Global isomorphism between the Lennard-Jones fluids and the Ising model

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The simple geometrical picture of global isomorphism between Ising model (Lattice Gas) and simple fluid is proposed. It is naturally incorporates two classical linear laws know more than a century for the molecular liquids. They are the linear character of the diameter for the binodal

$$n_{\mathrm{d}} = \frac{n_{\mathrm{l}} + n_{\mathrm{g}}}{2n_{\mathrm{c}}} = 1 + A\,\frac{T_{\mathrm{c}} - T}{T_{\mathrm{c}}} + \cdots$$

and the linear character of the Zeno-line (unit compressibility line $Z = P/(n\,T) = 1$) also called Batschinski law. The correct definition of the Zeno-element is given and its relation with the locus of the critical point is derived within this approach. We show that the liquid-vapor part of the phase diagram of the molecular fluids can be described as the isomorphic image of the phase diagram of the Lattice Gas. The relations between the thermodynamic functions are obtained. The position of the critical points of the fluids of the Lennard-Jones type are determined basing on the scaling symmetry. In case of two dimensions we use the exact Onsager result to construct the binodal of the corresponding Lennard-Jones fluid and compare it with the results of the simulations. In the three dimensional case we use available numerical results for the Ising model for the corresponding mapping. As a sequence the explanation of the well known fact about "global" cubic character of the coexistence curve of the molecular fluids is proposed. The microscopic approach to the construction of the isomorphism based on the Hamiltonian is discussed.