Lviv, 7 IV 2011

Residual diamagnetism driven by the superconducting fluctuations

T. Domański

M. Curie-Skłodowska University, Lublin, Poland

http://kft.umcs.lublin.pl/doman/lectures

Lviv, 7 IV 2011

Residual diamagnetism driven by the superconducting fluctuations

T. Domański

M. Curie-Skłodowska University, Lublin, Poland

⇒ **Collaboration :**

J. Ranninger (Grenoble), M. Zapalska (Lublin)

http://kft.umcs.lublin.pl/doman/lectures

Outline

Outline

- **Preliminaries**

/ Cooper pairing & Higgs mechanism /

Outline

★ **Preliminaries**

/ Cooper pairing & Higgs mechanism /

★ **Motivation**

/ pre-pairing for BE condensation /

Outline

- **Preliminaries**

  / Cooper pairing & Higgs mechanism /

- **Motivation**

  / pre-pairing for BE condensation /

- **Scenario & methodology**

Outline

- **Preliminaries**

  / Cooper pairing & Higgs mechanism /

- **Motivation**

  / pre-pairing for BE condensation /

- **Scenario & methodology**

- **Results**

Outline

- ⋆ **Preliminaries**

  / Cooper pairing & Higgs mechanism /

- ⋆ **Motivation**

  / pre-pairing for BE condensation /

- ⋆ **Scenario & methodology**

- ⋆ **Results**

$\Rightarrow$ *Bogoliubov quasiparticles above* T_c

Outline

- ★ **Preliminaries**

  / Cooper pairing & Higgs mechanism /

- ★ **Motivation**

  / pre-pairing for BE condensation /

- ★ **Scenario & methodology**

- ★ **Results**

  - *Bogoliubov quasiparticles above* T_c

  ⇒ *Diamagnetism above* T_c

Outline

★ **Preliminaries**

/ Cooper pairing & Higgs mechanism /

★ **Motivation**

/ pre-pairing for BE condensation /

★ **Scenario & methodology**

★ **Results**

- *Bogoliubov quasiparticles above* T_c
- *Diamagnetism above* T_c

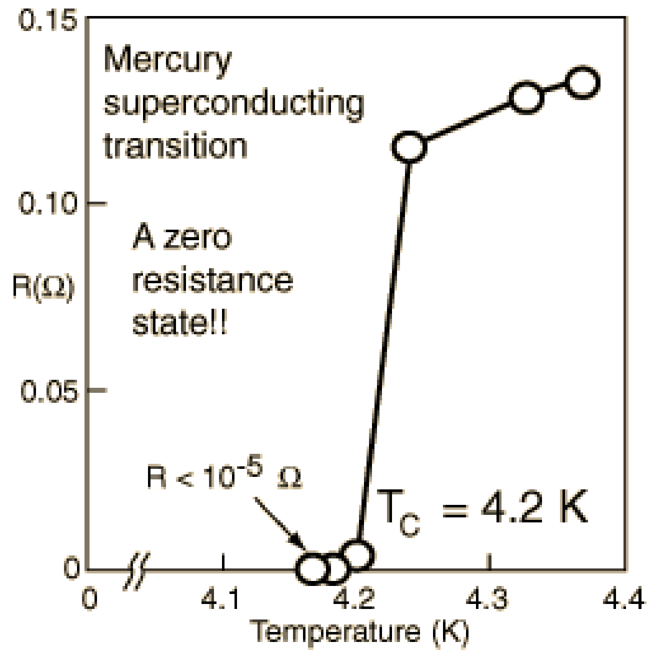
★ **Summary**

Preliminaries

Superconducting state – properties

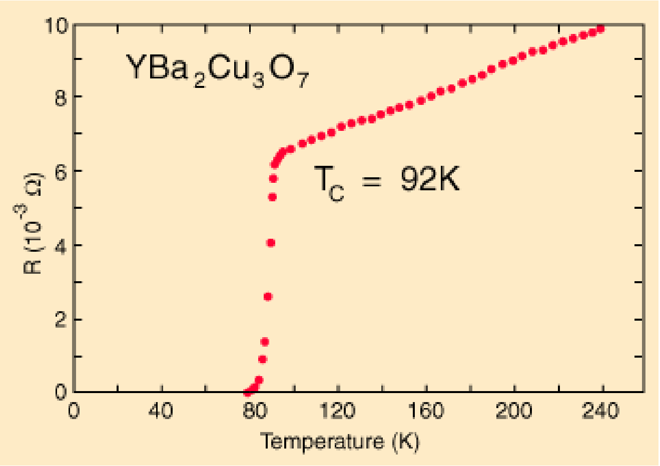
Superconducting state – properties

- ideal d.c. conductance



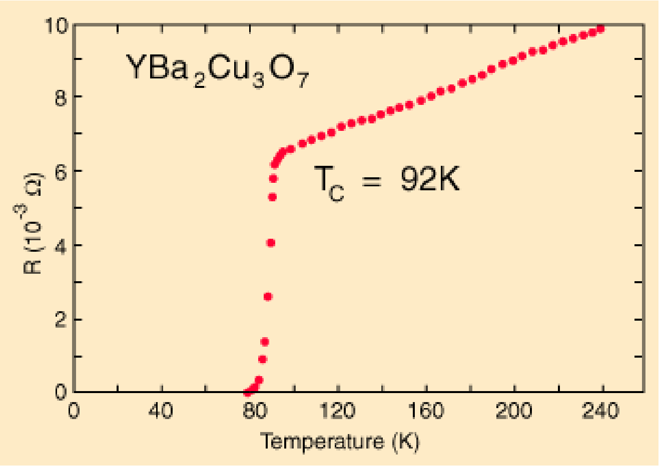
Superconducting state – properties

★ **ideal d.c. conductance**



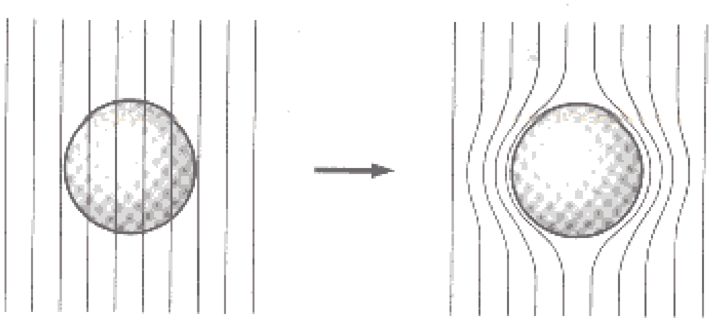
Superconducting state – properties

★ **ideal d.c. conductance**



★ **ideal diamagnetism**

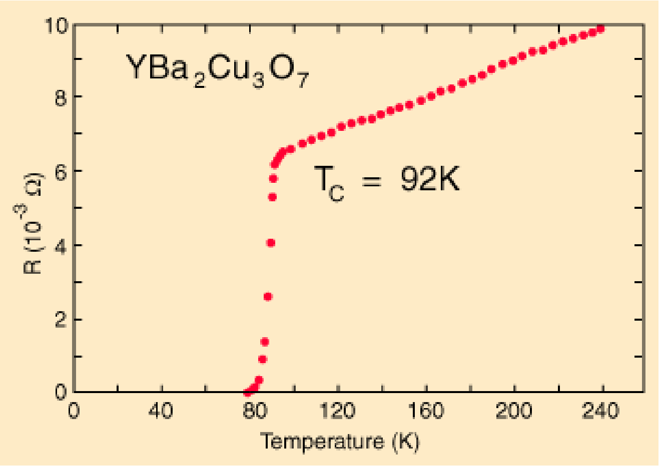
/perfect screening of the external magnetic field/



$T > T_c$ $\qquad$ $T < T_c$

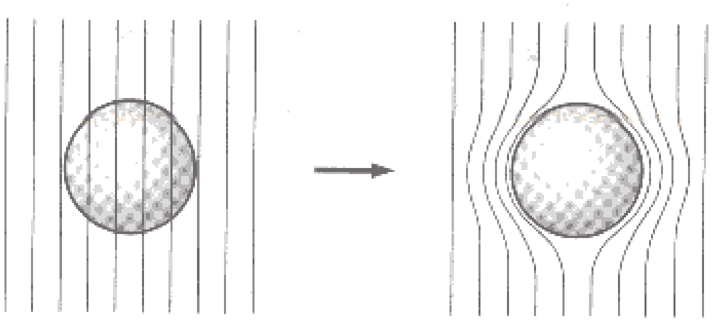
Superconducting state – properties

★ **ideal d.c. conductance**



★ **ideal diamagnetism**

/perfect screening of the external magnetic field/



⇒ **Both features originate from the pairing of fermions.**

Pairing is a phenomenon which occurs between various kinds of fermions such as: electrons, quarks, nucleons or atoms.

Pairing is a phenomenon which occurs between various kinds of fermions such as: electrons, quarks, nucleons or atoms.

Pairing mechanism can be driven by:

Pairing is a phenomenon which occurs between various kinds of fermions such as: electrons, quarks, nucleons or atoms.

Pairing mechanism can be driven by:

1. **exchange of phonons**

   / classical superconductors, MgB_2, ... /

Pairing is a phenomenon which occurs between various kinds of fermions such as: electrons, quarks, nucleons or atoms.

Pairing mechanism can be driven by:

1. **exchange of phonons**

   / classical superconductors, MgB_2*, ... /*

2. **exchange of magnons**

   / heavy fermion compounds /

Pairing is a phenomenon which occurs between various kinds of fermions such as: electrons, quarks, nucleons or atoms.

Pairing mechanism can be driven by:

1. **exchange of phonons**

   / classical superconductors, MgB_2, ... /

2. **exchange of magnons**

   / heavy fermion compounds /

3. **strong correlations**

   / high T_c superconductors /

Pairing is a phenomenon which occurs between various kinds of fermions such as: electrons, quarks, nucleons or atoms.

Pairing mechanism can be driven by:

1. **exchange of phonons**

   / classical superconductors, MgB_2, ... /

2. **exchange of magnons**

   / heavy fermion compounds /

3. **strong correlations**

   / high T_c superconductors /

4. **Feshbach resonance**

   / ultracold superfluid atoms /

Pairing is a phenomenon which occurs between various kinds of fermions such as: electrons, quarks, nucleons or atoms.

Pairing mechanism can be driven by:

1. **exchange of phonons**

   / classical superconductors, MgB_2, ... /

2. **exchange of magnons**

   / heavy fermion compounds /

3. **strong correlations**

   / high T_c superconductors /

4. **Feshbach resonance**

   / ultracold superfluid atoms /

5. **other**

   / pairing in nuclei, gluon-quark plasma /

Pairing is a phenomenon which occurs between various kinds of fermions such as: electrons, quarks, nucleons or atoms.

Pairing mechanism can be driven by:

1. **exchange of phonons**

   / classical superconductors, MgB$_2$, ... /

2. **exchange of magnons**

   / heavy fermion compounds /

3. **strong correlations**

   / high T_c superconductors /

4. **Feshbach resonance**

   / ultracold superfluid atoms /

5. **other**

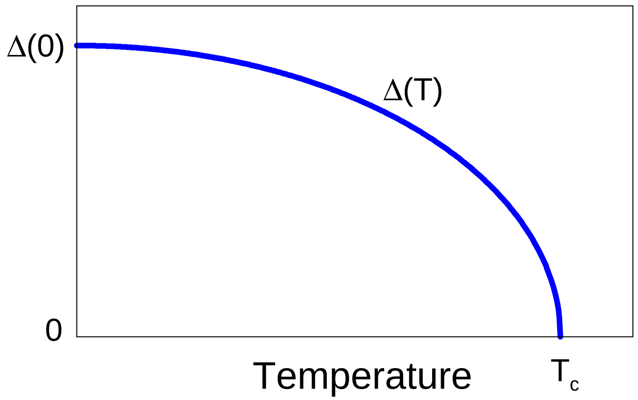
   / pairing in nuclei, gluon-quark plasma /

Appearance of fermion pairs usually goes hand in hand with **superconductivity/superfluidity** but it needn't be the rule.

