^{\rm int}h^{\rm int}}(\boldsymbol{k};z)$ depending on $\boldsymbol{k}$ and ω.

The system of equations (3.93) for time correlation functions allows an extended description of collective modes in liquids, which includes both hydrodynamic and kinetic processes. Including on the basis of functions $\phi_\nu(\xi)$ (3.96) some additional functions,

$$\psi^l_Q(\boldsymbol{\xi})=\frac{1}{5}(\xi^2-5)\xi_l, \qquad \psi^{lj}_\Pi(\boldsymbol{\xi})=\frac{\sqrt{2}}{2}(\xi_l\xi_j-\frac{1}{3}\xi^2\delta_{lj}), \qquad (3.124)$$

which corresponds to a 13-moments approximation of Grad, one obtains a system of equations for time correlation functions of an extended set of hydrodynamic variables $\{\hat{n}_{\boldsymbol{k}},\,\hat{\boldsymbol{p}}_{\boldsymbol{k}},\,\hat{h}^{\rm kin}_{\boldsymbol{k}},\,\hat{\overleftrightarrow{\Pi}}_{\boldsymbol{k}},\,\hat{\boldsymbol{Q}}_{\boldsymbol{k}},\,\hat{h}^{\rm int}_{\boldsymbol{k}}\}$:

$$z\tilde{\Phi}^{\rm H}(\boldsymbol{k};z)-\mathrm{i}\tilde{\Omega}^{\rm H}(\boldsymbol{k})\tilde{\Phi}^{\rm H}(\boldsymbol{k};z)+\tilde{\varphi}^{\rm H}(\boldsymbol{k};z)\tilde{\Phi}^{\rm H}(\boldsymbol{k};z)=-\tilde{\Phi}^{\rm H}(\boldsymbol{k}), \qquad (3.125)$$

where

$$\tilde{\Phi}^{\rm H}(\boldsymbol{k};z) = \begin{bmatrix} \Phi_{nn} & \Phi_{np} & \Phi_{nh^{\rm kin}} & \Phi_{n\Pi} & \Phi_{nQ} & \Phi_{nh^{\rm int}} \\ \Phi_{pn} & \Phi_{pp} & \Phi_{ph^{\rm kin}} & \Phi_{p\Pi} & \Phi_{pQ} & \Phi_{ph^{\rm int}} \\ \Phi_{h^{\rm kin}n} & \Phi_{h^{\rm kin}p} & \Phi_{h^{\rm kin}h^{\rm kin}} & \Phi_{h^{\rm kin}\Pi} & \Phi_{h^{\rm kin}Q} & \Phi_{h^{\rm kin}h^{\rm int}} \\ \Phi_{\Pi n} & \Phi_{\Pi p} & \Phi_{\Pi h^{\rm kin}} & \Phi_{\Pi\Pi} & \Phi_{\Pi Q} & \Phi_{\Pi h^{\rm int}} \\ \Phi_{Qn} & \Phi_{Qp} & \Phi_{Qh^{\rm kin}} & \Phi_{Q\Pi} & \Phi_{QQ} & \Phi_{Qh^{\rm int}} \\ \Phi_{h^{\rm int}n} & \Phi_{h^{\rm int}p} & \Phi_{h^{\rm int}h^{\rm kin}} & \Phi_{h^{\rm int}\Pi} & \Phi_{h^{\rm int}Q} & \Phi_{h^{\rm int}h^{\rm int}} \end{bmatrix}_{(\boldsymbol{k};z)} \tag{3.126}$$

is a matrix of Laplace images of the time correlation functions, $\hat{\overset{\leftrightarrow}{\Pi}}_{\boldsymbol{k}}=\int{\rm d}\boldsymbol{\xi}\varphi_{\Pi}(\boldsymbol{\xi})\hat{n}_{\boldsymbol{k}}(\boldsymbol{\xi})$, $\hat{\boldsymbol{Q}}_{\boldsymbol{k}} = \int {\rm d}\boldsymbol{\xi}\ \varphi_{Q}(\boldsymbol{\xi})\hat{n}_{\boldsymbol{k}}(\boldsymbol{\xi})$,

