Optimal dynamics for quantum information transfer and effective entangling gate through homogeneous quantum wires

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with

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and

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Overview

Goal $\longrightarrow$ information transmission between distant parts

- *Spin chain data-bus*

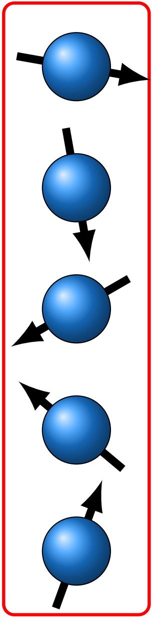
Tool $\longrightarrow$ Optimal dynamics

- *induce a coherent (ballistic) dynamics in a dispersive chain*

Results:

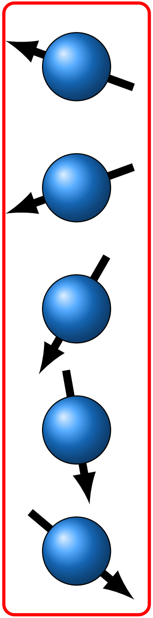
- High quality state transmission
- High quality generation of long-distance entanglement

Quantum data-bus

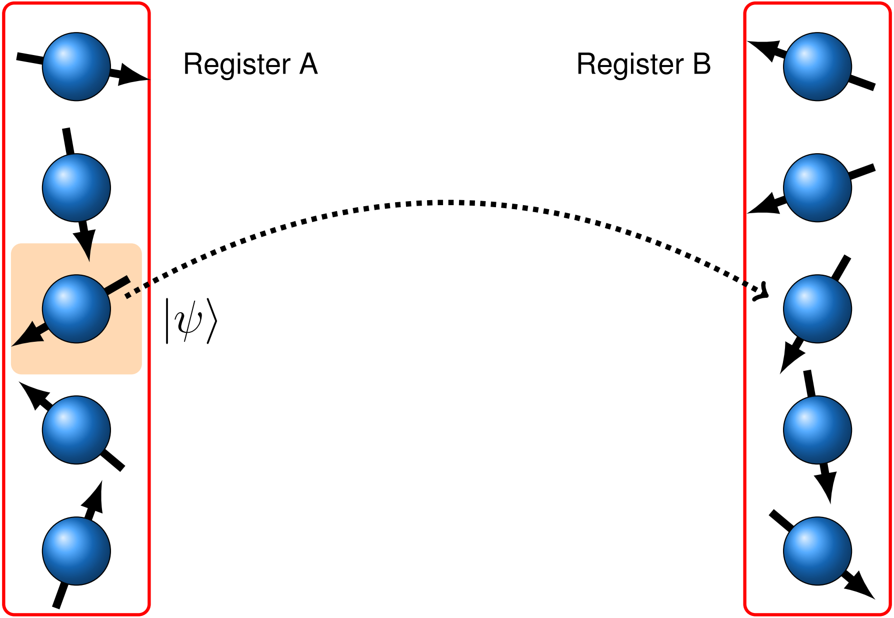


Register A

Register B



Quantum data-bus



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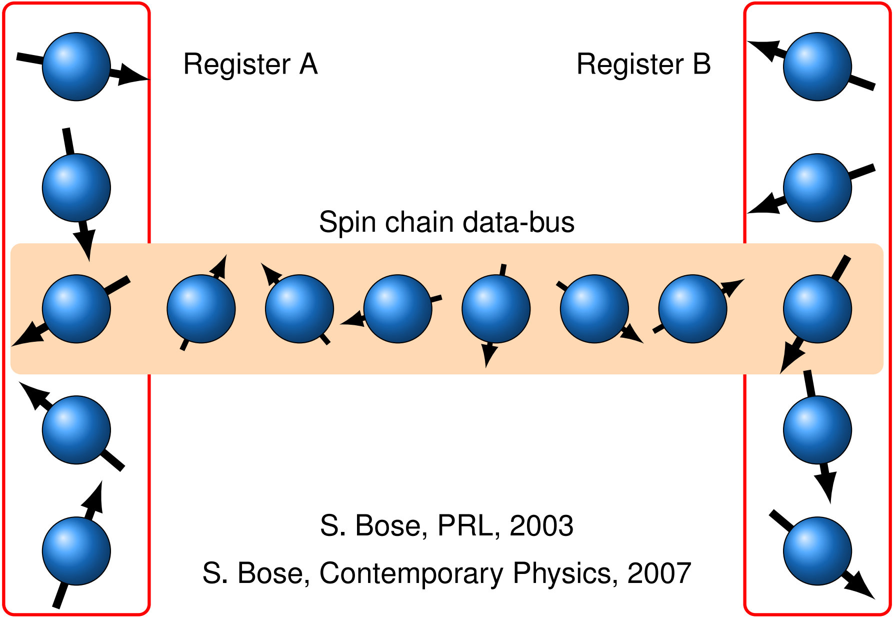
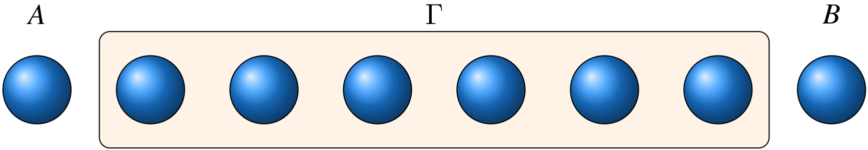