Conventional superconductors

Conventional superconductors

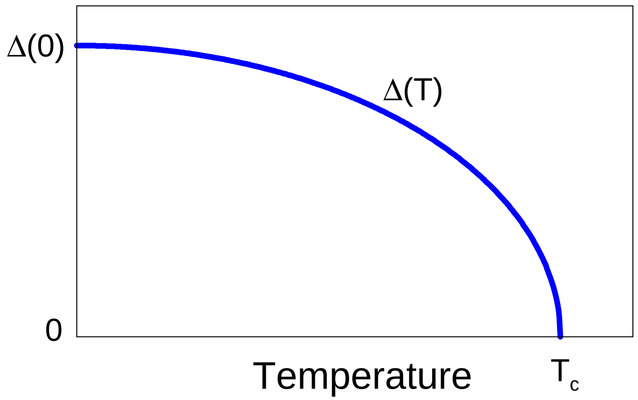
Pair formation coincides with an onset of coherence at T_c



Pairing causes the energy gap $\Delta(T)$ in a single particle spectrum

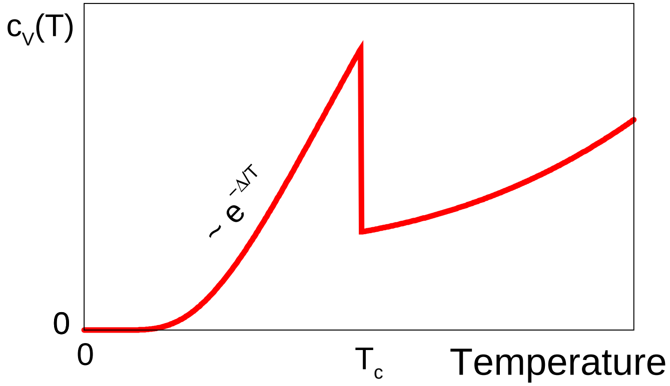
Conventional superconductors

Pair formation coincides with an onset of coherence at T_c



Pairing causes the energy gap $\Delta(T)$ in a single particle spectrum

2-nd order phase transition



Below T_c there appears the order parameter (ODLRO)

$\chi \propto \Delta(T)$

Conventional superconductors – meaning of T_c

Conventional superconductors – meaning of T_c

Electrons near the Fermi surface:

Conventional superconductors – meaning of T_c

Electrons near the Fermi surface:

$\Rightarrow$ **form the Cooper pairs**

Conventional superconductors – meaning of T_c

Electrons near the Fermi surface:

$\Rrightarrow$ **form the Cooper pairs**

$\Rrightarrow$ **and behave as a super-atom consisting**

Conventional superconductors – meaning of T_c

Electrons near the Fermi surface:

$\Rightarrow$ **form the Cooper pairs**

$\Rightarrow$ **and behave as a super-atom consisting of $\sim 10^{23}$ objects in identical state.**

Conventional superconductors – meaning of T_c

Electrons near the Fermi surface:

$\Longrightarrow$ form the Cooper pairs

$\Longrightarrow$ and behave as a super-atom consisting of $\sim 10^{23}$ objects in identical state.

Bose-Einstein condensate of these Cooper pairs is described by a common wave-function

$$\chi(\vec{r}, t)$$

Formal issues – generalities

Formal issues – generalities

The order parameter

$$\chi(\vec{r},t) \equiv \int d\vec{\rho} \;\; \langle \hat{c}_{\downarrow}(\vec{r}+\frac{\vec{\rho}}{2})\hat{c}_{\uparrow}(\vec{r}-\frac{\vec{\rho}}{2})\rangle$$

Formal issues – generalities

The order parameter

$$\chi(\vec{r},t) \equiv \int d\vec{\rho} \ \ \langle \hat{c}_{\downarrow}(\vec{r}+\frac{\vec{\rho}}{2})\hat{c}_{\uparrow}(\vec{r}-\frac{\vec{\rho}}{2})\rangle$$

is a complex quantity

$$\chi = |\chi| \ e^{i\theta}$$

Formal issues – generalities

The order parameter

$$\chi(\vec{r},t) \equiv \int d\vec{\rho} \ \langle \hat{c}_{\downarrow}(\vec{r}+\tfrac{\vec{\rho}}{2})\hat{c}_{\uparrow}(\vec{r}-\tfrac{\vec{\rho}}{2})\rangle$$

is a complex quantity

$$\chi = |\chi| \ e^{i\theta}$$

with the following physical implications:

Formal issues – generalities

The order parameter

$$\chi(\vec{r},t) \equiv \int d\vec{\rho}\ \langle \hat{c}_{\downarrow}(\vec{r}+\tfrac{\vec{\rho}}{2})\hat{c}_{\uparrow}(\vec{r}-\tfrac{\vec{\rho}}{2})\rangle$$

is a complex quantity

$$\chi = |\chi|\ e^{i\theta}$$

with the following physical implications:

$|\chi| \neq 0 \longrightarrow$ **amplitude causes the energy gap**

Formal issues – generalities

The order parameter

$$\chi(\vec{r},t) \equiv \int d\vec{\rho}\ \ \langle \hat{c}_{\downarrow}(\vec{r}+\tfrac{\vec{\rho}}{2})\hat{c}_{\uparrow}(\vec{r}-\tfrac{\vec{\rho}}{2})\rangle$$

is a complex quantity

$$\chi = |\chi|\ e^{i\theta}$$

with the following physical implications:

$|\chi| \neq 0 \longrightarrow$ **amplitude causes the energy gap**

$\nabla\theta \neq 0 \longrightarrow$ **phase slippage induces supercurrents**

Anderson-Higgs mechanism – a brief outlook

Low energy excitations of the BE condensed pairs are characterized by the collective (Goldstone) phasal mode.

Anderson-Higgs mechanism – a brief outlook

Low energy excitations of the BE condensed pairs are characterized by the collective (Goldstone) phasal mode.

For charged particles (e.g. electron pairs) the phase θ of the order parameter χ couples to the vector potential $\vec{A}$

Anderson-Higgs mechanism – a brief outlook

Low energy excitations of the BE condensed pairs are characterized by the collective (Goldstone) phasal mode.

For charged particles (e.g. electron pairs) the phase θ of the order parameter χ couples to the vector potential $\vec{A}$

$$S[\vec{A},\theta] = \int d\vec{r} \left[\frac{n_s}{m} \left(\nabla\theta - \vec{A} \right)^2 + \left(\mathrm{rot}\vec{A} \right)^2 \right]$$

Anderson-Higgs mechanism – a brief outlook

Low energy excitations of the BE condensed pairs are characterized by the collective (Goldstone) phasal mode.

For charged particles (e.g. electron pairs) the phase θ of the order parameter χ couples to the vector potential $\vec{A}$

$$S[\vec{A},\theta] = \int d\vec{r} \left[\frac{n_s}{m} \left(\nabla\theta - \vec{A} \right)^2 + \left(\mathrm{rot}\vec{A} \right)^2 \right]$$

Using gauge transformation $\vec{A} \longrightarrow \vec{A} + \nabla\phi$, $\theta \longrightarrow \theta + \phi$ such phase can be elliminated generating a massive term

Anderson-Higgs mechanism – a brief outlook

Low energy excitations of the BE condensed pairs are characterized by the collective (Goldstone) phasal mode.

For charged particles (e.g. electron pairs) the phase θ of the order parameter χ couples to the vector potential $\vec{A}$

$$S[\vec{A},\theta] = \int d\vec{r} \left[\frac{n_s}{m} \left(\nabla\theta - \vec{A} \right)^2 + \left(\mathrm{rot}\vec{A} \right)^2 \right]$$

Using gauge transformation $\vec{A} \longrightarrow \vec{A} + \nabla\phi$, $\theta \longrightarrow \theta + \phi$ such phase can be elliminated generating a massive term

$$S[\vec{A}] = \int d\vec{r} \left(\frac{n_s}{m} + q^2 \right) \vec{A}_{\vec{q}} \cdot \vec{A}_{-\vec{q}}$$

Anderson-Higgs mechanism – a brief outlook

Low energy excitations of the BE condensed pairs are characterized by the collective (Goldstone) phasal mode.

For charged particles (e.g. electron pairs) the phase θ of the order parameter χ couples to the vector potential $\vec{A}$

$$S[\vec{A},\theta] = \int d\vec{r} \left[\frac{n_s}{m} \left(\nabla\theta - \vec{A} \right)^2 + \left(\mathrm{rot}\vec{A} \right)^2 \right]$$

Using gauge transformation $\vec{A} \rightarrow \vec{A} + \nabla\phi$, $\theta \rightarrow \theta + \phi$ such phase can be elliminated generating a massive term

$$S[\vec{A}] = \int d\vec{r} \left(\frac{n_s}{m} + q^2 \right) \vec{A}_{\vec{q}} \cdot \vec{A}_{-\vec{q}}$$

This implies the London equation (Meissner effect)

$$\left(\frac{n_s}{m} - \nabla^2 \right) \vec{A}(\vec{r}) = 0$$

Anderson-Higgs mechanism – a brief outlook

Low energy excitations of the BE condensed pairs are characterized by the collective (Goldstone) phasal mode.

For charged particles (e.g. electron pairs) the phase θ of the order parameter χ couples to the vector potential $\vec{A}$

$$S[\vec{A},\theta] = \int d\vec{r} \left[\frac{n_s}{m} \left(\nabla\theta - \vec{A} \right)^2 + \left(\mathrm{rot}\vec{A} \right)^2 \right]$$

Using gauge transformation $\vec{A} \longrightarrow \vec{A} + \nabla\phi$, $\theta \longrightarrow \theta + \phi$ such phase can be elliminated generating a massive term

$$S[\vec{A}] = \int d\vec{r} \left(\frac{n_s}{m} + q^2 \right) \vec{A}_{\vec{q}} \cdot \vec{A}_{-\vec{q}}$$

This implies the London equation (Meissner effect)

$$\left(\frac{n_s}{m} - \nabla^2 \right) \vec{A}(\vec{r}) = 0$$

$\Rightarrow$ **Can a piece of this mechanism survive above T_c ?**

Motivation

Phase transitions – classification

Phase transitions – classification

The complex order parameter

$$\chi = |\chi| \, e^{i\theta}$$

Phase transitions – classification

The complex order parameter

$$\chi = |\chi| \, e^{i\theta}$$

vanishes at $T \longrightarrow T_c$ either by:

Phase transitions – classification

The complex order parameter

$$\chi = |\chi| \, e^{i\theta}$$

vanishes at $T \longrightarrow T_c$ either by:

1. **closing the gap** . . . . *(BCS superconductors)*

$$\lim_{T \to T_c} |\chi| = 0$$

Phase transitions – classification

The complex order parameter

$$\chi = |\chi| \, e^{i\theta}$$

vanishes at $T \longrightarrow T_c$ either by:

1. **closing the gap** . . . . *(BCS superconductors)*

$$\lim_{T \to T_c} |\chi| = 0$$

2. **disordering the phase** . . . . *(HTSC compounds)*

$$\lim_{T \to T_c} \langle \theta \rangle = 0$$

Historical remark

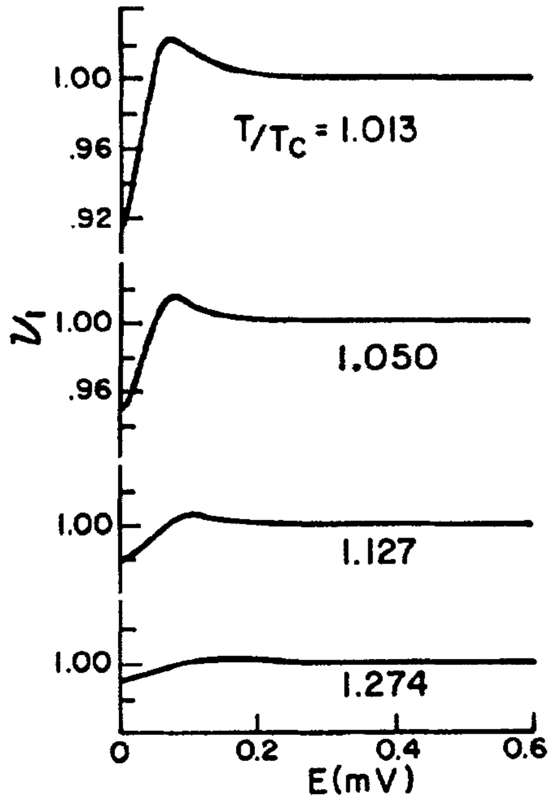
Historical remark

First empirical evidence for the sc fluctuations above T_c has been observed in the **granular aluminium** films.

Historical remark

First empirical evidence for the sc fluctuations above T_c has been observed in the **granular aluminium** films.

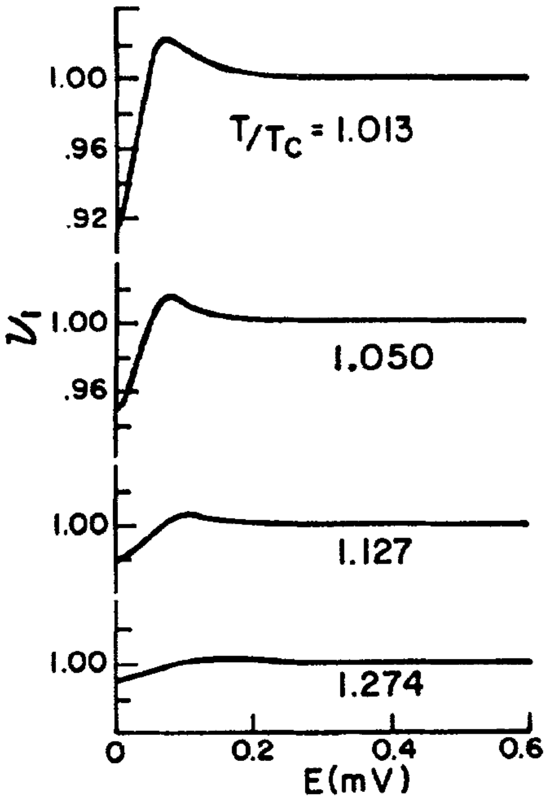
Tunneling conductance revealed a pseudogap surviving above T_c.