$$\mathrm{i}\tilde{\Omega}^{\rm H}(\boldsymbol{k}) = \begin{bmatrix} 0 & \mathrm{i}\Omega_{np} & 0 & 0 & 0 & 0 \\ \mathrm{i}\Omega_{pn} & 0 & \mathrm{i}\Omega_{ph^{\rm kin}} & \mathrm{i}\Omega_{p\Pi} & 0 & \mathrm{i}\Omega_{ph^{\rm int}} \\ 0 & \mathrm{i}\Omega_{h^{\rm kin}p} & 0 & 0 & \mathrm{i}\Omega_{h^{\rm kin}Q} & 0 \\ 0 & \mathrm{i}\Omega_{\Pi p} & 0 & 0 & \mathrm{i}\Omega_{\Pi Q} & 0 \\ 0 & 0 & \mathrm{i}\Omega_{Qh^{\rm kin}} & \mathrm{i}\Omega_{Q\Pi} & 0 & \mathrm{i}\Omega_{Qh^{\rm int}} \\ 0 & \mathrm{i}\Omega_{h^{\rm int}p} & 0 & 0 & \mathrm{i}\Omega_{h^{\rm int}Q} & 0 \end{bmatrix}_{(\boldsymbol{k})} \tag{3.127}$$

is an extended hydrodynamic frequency matrix,

$$\tilde{\varphi}^{\rm H}(\boldsymbol{k};z) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & D^{H}_{pp} & D^{H}_{ph^{\rm kin}} & D^{H}_{p\Pi} & D^{H}_{pQ} & D^{H}_{ph^{\rm int}} \\ 0 & D_{h^{\rm kin}p} & D_{h^{\rm kin}h^{\rm kin}} & D_{h^{\rm kin}\Pi} & D_{h^{\rm kin}Q} & D_{h^{\rm kin}h^{\rm int}} \\ 0 & D_{\Pi p} & D_{\Pi h^{\rm kin}} & D_{\Pi\Pi} & D_{\Pi Q} & D_{\Pi h^{\rm int}} \\ 0 & D_{Qp} & D_{Qh^{\rm kin}} & D_{Q\Pi} & D_{QQ} & D_{Qh^{\rm int}} \\ 0 & D_{h^{\rm int}p} & D_{h^{\rm int}h^{\rm kin}} & D_{h^{\rm int}\Pi} & D_{h^{\rm int}Q} & D_{h^{\rm int}h^{\rm int}} \end{bmatrix}_{(\boldsymbol{k};z)} \tag{3.128}$$

is a matrix of generalized memory functions, elements of which are transport kernels (3.94) projected on the basis of functions $\phi_{\nu}(\boldsymbol{\xi})$ (3.96), (3.124). For such a description, the spectrum of generalized collective modes of the system is determined for intermediate $\boldsymbol{k}$ and ω by the relation $\det\left|z\tilde{I} - \mathrm{i}\tilde{\Omega}^{\rm H}(\boldsymbol{k}) + \tilde{\varphi}^{\rm H}(\boldsymbol{k})\right| = 0$ which takes into account kinetic and hydrodynamic processes. In the hydrodynamic limit $k \to 0$, $\omega \to 0$, when the contribution of cross dissipative correlations between the kinetic and hydrodynamic processes practically vanishes, the system of equations for the time correlation function (3.125), after some transformations, can be reduced to a system of equations for time corelation functions of densities for the number of particles $\hat{n}_{\boldsymbol{k}}$, momentum $\hat{\boldsymbol{p}}_{\boldsymbol{k}}$, total enthalpy $\hat{h}_{\boldsymbol{k}}$, the generalized stress tensor $\hat{\overset{\leftrightarrow}{\pi}}_{\boldsymbol{k}} = (1 - \mathscr{P}_{\rm H})\mathrm{i}L_N\hat{\boldsymbol{p}}_{\boldsymbol{k}}$ and the generalized enthalpy flow $\hat{\boldsymbol{q}}_{\boldsymbol{k}} = (1-\mathscr{P}_{\rm H})\mathrm{i}L_N\hat{h}_{\boldsymbol{k}}$, where $\mathscr{P}_{\rm H}$ is the Mori operator constructed on the dynamical variables $\{\hat{n}_{\boldsymbol{k}}, \hat{\boldsymbol{p}}_{\boldsymbol{k}}, \hat{h}_{\boldsymbol{k}}\}$. For such a system of equations, the spectrum of collective excitations is determined from [139,141]:

$$\begin{vmatrix} z & \mathrm{i}\Omega_{np} & 0 & 0 & 0 \\ \mathrm{i}\Omega_{pn} & z & \mathrm{i}\Omega_{ph} & \mathrm{i}\Omega_{p\pi} & 0 \\ 0 & \mathrm{i}\Omega_{hp} & z & 0 & 0 \\ 0 & \mathrm{i}\Omega_{\pi p} & 0 & z+\varphi_{\pi\pi} & \mathrm{i}\Omega_{\pi Q}+\varphi_{\pi Q} \\ 0 & 0 & 0 & \mathrm{i}\Omega_{Q\pi}+\varphi_{Q\pi} & \varphi_{QQ} \end{vmatrix}_{(\boldsymbol{k};z)} = 0. \tag{3.129}$$

In the hydrodynamic limit this gives: the heat mode

$$z_{\mathrm{H}}(k) = D_{\mathrm{T}}k^2 + \mathcal{O}(k^4), \tag{3.130}$$

two complex conjugated sound modes

$$z_{\pm}(k) = \pm\mathrm{i}\omega_{\mathrm{S}}(k) + z_{\mathrm{S}}(k), \tag{3.131}$$

where $\omega_{\mathrm{S}}(k) = ck + o(k^3)$ is a frequency of sound propagation, $z_{\mathrm{S}}(k) = \Gamma k^2 + \mathcal{O}(k^4)$ is a frequency of sound damping with the damping coefficient Γ; two nonvanishing in the limit $k \to 0$ kinetic modes

$$\begin{aligned} z_{\pi}(k) &= \varphi_{\pi\pi}(0) + \mathcal{O}(k^2), \\ z_{Q}(k) &= \varphi_{QQ}(0) + \mathcal{O}(k^2). \end{aligned} \tag{3.132}$$

Here D_{T} denotes a thermal diffusion coefficient

$$D_{\mathrm{T}} = \frac{v_{\mathrm{T}Q}^2}{\gamma\varphi_{QQ}(0)} = \frac{\lambda}{mnc_p}, \quad v_{\mathrm{T}Q}^2 = \frac{m\Phi_{QQ} - h^2}{nc_V}, \quad \gamma = c_p/c_V$$

c_p and c_V are, correspondingly, thermodynamic values of specific heats at the constant pressure and volume, λ is a thermal conductivity coefficient, h denotes a thermodynamic value of enthalpy, $c = \gamma/mnS(k=0)$ denotes an adiabatic sound velocity. In (3.131)

$$\Gamma = \frac{1}{2}(\gamma - 1)D_{\mathrm{T}} + \frac{1}{2}\eta^{||} \tag{3.133}$$

is a sound damping coefficient with

$$\eta^{||} = \frac{v_{p\pi}^2}{\varphi_{\pi\pi}(0)} = \left(\frac{4}{3}\eta + \kappa\right)/nm, \qquad v_{p\pi}^2 = \frac{mS(0)\Phi_{\pi\pi}(0) - \gamma}{mnS(0)},$$