S. Bose, PRL, 2003

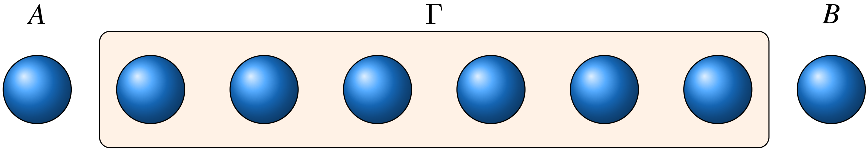
S. Bose, Contemporary Physics, 2007

State transmission



- Out of equilibrium initial state: $|\Psi_{\text{in}}\rangle = |\psi_A\rangle\,|\psi_\Gamma\rangle\,|\psi_B\rangle$

State transmission



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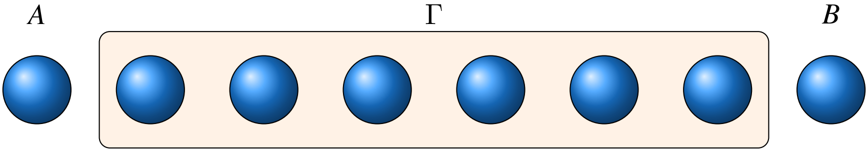
- State of B at time t:

$$\rho_B(t) = \operatorname*{Tr}_{A\Gamma}\left[e^{-it\mathcal{H}_{A\Gamma B}}\,|\Psi_{\text{in}}\rangle\,\langle\Psi_{\text{in}}|\,e^{it\mathcal{H}_{A\Gamma B}}\right]$$

State transmission



- Out of equilibrium initial state: $|\Psi_{\rm in}\rangle = |\psi_A\rangle\,|\psi_\Gamma\rangle\,|\psi_B\rangle$
- State of B at time t:

$$\rho_B(t) = \operatorname*{Tr}_{A\Gamma}\left[e^{-it\mathcal{H}_{A\Gamma B}}\,|\Psi_{\rm in}\rangle\,\langle\Psi_{\rm in}|\,e^{it\mathcal{H}_{A\Gamma B}}\right]$$

- Measure of the transmission quality: minimum fidelity

$$\mathcal{F}(t) = \min_{|\psi_A\rangle}\,\langle\psi_A|\,\rho_B(t)\,|\psi_A\rangle$$

Optimal dynamics

Requirement: quadratic Hamiltonian in terms of *local* creation and annihilation operators

$$\mathcal{H}_{A\Gamma B} = \sum_{ij} \left(M_{ij}\, a_i^\dagger a_j + N_{ij}\, a_i^\dagger a_j^\dagger + \text{h.c.} \right)$$

- Normal modes and dispersion relation ω_k

$$\mathcal{H}_{A\Gamma B} = \sum_k \omega_k\, d_k^\dagger d_k$$

- Density of excitations in the initial state

$$\rho(k) = \langle \Psi_{\text{in}} |\, d_k^\dagger d_k \,| \Psi_{\text{in}} \rangle$$

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Optimal dynamics

Induce a coherent (ballistic) dynamics in a dispersive chain

$\Longrightarrow$ Make $\rho(k)$ mostly peaked around the linear zone of ω_k through a **minimal** engineering of the interactions