Historical remark

First empirical evidence for the sc fluctuations above T_c has been observed in the **granular aluminium** films.

Tunneling conductance revealed a pseudogap surviving above T_c.



R.W. Cohen and B. Abels, Phys. Rev. **168**, *444 (1968).*

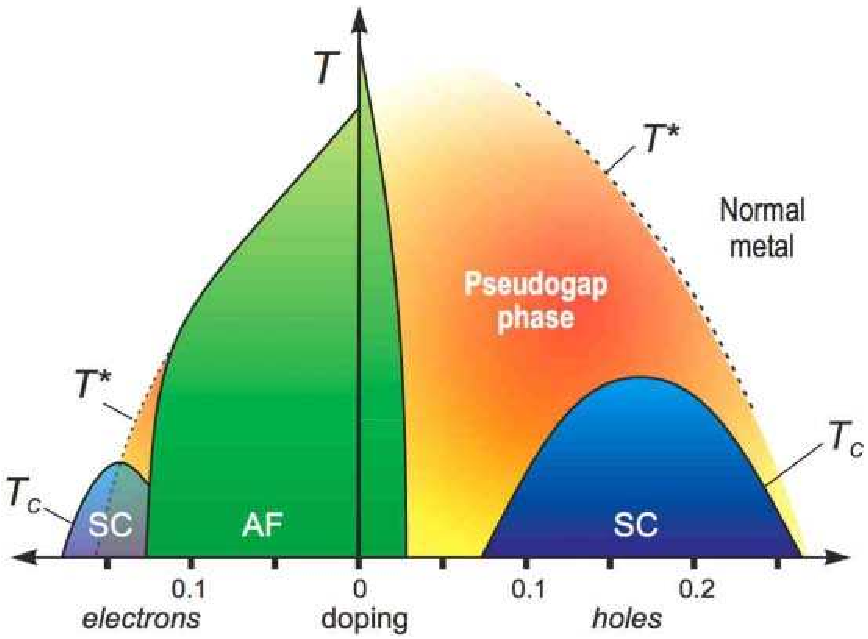
HTSC materials – phase diagram

Superconductivity appears upon doping the Mott insulator by

HTSC materials – phase diagram

Superconductivity appears upon doping the Mott insulator by

electrons or *holes*

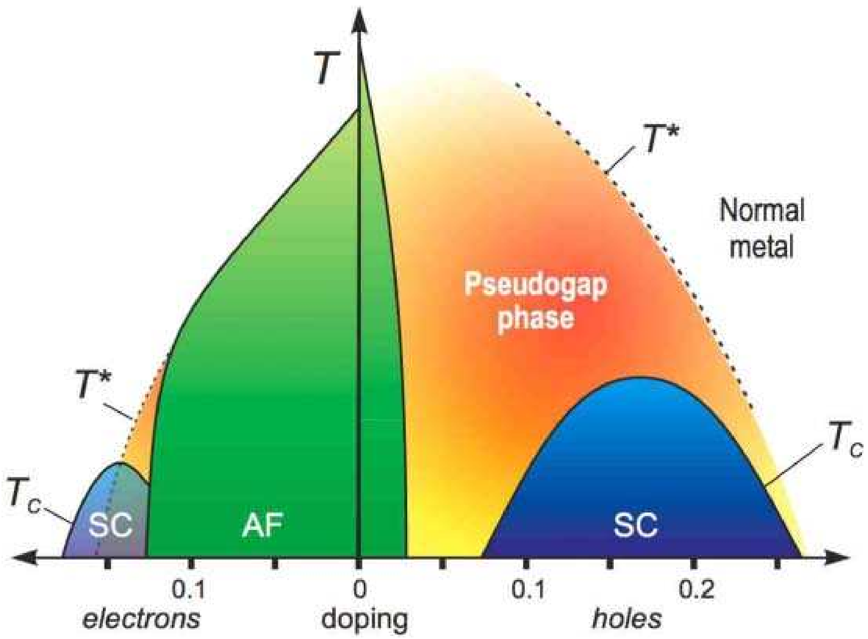


O. Fisher et al, *Rev. Mod. Phys.* **79**, *353 (2007).*

HTSC materials – phase diagram

Superconductivity appears upon doping the Mott insulator by

electrons or *holes*



O. Fisher et al, *Rev. Mod. Phys.* **79**, *353 (2007).*

Unresolved problem:

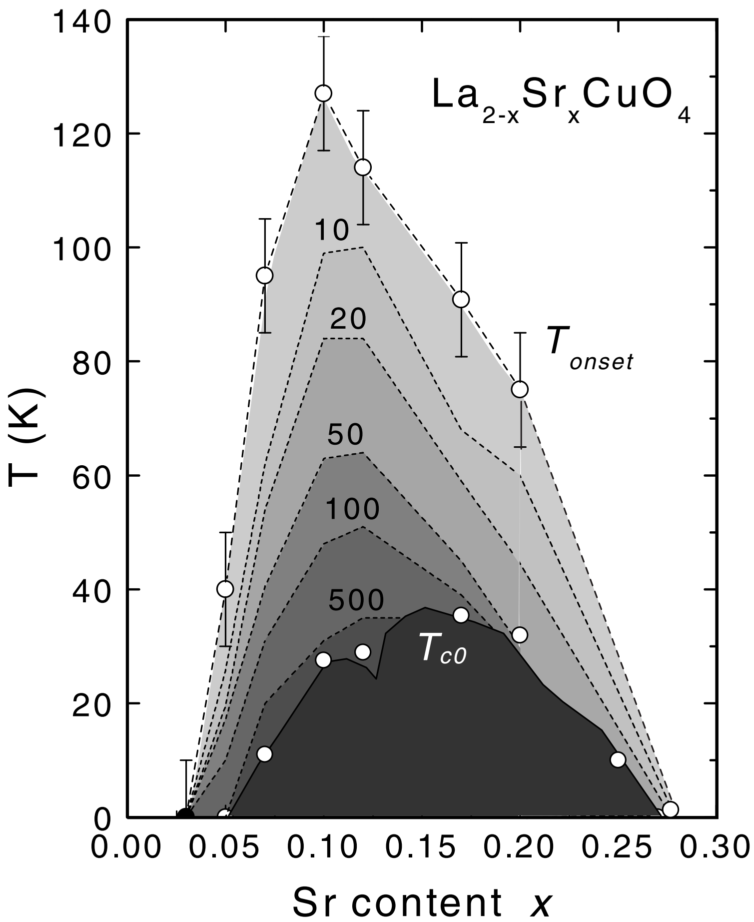
What causes the pseudogap ?

Incoherent pairs above T_c

experimental fact $\#$ 1

Incoherent pairs above T_c

experimental fact # 1

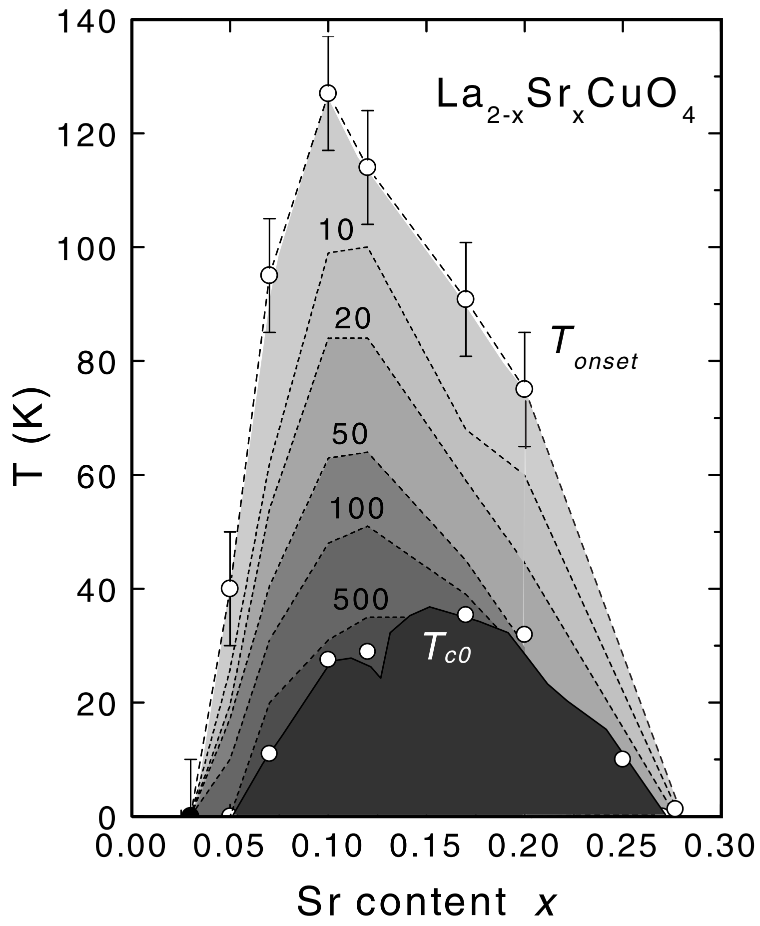


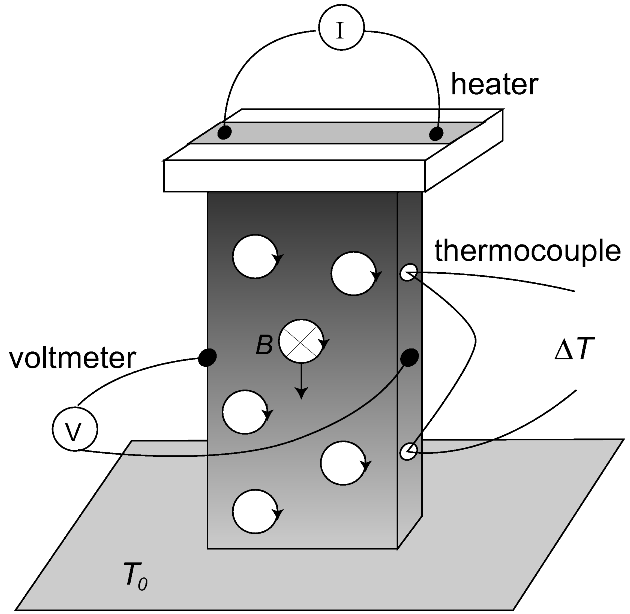
Phase slippage detected in the large Nernst effect.

Y. Wang et al, *Science* **299**, *86 (2003).*

Incoherent pairs above T_c

experimental fact # 1





Phase slippage detected in the large Nernst effect.

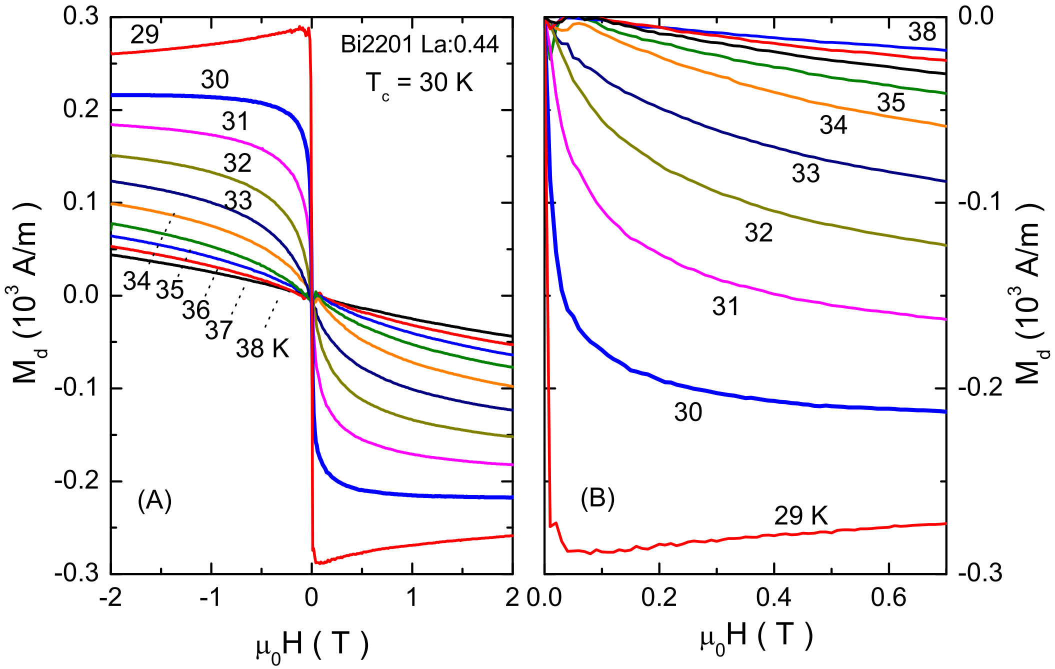
Y. Wang et al, *Science* **299**, *86 (2003).*