where η and κ are shear and bulk viscosities. This spectrum coincides with the results of papers [139,141,142]. However, at fixed values of $\boldsymbol{k}$ and ω, the transport kernels $\varphi_{\pi\pi}$, $\varphi_{\pi Q}$, $\varphi_{Q\pi}$, φ_{QQ} are expressed via the generalized transport kernels $D_{\nu\mu}(\boldsymbol{k};z)$ of matrix (3.128), i.e. via $\varphi_{nn}(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';t,t')$, $\varphi_{nh}(\boldsymbol{k},\boldsymbol{p};t,t')$, $\varphi_{hn}(\boldsymbol{k},\boldsymbol{p}';t,t')$, $\varphi_{hh}(\boldsymbol{k};t,t')$ (3.50)–(3.53), according to the definition $D_{\nu\mu}(\boldsymbol{k};z)$ (3.94). Here, it is important to point out that passing from the system of transport equations of a self-consistent description of kinetics and hydrodynamics to the equations of generalized hydrodynamics, we can connect the generalized transport kernels (3.50)–(3.53) with the hydrodynamic transport kernels $D_{\nu\mu}(\boldsymbol{k};z)$ in (3.123) or (3.128). Therefore, we gain the aim of the present section, namely, to connect transport kernels of a self-consistent description of kinetics and hydrodynamics with hydrodynamic transport kernels for time correlation functions for densities of the number of particles $\hat{n}_{\boldsymbol{k}}$, momentum $\hat{\boldsymbol{p}}_{\boldsymbol{k}}$, enthalpy $\hat{h}_{\boldsymbol{k}}$, which can be calculated by using the MD method [142–152,159], as well as by experiments on light scattering for different real systems. Evidently, the most important problem is model calculations of transport kernels (3.50)–(3.53). Such calculations must take into account a type of interaction of particles at short and long distances, as well as their structural distribution. From this point of view, of significant interest is the modelling of transport kernels (3.50)–(3.53) for a system of charged hard spheres, since for a system of hard spheres a transport kernel of the type $\varphi_{nn}(\boldsymbol{k};\boldsymbol{p},\boldsymbol{p'};t,t')$ (3.50) was computed via the Enskog collision integral [120,121].

4. Conclusion

We have presented one of the approaches for a self-consistent description of kinetic and hydrodynamic processes in systems of interacting particles, formulated by the NSO method. It is based on a modification of the boundary condition to the Liouville equation, which takes into account a nonequilibrity of the one-particle distribution function, as well as the local conservation laws which constitute a basis for the hydrodynamic description. Using such a description, generalized kinetic equations for dense gases and liquids can be derived. The result obtained can be extended to quantum systems of interacting particles [16–18]. In particular, it can be applied to nuclear matter and chemically reacting systems, where kinetic processes play an important role together with hydrodynamic ones. The method applied can be extended to the investigations of nonlinear and hydrodynamic fluctuations in polar, ionic and magnetic liquids and electrolyte solutions.

There is a significant interest in investigations of collective excitations, time correlation functions and transport coefficients on the basis of equations (3.93), (3.94), (3.123), (3.128) at the presence of transport kernels $\varphi(\boldsymbol{k};z)$ which describe an interference between kinetic and hydrodynamic processes. In particular, the transport kernel $\varphi(\boldsymbol{k},\boldsymbol{p},\boldsymbol{p}';t)$ can take into account processes of interaction of particles at short and long distances (at Coulomb and dipole interactions of particles). To do this, we can apply an approach of collective modes [142–152].

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[2]Engl. transl. reads "Journal for Technical Physics"
[3]Engl. transl. reads "Heat Physics of High Temperatures"

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Узгоджений опис кінетики та гідродинаміки систем взаємодіючих частинок методом нерівноважного статистичного оператора

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Представлено один із статистичних підходів узгодженого опису кінетичних та гідродинамічних процесів систем взаємодіючих частинок, що сформульований на основі методу нерівноважного статистичного оператора Д.М.Зубарєва. Показано, як із ланцюжка рівнянь ББГКІ з модифікованими граничними умовами отримуються кінетичне рівняння ревізованої теорії Енскога для моделі твердих сфер, кінетичне рівняння для багатосходинкового потенціалу та кінетичне рівняння Енскога-Ландау для моделі заряджених твердих сфер. Проаналізовано ланцюжки рівнянь ББГКІ на основі модифікованих групових розкладів. Отримано узагальнені рівняння переносу узгодженого опису кінетики та гідродинаміки. Для випадку слабо нерівноважних систем класичних взаємодіючих частинок при взаємному врахуванні кінетичних та гідродинамічних процесів досліджено часові кореляційні функції, спектр колективних збуджень та узагальнені коефіцієнти переносу.

Ключові слова: *кінетика, гідродинаміка, кінетичні рівняння, коефіцієнти переносу, (часові) кореляційні функції*

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