Consequences of the optimal dynamics

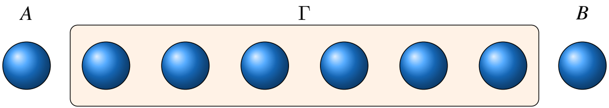
Optimal dynamics

Induce a coherent (ballistic) dynamics in a dispersive chain

$\Longrightarrow$ Make $\rho(k)$ mostly peaked around the linear zone of ω_k through a **minimal** engineering of the interactions

- All relevant excitations (with high $\rho(k)$) have constant group velocity $v_k = \partial_k \omega_k$
- High quality "wave-packet" transmission
- $\mathcal{F}(t^*) \simeq 1$
- Ballistic transport with arrival time $t^* \simeq vN$

Optimal dynamics with the XY model



XX model

$$\mathcal{H}_\Gamma = J \sum_{n=1}^{N-1} \left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y\right)$$

$$\mathcal{H}_{A\Gamma} = J_0 \left(\sigma_0^x \sigma_1^x + \sigma_0^y \sigma_1^y\right)$$

$$\mathcal{H}_{\Gamma B} = J_0 \left(\sigma_N^x \sigma_{N+1}^x + \sigma_N^y \sigma_{N+1}^y\right)$$

Optimal dynamics by setting J_0 to a properly tuned value

Banchi, Apollaro, Cuccoli, Vaia, Verrucchi, PRA, 2010

Optimal dynamics with the XX model

- Dispersion relation ($J \equiv 1$)

$$\omega_k = \cos k$$

- Density of excitations

$$\rho(k) \approx \frac{1}{\Delta^2 + \cot^2 k} \qquad\qquad \Delta = \frac{J_0^2}{2 - J_0^2}$$

peaked around the inflection point $k = \pi/2$ with width Δ

- Normal modes ($n = 1, \ldots, N+2$)

$$k_n = \frac{\pi n + \varphi_{k_n}}{N+3} \qquad\qquad \varphi_k = 2k - 2\cot^{-1}\left(\frac{2 - J_0^2}{J_0^2} \cot k\right)$$

Banchi, Apollaro, Cuccoli, Vaia, Verrucchi, in preparation

Optimal dynamics with the XX model

Group velocity of the “wave-packet” close to the inflection point

$$v_k = \partial_k \omega_k \simeq \text{const.} \times \left[1 + \left(\frac{2}{N J_0^6} - \frac{1}{2} \right) k^2 + O(k^4) \right]$$

Optimal values of $J_0^{\text{opt.}}$:

$$J_0^{\text{opt.}} \simeq 1.05\, N^{-\frac{1}{6}}$$

Non-weak coupling regime and ballistic transport

N	25	51	101	251	501	1001	2501	5001
$J_0^{\text{opt.}}$	0.63	0.56	0.49	0.42	0.37	0.33	0.28	0.25

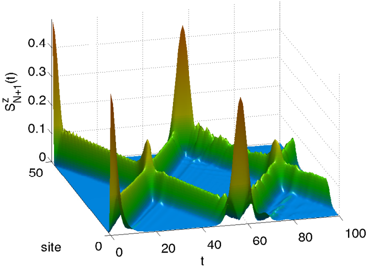
t^*	32.5	60.2	112	266	520	1025	2533	5041

Movie

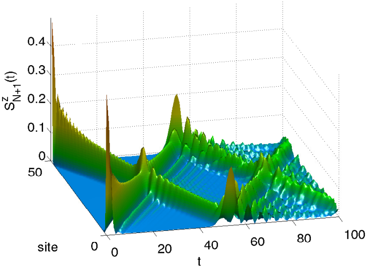
Coherent transport: $N = 50$

$$|\Psi_{\rm in}\rangle = |{\uparrow}\rangle|\Omega_\Gamma\rangle|{\uparrow}\rangle$$