Incoherent pairs above T_c

experimental fact $\#$ 2

Incoherent pairs above T_c

experimental fact # 2



The Van Vleck background $(A + BT)H$ *is subracted*

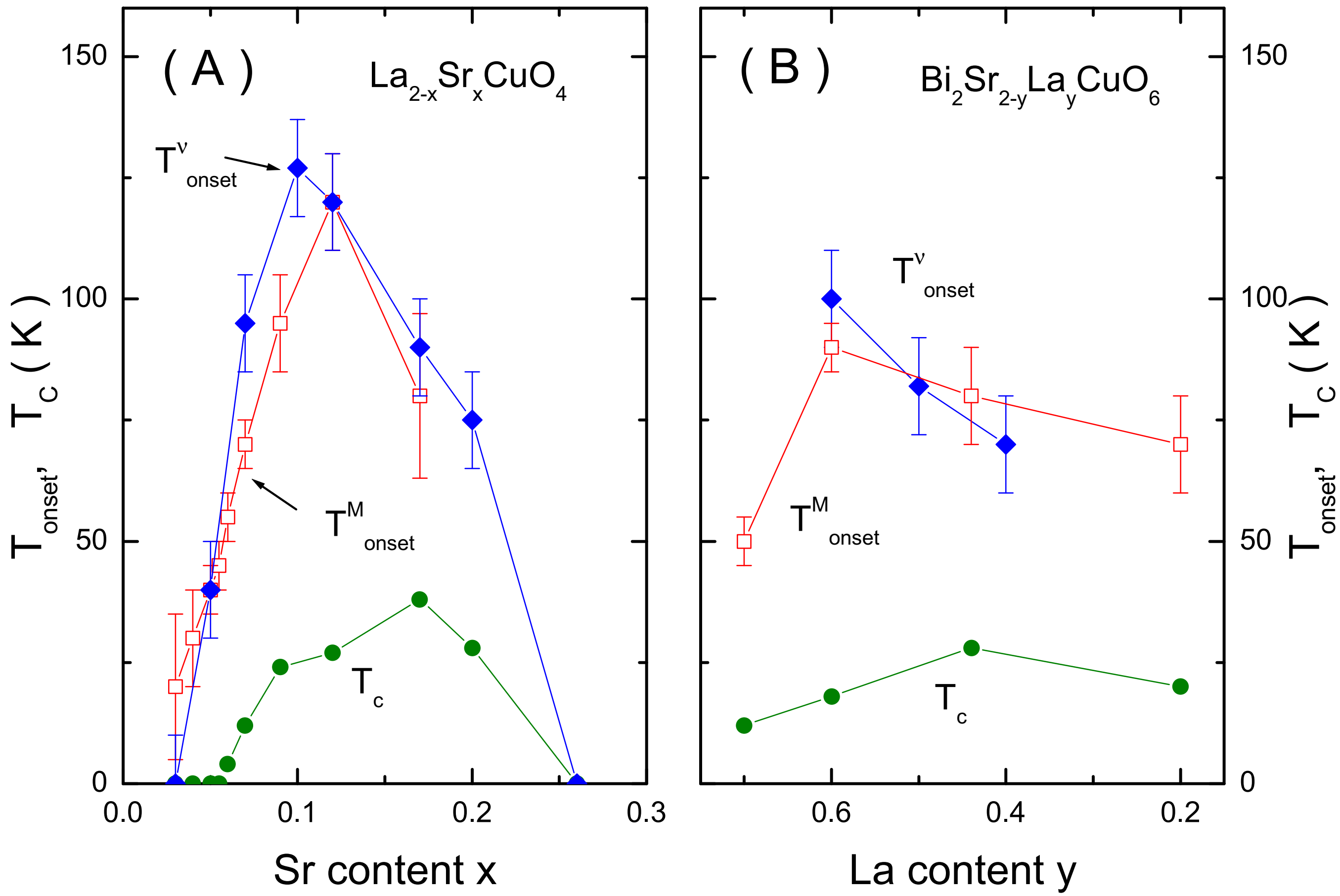
$$T_c = 30K$$

Enhanced diamagnetic response revealed above T_c by the ultrahigh precission torque magnetometry.

L. Li et al *and N.P. Ong, Phys. Rev. B* **81**, *054510 (2010).*

Incoherent pairs above T_c

experimental fact # 2



T^{ν} **– onset of the Nernst effect**

T^{M} **– onset of the diamagnetism**

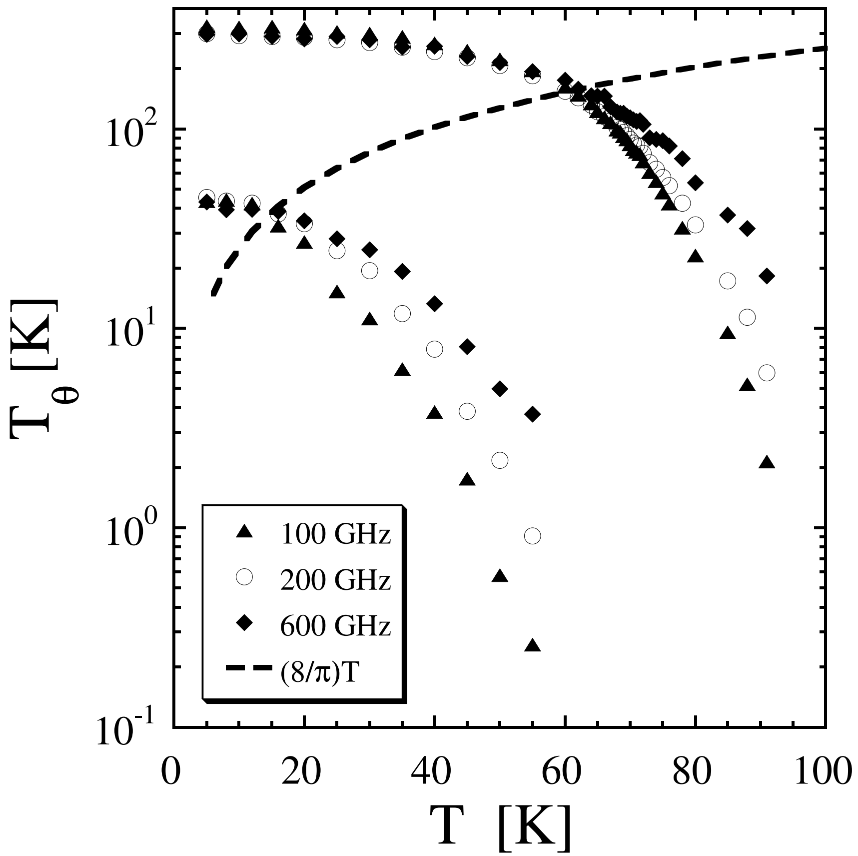
L. Li et al *and N.P. Ong, Phys. Rev. B* **81**, *054510 (2010).*

Incoherent pairs above T_c

experimental fact $\#$ 3

Incoherent pairs above T_c

experimental fact # 3



Dynamic phase-stiffness $\quad T_\theta = \omega \; \mathrm{Im}\sigma(\omega, T)/\sigma_Q$

observed at the ultrafast (teraHz) external ac fields.

J. Corson et al, *Nature* **398**, *221 (1999).*

Incoherent pairs above T_c

... continued

Incoherent pairs above T_c

... continued

⇒ **Josephson-like features seen above T_c in the tunneling**

N. Bergeal et al, *Nature Phys.* **4**, *608 (2008).*

Incoherent pairs above T_c

... continued

⇒ **Josephson-like features seen above T_c in the tunneling**

N. Bergeal et al, *Nature Phys.* **4**, *608 (2008).*

⇒ **smooth evolution of the electronic spectrum observed by ARPES near the superconductor–insulator transition**

U. Chatterjee et al, *Nature Phys.* **5**, *1456 (2009).*

Incoherent pairs above T_c ... *continued*

⇒ **Josephson-like features seen above T_c in the tunneling**

N. Bergeal et al, *Nature Phys.* **4**, *608 (2008).*

⇒ **smooth evolution of the electronic spectrum observed by ARPES near the superconductor–insulator transition**

U. Chatterjee et al, *Nature Phys.* **5**, *1456 (2009).*

⇒ **spectroscopic fingerprints of the Bogoliubov QPs seen by the unique octet patterns which survive up to $1.5T_c$**

J. Lee, ... and J.C. Davis, Science **325**, *1099 (2009).*

Scenario & methodology

Boson-Fermion model

[in a lattice representation]

$$\hat{H} = \sum_{i,j,\sigma} (t_{ij} - \mu\, \delta_{i,j})\, \hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} + \sum_{l} \left(E_l^{(B)} - 2\mu \right) \hat{b}^{\dagger}_l \hat{b}_l + \sum_{i,j} g_{ij} \left[\hat{b}^{\dagger}_l \hat{c}_{i,\downarrow} \hat{c}_{j,\uparrow} + \text{h.c.} \right]$$

Boson-Fermion model

[in a lattice representation]

$$\hat{H} = \sum_{i,j,\sigma} (t_{ij} - \mu\, \delta_{i,j})\, \hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} + \sum_{l} \left(E_l^{(B)} - 2\mu \right) \hat{b}_l^{\dagger} \hat{b}_l + \sum_{i,j} g_{ij} \left[\hat{b}_l^{\dagger} \hat{c}_{i,\downarrow} \hat{c}_{j,\uparrow} + \text{h.c.} \right]$$

$$\vec{R}_l = (\vec{r}_i + \vec{r}_j)/2$$

Boson-Fermion model

[in a lattice representation]

$$\hat{H} = \sum_{i,j,\sigma} (t_{ij} - \mu\, \delta_{i,j})\, \hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} + \sum_{l} \left(E_l^{(B)} - 2\mu \right) \hat{b}^{\dagger}_l \hat{b}_l$$

$$+ \sum_{i,j} g_{ij} \left[\hat{b}^{\dagger}_l \hat{c}_{i,\downarrow} \hat{c}_{j,\uparrow} + \text{h.c.} \right]$$

$$\vec{R}_l = (\vec{r}_i + \vec{r}_j)/2$$

describes a two-component system consisting of:

Boson-Fermion model

[in a lattice representation]

$$\hat{H} = \sum_{i,j,\sigma} (t_{ij} - \mu\, \delta_{i,j})\, \hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} + \sum_{l} \left(E_l^{(B)} - 2\mu \right) \hat{b}^{\dagger}_l \hat{b}_l$$

$$+ \sum_{i,j} g_{ij} \left[\hat{b}^{\dagger}_l \hat{c}_{i,\downarrow} \hat{c}_{j,\uparrow} + \text{h.c.} \right]$$

$$\vec{R}_l = (\vec{r}_i + \vec{r}_j)/2$$

describes a two-component system consisting of:

$\hat{c}^{(\dagger)}_{i\sigma}$ **itinerant fermions ....** *(e.g. holes near the Mott insulator)*

Boson-Fermion model

[in a lattice representation]

$$\hat{H} = \sum_{i,j,\sigma} (t_{ij} - \mu\, \delta_{i,j})\, \hat{c}^\dagger_{i\sigma} \hat{c}_{j\sigma} + \sum_l \left(E_l^{(B)} - 2\mu\right) \hat{b}^\dagger_l \hat{b}_l$$

$$+ \sum_{i,j} g_{ij} \left[\hat{b}^\dagger_l \hat{c}_{i,\downarrow} \hat{c}_{j,\uparrow} + \text{h.c.}\right]$$

$$\vec{R}_l = (\vec{r}_i + \vec{r}_j)/2$$

describes a two-component system consisting of:

$\hat{c}^{(\dagger)}_{i\sigma}$ **itinerant fermions** ....*(e.g. holes near the Mott insulator)*

$\hat{b}^{(\dagger)}_l$ **immobile local pairs** ......*(RVB defines them on the bonds)*

Boson-Fermion model

[in a lattice representation]

$$\hat{H} = \sum_{i,j,\sigma} (t_{ij} - \mu\, \delta_{i,j})\, \hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} + \sum_{l} \left(E_l^{(B)} - 2\mu \right) \hat{b}^{\dagger}_l \hat{b}_l$$
$$+ \sum_{i,j} g_{ij} \left[\hat{b}^{\dagger}_l \hat{c}_{i,\downarrow} \hat{c}_{j,\uparrow} + \text{h.c.} \right]$$

$$\vec{R}_l = (\vec{r}_i + \vec{r}_j)/2$$

describes a two-component system consisting of:

$\hat{c}^{(\dagger)}_{i\sigma}$ **itinerant fermions** ....*(e.g. holes near the Mott insulator)*

$\hat{b}^{(\dagger)}_{l}$ **immobile local pairs** ......*(RVB defines them on the bonds)*

interacting via:

Boson-Fermion model

[in a lattice representation]

$$\hat{H} = \sum_{i,j,\sigma} (t_{ij} - \mu\,\delta_{i,j})\,\hat{c}^{\dagger}_{i\sigma}\hat{c}_{j\sigma} + \sum_{l} \left(E_l^{(B)} - 2\mu\right)\hat{b}^{\dagger}_l \hat{b}_l$$

$$+ \sum_{i,j} g_{ij}\left[\hat{b}^{\dagger}_l \hat{c}_{i,\downarrow}\hat{c}_{j,\uparrow} + \text{h.c.}\right]$$

$$\vec{R}_l = (\vec{r}_i + \vec{r}_j)/2$$

describes a two-component system consisting of:

$\hat{c}^{(\dagger)}_{i\sigma}$ **itinerant fermions** ....*(e.g. holes near the Mott insulator)*

$\hat{b}^{(\dagger)}_{l}$ **immobile local pairs** ......*(RVB defines them on the bonds)*

interacting via:

$\hat{b}^{\dagger}_l\ \hat{c}_{i,\downarrow}\hat{c}_{j,\uparrow} + h.c.$ ....*(the Andreev-type scattering)*

Boson-Fermion model

[in a lattice representation]

$$\hat{H} = \sum_{i,j,\sigma} (t_{ij} - \mu\, \delta_{i,j})\, \hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} + \sum_{l} \left(E_l^{(B)} - 2\mu \right) \hat{b}^{\dagger}_l \hat{b}_l + \sum_{i,j} g_{ij} \left[\hat{b}^{\dagger}_l \hat{c}_{i,\downarrow} \hat{c}_{j,\uparrow} + \text{h.c.} \right]$$

$$\vec{R}_l = (\vec{r}_i + \vec{r}_j)/2$$

describes a two-component system consisting of:

$\hat{c}^{(\dagger)}_{i\sigma}$ **itinerant fermions** ....*(e.g. holes near the Mott insulator)*

$\hat{b}^{(\dagger)}_l$ **immobile local pairs** ......*(RVB defines them on the bonds)*

interacting via:

$\hat{b}^{\dagger}_l\ \hat{c}_{i,\downarrow} \hat{c}_{j,\uparrow}\ + h.c.$ ....*(the Andreev-type scattering)*

In the Lagrangian language we obtain this kind of physics upon applying the Hubbard-Stratonovich transformation !

Boson-Fermion model

[in the momentum space]

$$\hat{H} = \sum_{\mathbf{k}\sigma} (\varepsilon_{\mathbf{k}} - \mu)\, \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} + \sum_{\mathbf{q}} \left(E^{(B)} - 2\mu \right) \hat{b}^{\dagger}_{\mathbf{q}} \hat{b}_{\mathbf{q}}$$

$$+ \frac{1}{\sqrt{N}} \sum_{\mathbf{k},\mathbf{q}} g_{\mathbf{k},\mathbf{q}} \left[\hat{b}^{\dagger}_{\mathbf{q}} \hat{c}_{\mathbf{k},\downarrow} \hat{c}_{\mathbf{q}-\mathbf{k},\uparrow} + \text{h.c.} \right]$$

Boson-Fermion model

[in the momentum space]

$$\hat{H} = \sum_{\mathbf{k}\sigma} (\varepsilon_{\mathbf{k}} - \mu)\, \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} + \sum_{\mathbf{q}} \left(E^{(B)} - 2\mu \right) \hat{b}^{\dagger}_{\mathbf{q}} \hat{b}_{\mathbf{q}} + \frac{1}{\sqrt{N}} \sum_{\mathbf{k},\mathbf{q}} g_{\mathbf{k},\mathbf{q}} \left[\hat{b}^{\dagger}_{\mathbf{q}} \hat{c}_{\mathbf{k},\downarrow} \hat{c}_{\mathbf{q}-\mathbf{k},\uparrow} + \text{h.c.} \right]$$

This scenario has been investigated by various groups:

J. Ranninger *with coworkers* . . . . Grenoble

R. Micnas, S. Robaszkiewicz . . . . Poznań

T.D. Lee *with coworkers* . . . . New York

V.B. Geshkenbein, L.B. Ioffe, A.I. Larkin . . . .

E. Altman & A. Auerbach . . . . Technion

A. Griffin *with coworkers* . . . . Toronto

K. Levin *with coworkers* . . . . Chicago

and many others.

Outline of the procedure

Outline of the procedure

For studying the many-body effects we construct the sequence

Outline of the procedure

For studying the many-body effects we construct the sequence of unitary transformations

Outline of the procedure

For studying the many-body effects we construct the sequence of unitary transformations

$$\hat{H} \longrightarrow \hat{H}(l_1) \longrightarrow \hat{H}(l_2) \longrightarrow \ldots \longrightarrow \hat{H}(\infty)$$

Outline of the procedure

For studying the many-body effects we construct the sequence of unitary transformations

$$\hat{H} \longrightarrow \hat{H}(l_1) \longrightarrow \hat{H}(l_2) \longrightarrow \ldots \longrightarrow \hat{H}(\infty)$$

decoupling the boson from fermion degrees of freedom.

Outline of the procedure

For studying the many-body effects we construct the sequence of unitary transformations

$$\hat{H} \longrightarrow \hat{H}(l_1) \longrightarrow \hat{H}(l_2) \longrightarrow \ldots \longrightarrow \hat{H}(\infty)$$

decoupling the boson from fermion degrees of freedom.

F. Wegner (1994); K.G. Wilson (1994) - inventors of this RG-like scheme

Outline of the procedure

For studying the many-body effects we construct the sequence of unitary transformations

$$\hat{H} \longrightarrow \hat{H}(l_1) \longrightarrow \hat{H}(l_2) \longrightarrow \ldots \longrightarrow \hat{H}(\infty)$$

decoupling the boson from fermion degrees of freedom.

F. Wegner (1994); K.G. Wilson (1994) - inventors of this RG-like scheme

Hamiltonian at $l = 0$

$$\hat{H}_F + \hat{H}_B + \hat{V}_{BF}$$

Outline of the procedure

For studying the many-body effects we construct the sequence of unitary transformations

$$\hat{H} \longrightarrow \hat{H}(l_1) \longrightarrow \hat{H}(l_2) \longrightarrow \ldots \longrightarrow \hat{H}(\infty)$$

decoupling the boson from fermion degrees of freedom.

F. Wegner (1994); K.G. Wilson (1994) - inventors of this RG-like scheme

Hamiltonian at $0 < l < \infty$

$$\hat{H}_F(l) + \hat{H}_B(l) + \hat{V}_{BF}(l)$$

Outline of the procedure

For studying the many-body effects we construct the sequence of unitary transformations

$$\hat{H} \longrightarrow \hat{H}(l_1) \longrightarrow \hat{H}(l_2) \longrightarrow \ldots \longrightarrow \hat{H}(\infty)$$

decoupling the boson from fermion degrees of freedom.

F. Wegner (1994); K.G. Wilson (1994) - inventors of this RG-like scheme

Hamiltonian at $l = \infty$

$$\hat{H}_F(\infty) + \hat{H}_B(\infty) \; + \; 0$$

Outline of the procedure

For studying the many-body effects we construct the sequence of unitary transformations

$$\hat{H} \longrightarrow \hat{H}(l_1) \longrightarrow \hat{H}(l_2) \longrightarrow \ldots \longrightarrow \hat{H}(\infty)$$

decoupling the boson from fermion degrees of freedom.

F. Wegner (1994); K.G. Wilson (1994) - inventors of this RG-like scheme

Hamiltonian at $l = \infty$

$$\hat{H}_F(\infty) + \hat{H}_B(\infty) \; + \; 0$$

T. Domański and J. Ranninger, Phys. Rev. **B 63**, *134505 (2001).*

Continuous unitary transformation – algorithm

Let
$$\hat{H}(l) = \hat{S}(l)\ \hat{H}\ \hat{S}^{\dagger}(l)$$

Continuous unitary transformation – algorithm

Let
$$\hat{H}(l) = \hat{S}(l)\ \hat{H}\ \hat{S}^{\dagger}(l)$$

l – a continuous **flow parameter**.

Continuous unitary transformation – algorithm

Let
$$\hat{H}(l) = \hat{S}(l)\ \hat{H}\ \hat{S}^{\dagger}(l)$$

l – a continuous **flow parameter**.

The derivative

$$\frac{d\hat{H}(l)}{dl} = \frac{d\hat{S}(l)}{dl}\hat{H}\hat{S}^{\dagger}(l) + \hat{S}(l)\hat{H}\frac{d\hat{S}^{\dagger}(l)}{dl}$$
$$= \frac{d\hat{S}(l)}{dl}\hat{S}^{\dagger}(l)\hat{H}(l) + \hat{H}(l)\hat{S}(l)\frac{d\hat{S}^{\dagger}(l)}{dl}$$

Continuous unitary transformation – algorithm

Let $$\hat{H}(l) = \hat{S}(l)\ \hat{H}\ \hat{S}^{\dagger}(l)$$

l – a continuous **flow parameter**.

The derivative

$$\begin{aligned}\frac{d\hat{H}(l)}{dl} &= \frac{d\hat{S}(l)}{dl}\hat{H}\hat{S}^{\dagger}(l) + \hat{S}(l)\hat{H}\frac{d\hat{S}^{\dagger}(l)}{dl} \\ &= \frac{d\hat{S}(l)}{dl}\hat{S}^{\dagger}(l)\hat{H}(l) + \hat{H}(l)\hat{S}(l)\frac{d\hat{S}^{\dagger}(l)}{dl}\end{aligned}$$

Using the unitary transform. identity $\hat{S}(l)\hat{S}^{\dagger}(l) = 1$, so that $\frac{d\hat{S}(l)}{dl}\hat{S}^{\dagger}(l) + \hat{S}(l)\frac{d\hat{S}^{\dagger}(l)}{dl} = 0$ we obtain **the flow equation**

Continuous unitary transformation – algorithm

Let
$$\hat{H}(l) = \hat{S}(l)\ \hat{H}\ \hat{S}^{\dagger}(l)$$

l – a continuous **flow parameter**.

The derivative

$$\begin{aligned}\frac{d\hat{H}(l)}{dl} &= \frac{d\hat{S}(l)}{dl}\hat{H}\hat{S}^{\dagger}(l) + \hat{S}(l)\hat{H}\frac{d\hat{S}^{\dagger}(l)}{dl} \\ &= \frac{d\hat{S}(l)}{dl}\hat{S}^{\dagger}(l)\hat{H}(l) + \hat{H}(l)\hat{S}(l)\frac{d\hat{S}^{\dagger}(l)}{dl}\end{aligned}$$

Using the unitary transform. identity $\hat{S}(l)\hat{S}^{\dagger}(l) = 1$, so that $\frac{d\hat{S}(l)}{dl}\hat{S}^{\dagger}(l) + \hat{S}(l)\frac{d\hat{S}^{\dagger}(l)}{dl} = 0$ we obtain **the flow equation**

$$\frac{d\hat{H}(l)}{dl} = [\hat{\eta}(l), \hat{H}(l)]$$

Continuous unitary transformation – algorithm

Let
$$\hat{H}(l) = \hat{S}(l)\ \hat{H}\ \hat{S}^{\dagger}(l)$$

l – a continuous **flow parameter**.

The derivative

$$\frac{d\hat{H}(l)}{dl} = \frac{d\hat{S}(l)}{dl}\hat{H}\hat{S}^{\dagger}(l) + \hat{S}(l)\hat{H}\frac{d\hat{S}^{\dagger}(l)}{dl}$$
$$= \frac{d\hat{S}(l)}{dl}\hat{S}^{\dagger}(l)\hat{H}(l) + \hat{H}(l)\hat{S}(l)\frac{d\hat{S}^{\dagger}(l)}{dl}$$

Using the unitary transform. identity $\hat{S}(l)\hat{S}^{\dagger}(l) = 1$, so that $\frac{d\hat{S}(l)}{dl}\hat{S}^{\dagger}(l) + \hat{S}(l)\frac{d\hat{S}^{\dagger}(l)}{dl} = 0$ we obtain **the flow equation**

$$\frac{d\hat{H}(l)}{dl} = [\hat{\eta}(l), \hat{H}(l)]$$

where

$$\hat{\eta}(l) = \frac{d\hat{S}(l)}{dl}\hat{S}^{\dagger}(l) = -\hat{\eta}^{\dagger}(l).$$

Continuous unitary transformation – algorithm

How one can guess a diagonalizing generator $\eta(l)$?

Continuous unitary transformation – algorithm

How one can guess a diagonalizing generator $\eta(l)$?

For the Hamiltonian

$$\hat{H} = \hat{H}_{diag} + \hat{H}_{off}$$

one can choose

$$\hat{\eta}(l) = \left[\hat{H}_{diag}(l), \hat{H}_{off}(l)\right]$$

and then

$$\lim_{l \to \infty} \hat{H}_{off}(l) = 0$$

Continuous unitary transformation – algorithm

How one can guess a diagonalizing generator $\eta(l)$?

