

Time dependent spread of a generic 1D wavepacket

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In several problems concerning 1D dynamics, e.g., quantum-state transmission [1], one is faced with the dispersive evolution of an input wavepacket, whose Fourier-space components are determined by its initial shape. The evolution occurs along a ‘wire’ and is substantially ruled by its dispersion relation ω_k , which is usually a nonlinear function of k when the wire is realized by discrete arrays of physical objects. It is textbook knowledge that a Gaussian packet whose Fourier density is peaked at k_0 travels at the group velocity $v = \omega'_{k_0}$ and its variance broadens according to $\sigma^2(t) = \sigma_0^2 + (\omega''_{k_0}/2\sigma_0)^2 t^2$, as it happens for the wavefunction of a Schrödinger particle in 1D. However, this case is rather peculiar since its $\omega_k \propto k^2$ has no inflection points.

In order to preserve as much as possible the wavepacket shape one must avoid dispersion: it is therefore convenient to initialize the wavepacket such as k_0 sits on an inflection point of the dispersion relation, i.e., $\omega''_{k_0} = 0$, so that higher-order terms determine the dispersion. In Ref. [2] the role of the cubic nonlinearity of ω_k is accounted for in the case of a Gaussian packet. However, there are reasons to look for an extension of this result: besides the possibility that cubic terms could also vanish (e.g., by symmetry), one could be interested in a non-Gaussian initial shape of the wavepacket. In this work such an extension is obtained in terms of rather simple formulas. These permit to obtain an optimal initial width, which shows peculiar scaling as a function of the wire length.

[1] T. J. Osborne and N. Linden, Phys. Rev. A **69**, 052315 (2004).

[2] M. Miyagi and S. Nishida, Appl. Opt. **18**, 678 and 2237 (1979).