Wave packet propagation displayed by the magnetization $S^z(x,t)$



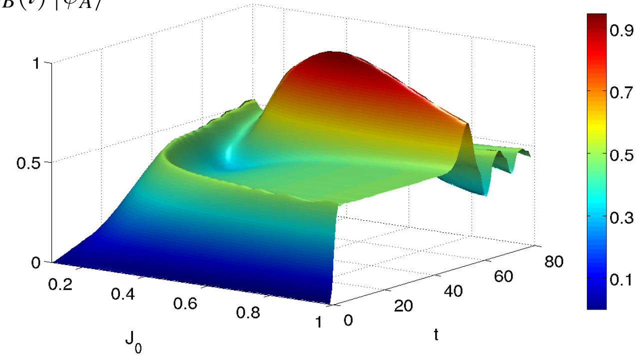
$J_0 = J_0^{\text{opt.}} \simeq 0.56\, J$



$J_0 = J$

State transmission quality

$$\mathcal{F}(t) = \min_{|\psi_A\rangle} \langle\psi_A|\, \rho_B(t)\, |\psi_A\rangle$$



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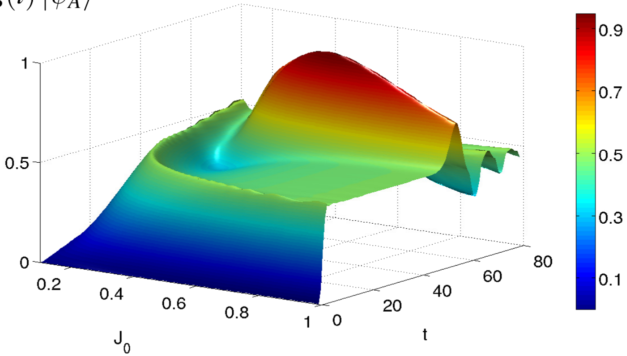

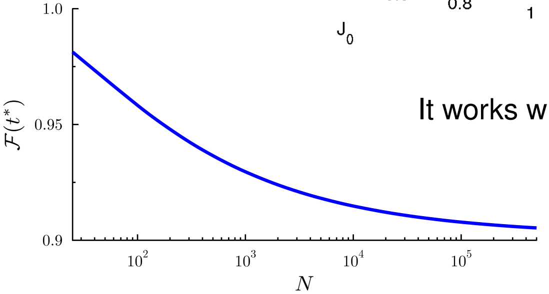

It works with very long channels!

Entangling quantum gates between distant qubits

$$|\uparrow\rangle_A|\uparrow\rangle_B \longrightarrow e^{i\phi_{\uparrow\uparrow}}|\uparrow\rangle_A|\uparrow\rangle_B$$
$$|\uparrow\rangle_A|\downarrow\rangle_B \longrightarrow e^{i\phi_{\uparrow\downarrow}}|\downarrow\rangle_A|\uparrow\rangle_B$$
$$|\downarrow\rangle_A|\uparrow\rangle_B \longrightarrow e^{i\phi_{\downarrow\uparrow}}|\uparrow\rangle_A|\downarrow\rangle_B$$
$$|\downarrow\rangle_A|\downarrow\rangle_B \longrightarrow e^{i\phi_{\downarrow\downarrow}}|\downarrow\rangle_A|\downarrow\rangle_B$$

Optimal dynamics effectively generates an entangling quantum gate

Fixing

$$|\Psi_{\text{in}}\rangle = (|\uparrow\rangle + |\downarrow\rangle)_A \otimes |\psi\rangle_\Gamma \otimes (|\uparrow\rangle + |\downarrow\rangle)_B$$

A and B get maximally entangled at t^*

Banchi, Bayat, Verrucchi, Bose, PRL in print

Interacting models and works in progress...

$$\mathcal{H} = \sum_{n=0}^{N} J_n \left[\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \Delta\, \sigma_n^z \sigma_{n+1}^z \right]$$

where $J_N = J_0$ and $J_n = J$ for $n \neq 0, N$

- Mean field approach for $\Delta \simeq 0$: mapping to a quadratic free-fermionic Hamiltonian
- Numerical optimization: finding J_0^{opt}

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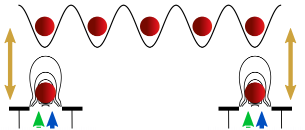
Complex scattering for strong Δ

- Scattering can favour transmission
- Optimize the initial state of the chain as well

Bayat, Banchi, Bose, Verrucchi, in preparation

Conclusions

- General idea for quadratic Hamiltonians
- High quality information transmission between distant parts
- Experimental feasibility (unmodulated interactions)
  - Cold atoms



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  - Other possible applications in solid state