For the Hamiltonian

$$\hat{H} = \hat{H}_{diag} + \hat{H}_{off}$$

one can choose

$$\hat{\eta}(l) = \left[\hat{H}_{diag}(l), \hat{H}_{off}(l)\right]$$

and then

$$\lim_{l\to\infty} \hat{H}_{off}(l) = 0$$

For more details see for instance:

S. Kehrein, Springer Tracts in Modern Physics **217**, (2006);

F. Wegner, J. Phys. A: Math. Gen. **39**, 8221 (2006).

Continuous unitary transformation – algorithm

Similar ideas have been also earlier developed also in the field of **control theory** under the names:

- *"double bracket flow"*

  R.W. Brockett, Lin. Alg. and its Appl. **146***, 79 (1991).*

- *"isospectral flow"*

  M.T. Chu and K.R. Driessel, J. Num. Anal. **27***, 1050 (1990).*

Continuous unitary transformation – algorithm

Similar ideas have been also earlier developed also in the field of **control theory** under the names:

- *"double bracket flow"*

  R.W. Brockett, Lin. Alg. and its Appl. **146**, *79 (1991).*

- *"isospectral flow"*

  M.T. Chu and K.R. Driessel, J. Num. Anal. **27**, *1050 (1990).*

Pedagogical study of the CUT method efficiency compared to the standard numerical procedures such as e.g. the Jacobi transformation, has been done by

Continuous unitary transformation – **algorithm**

Similar ideas have been also earlier developed also in the field of **control theory** under the names:

- *"double bracket flow"*

  R.W. Brockett, Lin. Alg. and its Appl. ***146****, 79 (1991).*

- *"isospectral flow"*

  M.T. Chu and K.R. Driessel, J. Num. Anal. ***27****, 1050 (1990).*

Pedagogical study of the CUT method efficiency compared to the standard numerical procedures such as e.g. the Jacobi transformation, has been done by

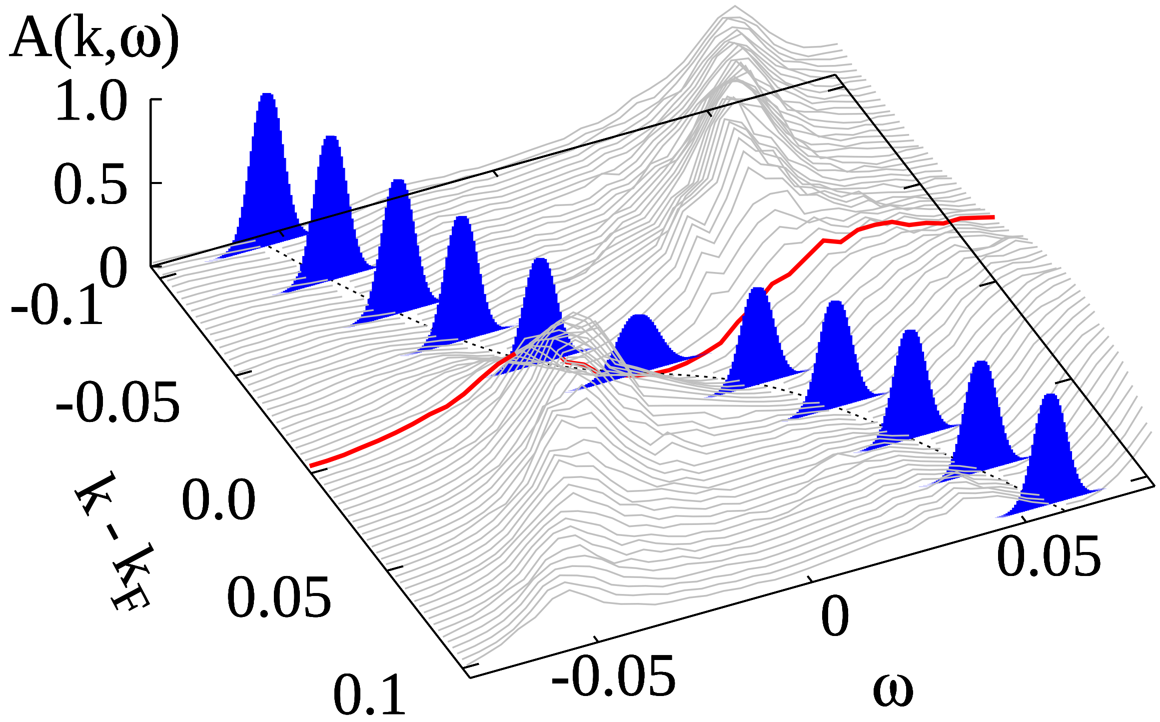
S.R. White, J. Chem. Phys. ***117****, 7472 (2002).*

Results :

1. Bogoliubov quasiparticles above T_c

Effective spectrum: BF model

$T_c < T < T^*$

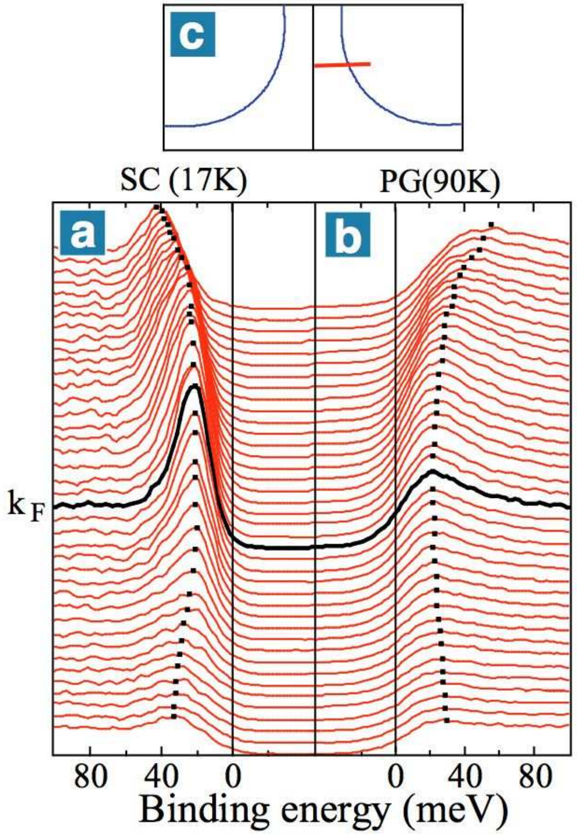


T. Domański and J. Ranninger, Phys. Rev. Lett. ***91****, 255301 (2003).*

Evidence for Bogoliubov QPs above T_c

Evidence for Bogoliubov QPs above T_c

J. Campuzano group (Chicago, USA)

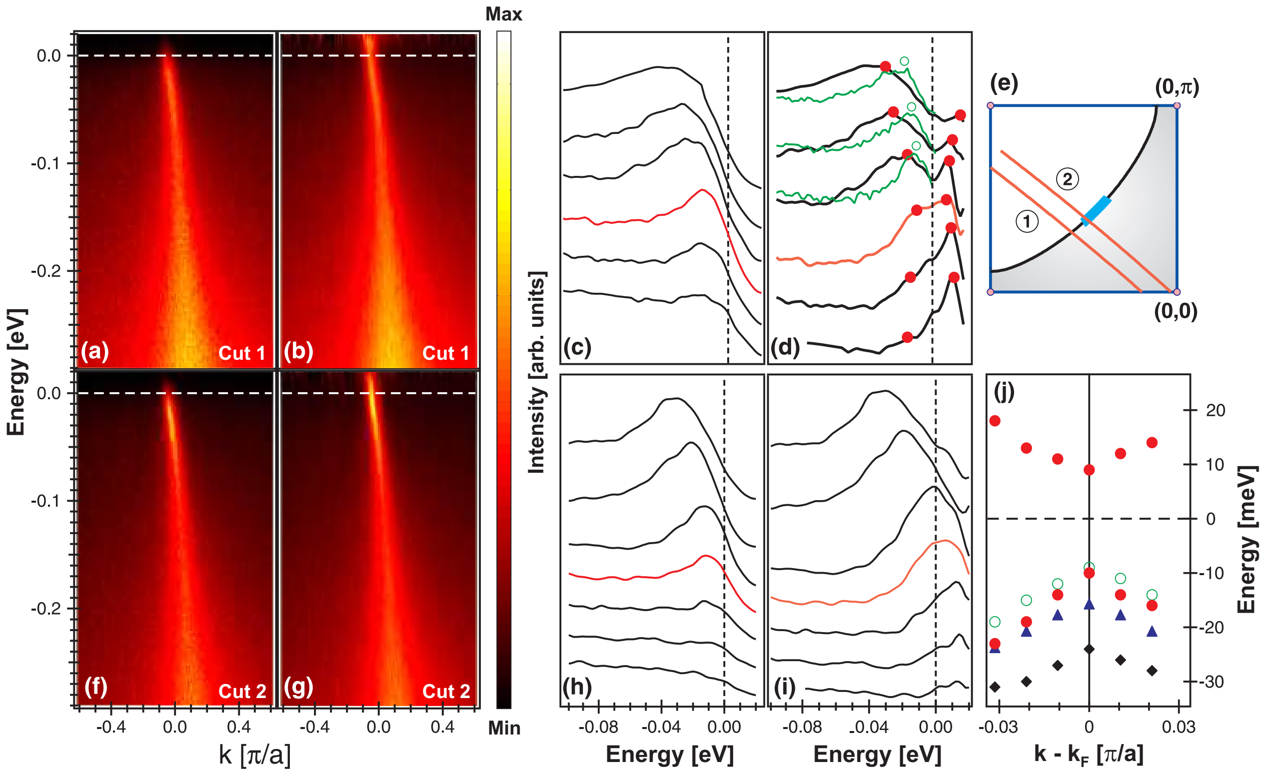


Results for: $Bi_2Sr_2CaCu_2O_8$

A. Kanigel et al, Phys. Rev. Lett. **101***, 137002 (2008).*

Evidence for Bogoliubov QPs above T_c

PSI group (Villigen, Switzerland)

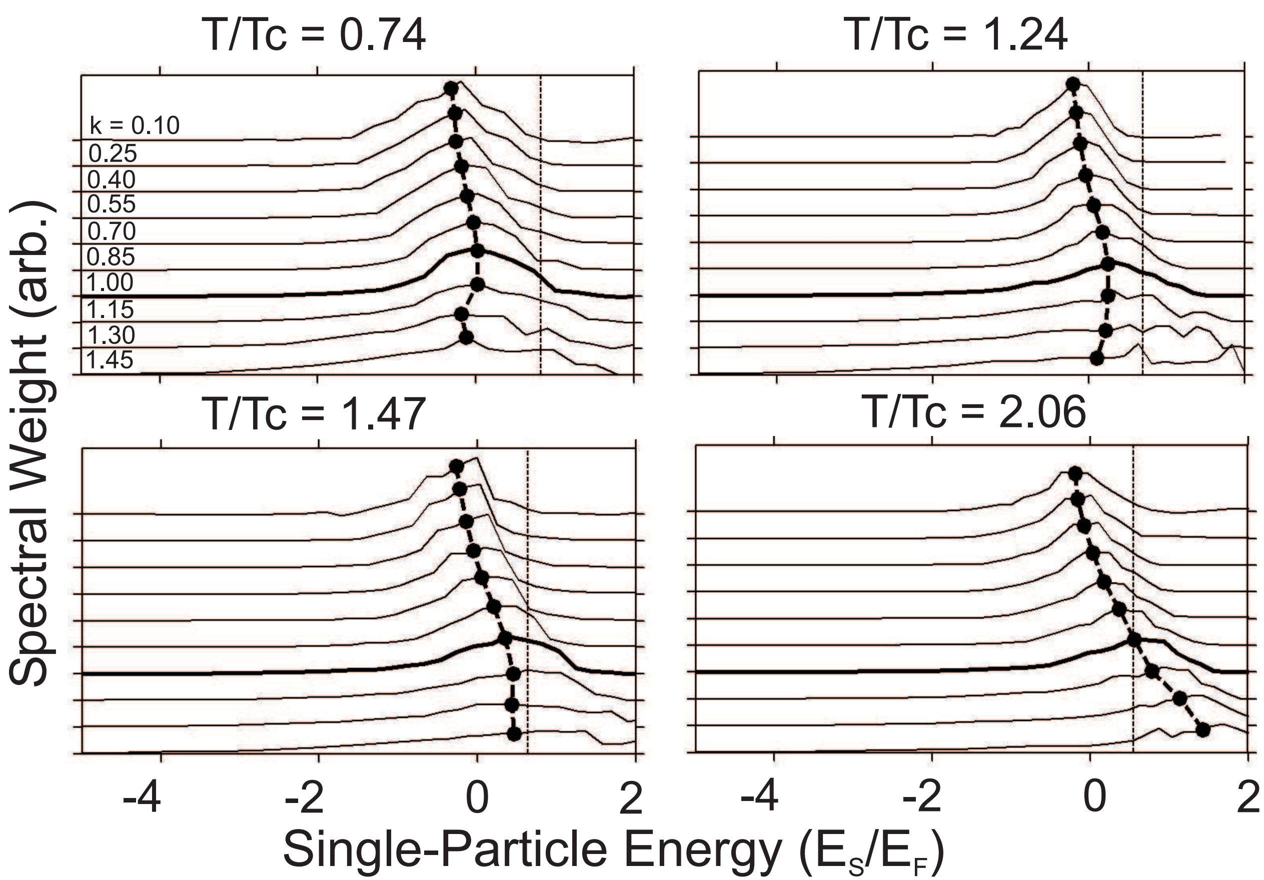


Results for: $La_{1.895}Sr_{0.105}CuO_4$

M. Shi et al, Eur. Phys. Lett. ***88****, 27008 (2009).*

Evidence for Bogoliubov QPs above T_c

D. Jin group (Boulder, USA)



Results for: ultracold ^{40}K atoms

J.P. Gaebler et al, Nature Phys. **6**, *569 (2010).*

Results :

2. Diamagnetism above T_c

Correlation functions

For studying the diamagnetic response (in the Kubo formalism) we have to determine the current-current correlation function

$$-\,\hat{T}_\tau \langle \hat{j}_{\mathbf{q}}(\tau)\ \hat{j}_{-\mathbf{q}}(0)\rangle$$

with statistical averaging defined as

$$\langle \ldots \rangle = \mathrm{Tr}\left\{ e^{-\beta \hat{H}} \ldots \right\} / \mathrm{Tr}\left\{ e^{-\beta \hat{H}} \right\}$$

and $\beta^{-1} = k_B T$.

This can be achieved using the following invariance

$$\begin{aligned}
\mathrm{Tr}\left\{ e^{-\beta\hat{H}} \hat{O} \right\} &= \mathrm{Tr}\left\{ e^{\hat{S}(l)} e^{-\beta\hat{H}} \hat{O} e^{-\hat{S}(l)} \right\} \\
&= \mathrm{Tr}\left\{ e^{\hat{S}(l)} e^{-\beta\hat{H}} e^{-\hat{S}(l)} e^{\hat{S}(l)} \hat{O} e^{-\hat{S}(l)} \right\} \\
&= \mathrm{Tr}\left\{ e^{-\beta\hat{H}(l)} \hat{O}(l) \right\}
\end{aligned}$$

where

$$\hat{H}(l) = e^{\hat{S}(l)} \hat{H} e^{-\hat{S}(l)} \qquad \hat{O}(l) = e^{\hat{S}(l)} \hat{O} e^{-\hat{S}(l)}$$

Correlation functions – some remarks

⋆ The easiest way for calculating $\langle \hat{j}_{\mathbf{q}}(\tau)\, \hat{j}_{-\mathbf{q}} \rangle$ is in a limit $l \longrightarrow \infty$ when $\hat{H}(\infty)$ becomes (block-)diagonal.

⋆ The operators $\hat{j}_q$ must however undergo the continuous transformation

$$\hat{j} \longrightarrow \ldots \longrightarrow \hat{j}(l) \longrightarrow \ldots \longrightarrow \hat{j}(\infty)$$

⋆ obeying **the flow equation**:

$$\frac{\partial \hat{j}_{\mathbf{q}}(l)}{\partial l} = \left[\hat{\eta}(l), \hat{j}_{\mathbf{q}}(l) \right]$$

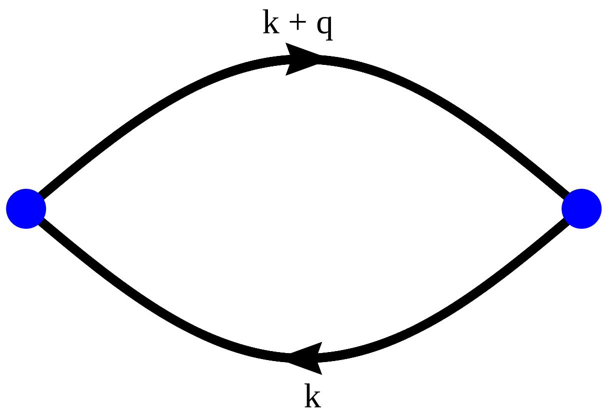
Diamagnetic response above T_c

Diamagnetic response above T_c

Main contributions to the current-current response function:

Diamagnetic response above T_c

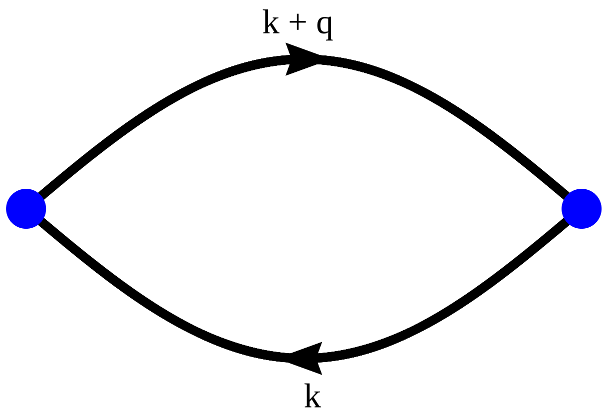
Main contributions to the current-current response function:



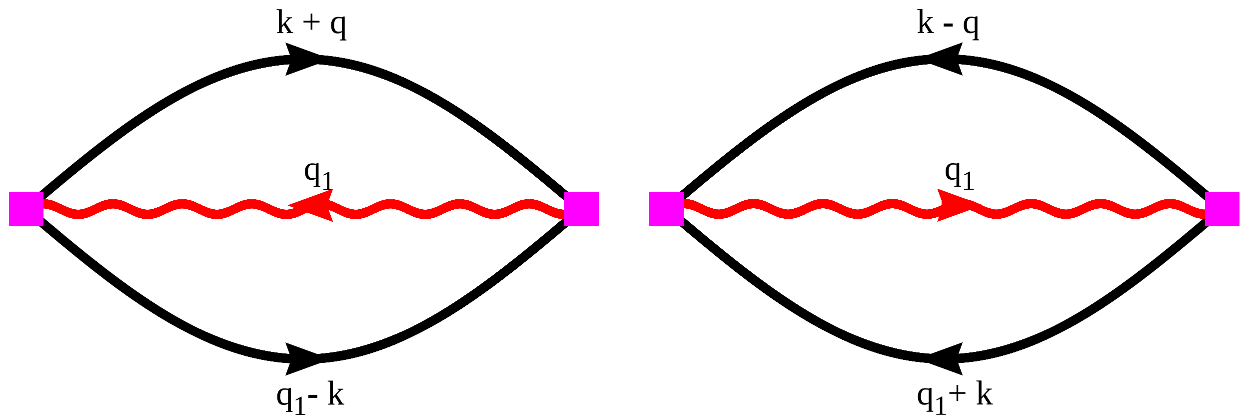
$\Longleftarrow$ **the usual bubble diagram**

Diagmagnetic response above T_c

Main contributions to the current-current response function:



$\Longleftarrow$ **the usual bubble diagram**



anomalous diagrams

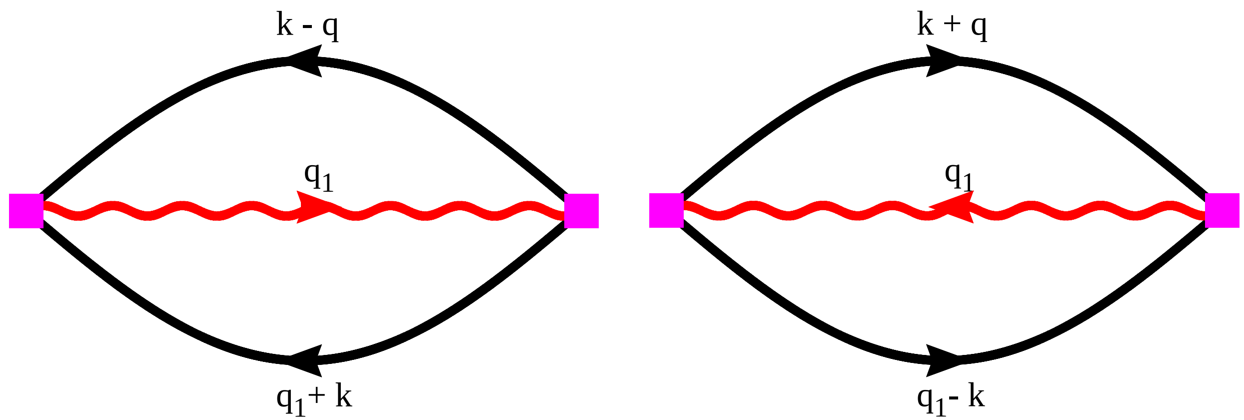
Each vertex has to be determined from the flow equations.

T. Domanski and J. Ranninger, (to be published).

Diamagnetic response above T_c

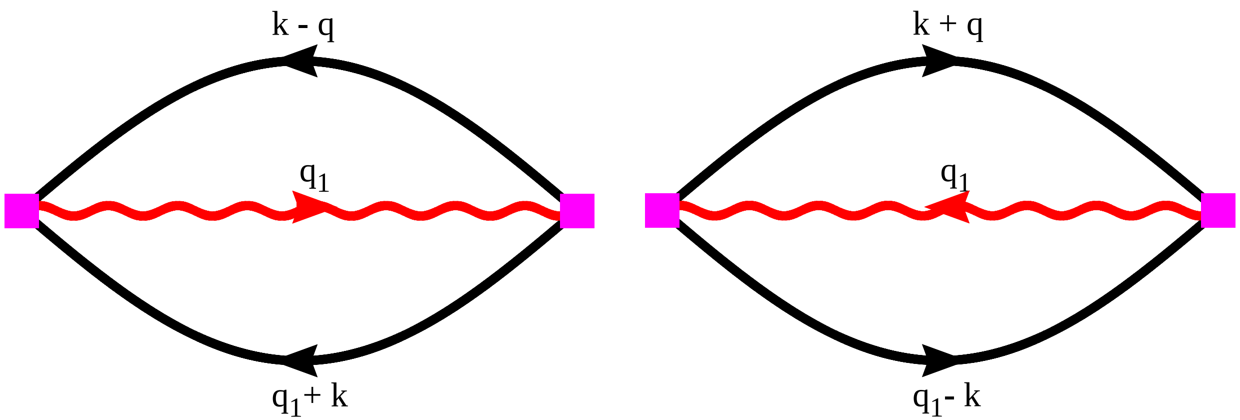
Diamagnetic response above T_c

The anomalous contributions

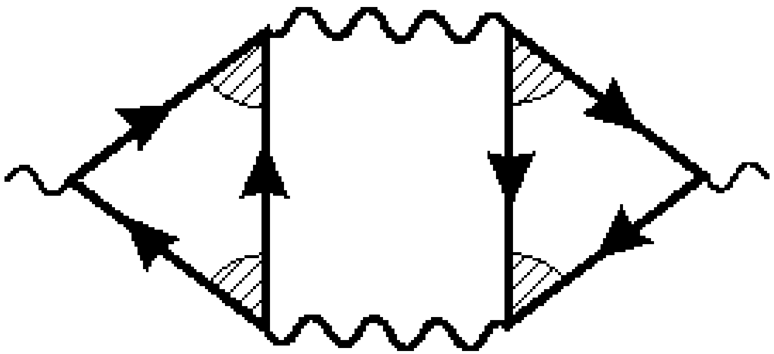


Diamagnetic response above T_c

The anomalous contributions

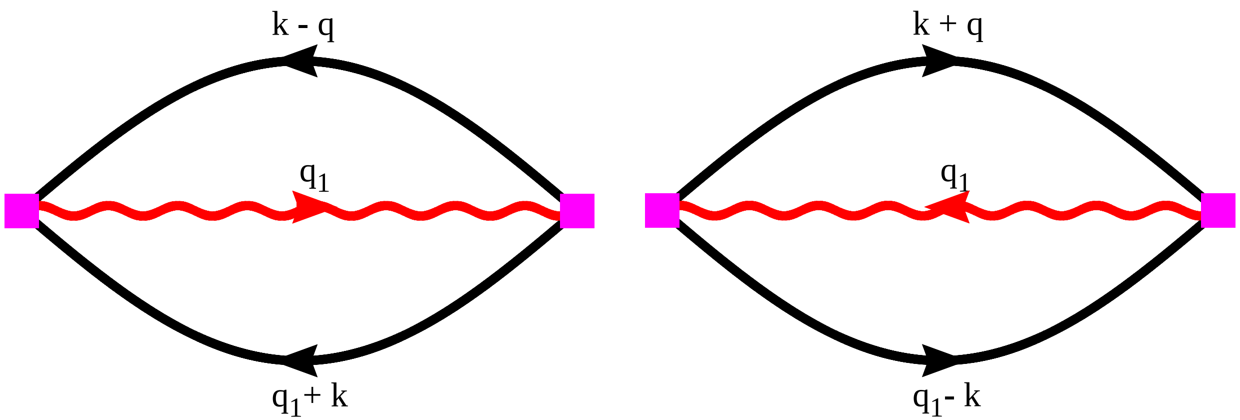


resemble the Aslamasov-Larkin diagram

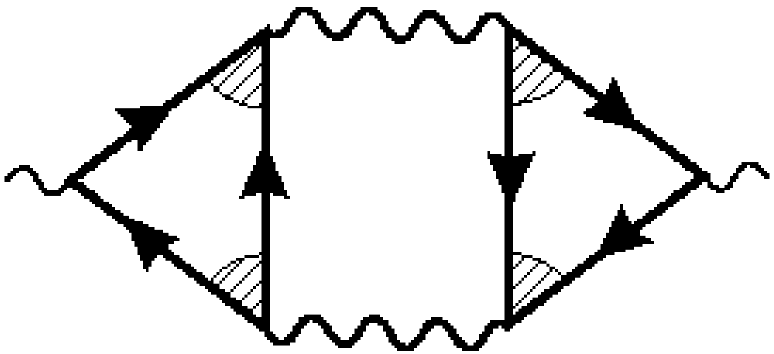


Diamagnetic response above T_c

The anomalous contributions



resemble the Aslamasov-Larkin diagram



enhancing the conductance/diamagnetism above T_c.

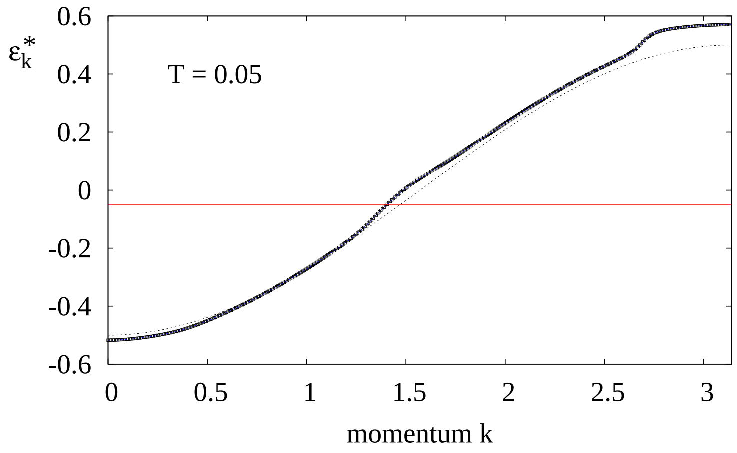
Onset of diamagnetism above T_c

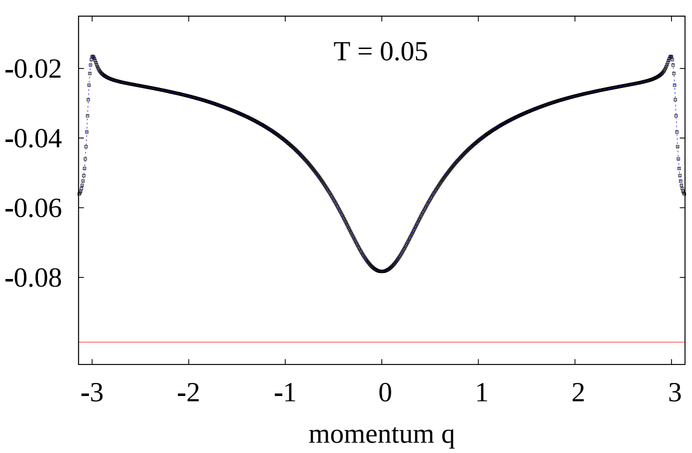
Onset of diamagnetism above T_c

Onset of the diamagnetism coincides with appearance of the collective features in the fermion/boson spectrum.

Onset of diamagnetism above T_c

Onset of the diamagnetism coincides with appearance of the collective features in the fermion/boson spectrum.



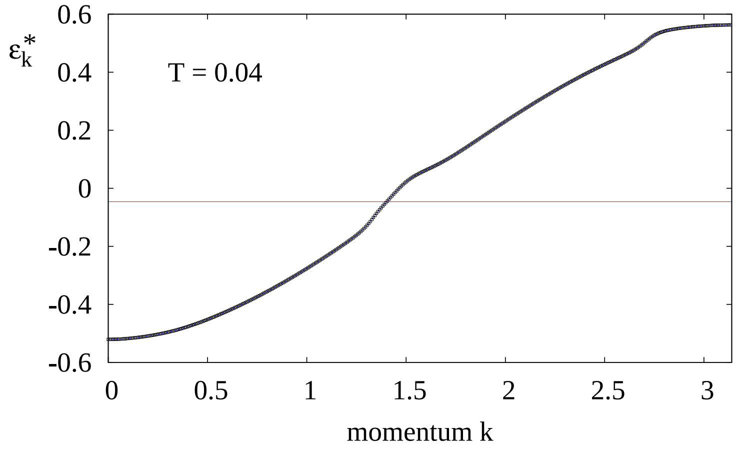


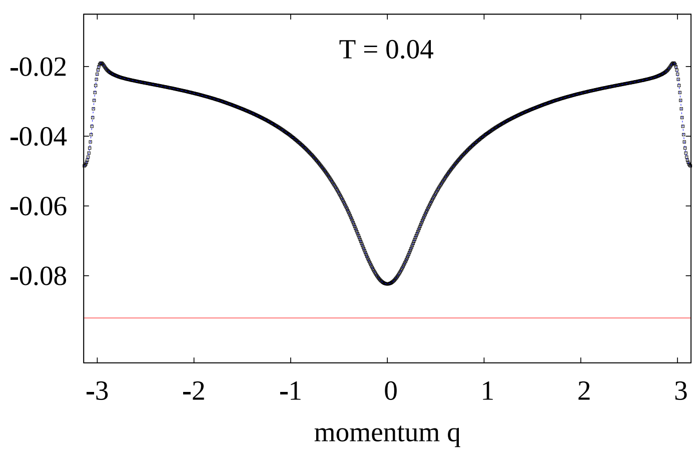
fermion dispersion

pair dispersion

Onset of diamagnetism above T_c

Onset of the diamagnetism coincides with appearance of the collective features in the fermion/boson spectrum.



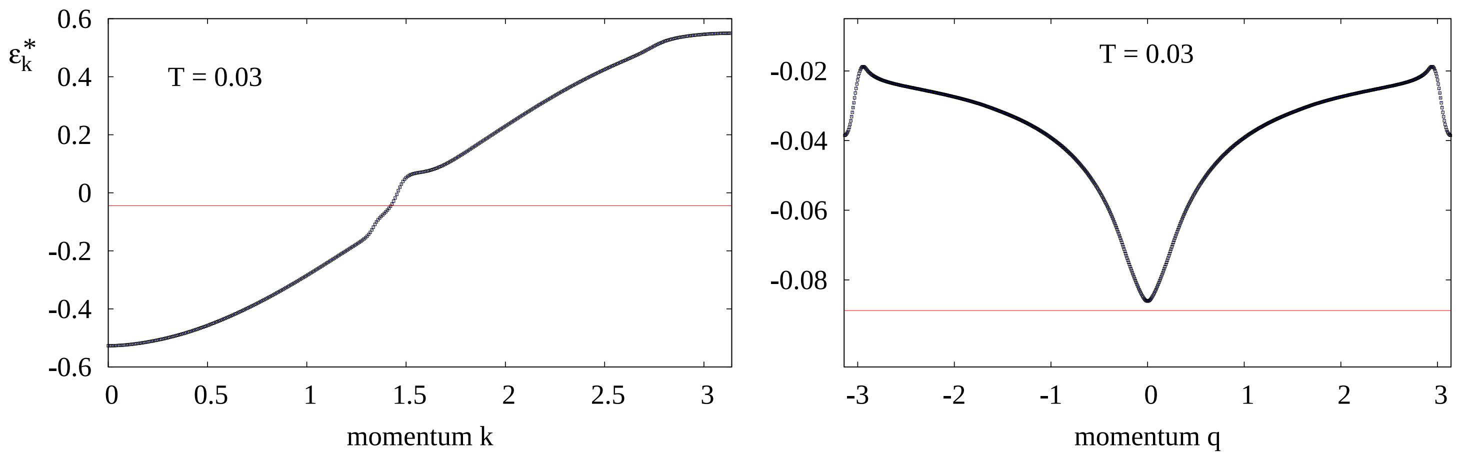


fermion dispersion

pair dispersion

Onset of diamagnetism above T_c

Onset of the diamagnetism coincides with appearance of the collective features in the fermion/boson spectrum.

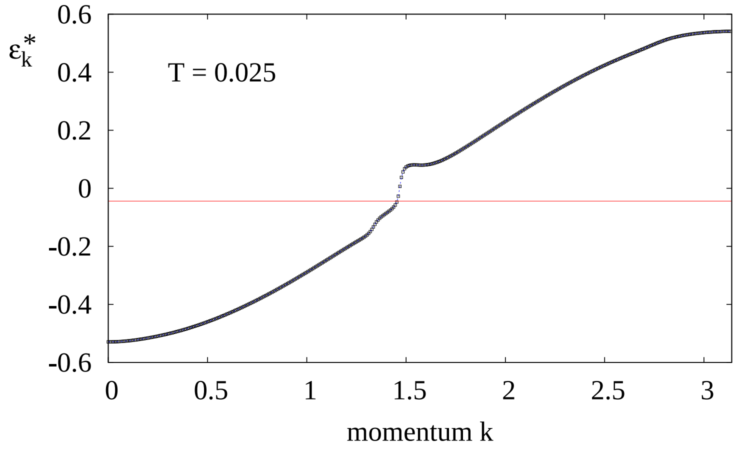


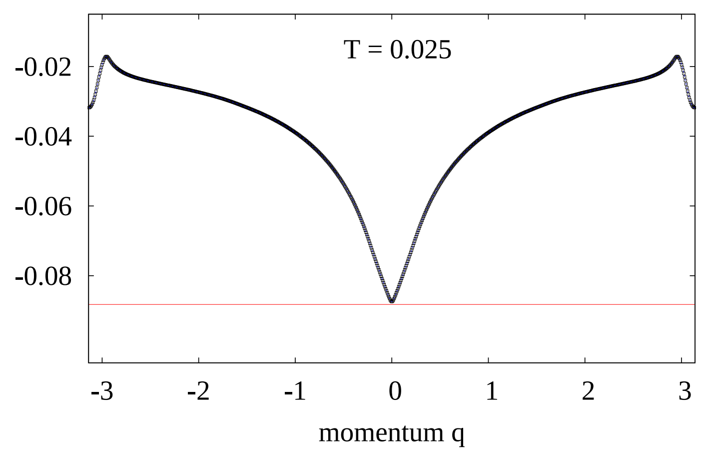
fermion dispersion

pair dispersion

Onset of diamagnetism above T_c

Onset of the diamagnetism coincides with appearance of the collective features in the fermion/boson spectrum.

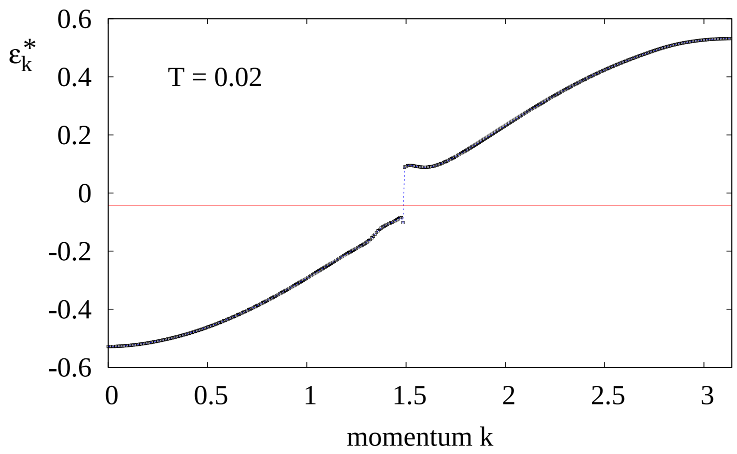




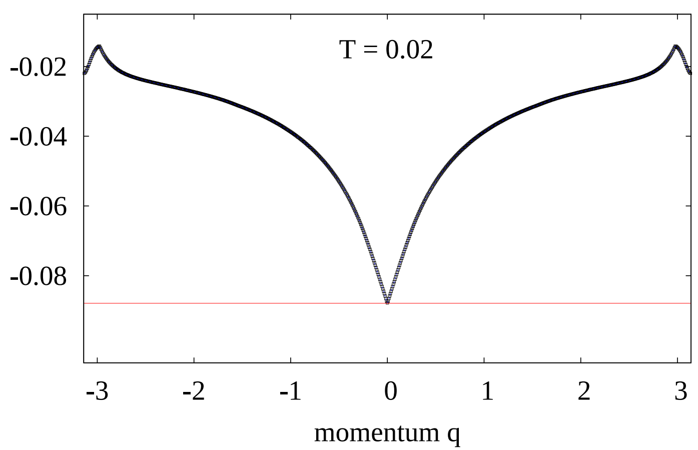
fermion dispersion **pair dispersion**

Onset of diamagnetism above T_c

Onset of the diamagnetism coincides with appearance of the collective features in the fermion/boson spectrum.



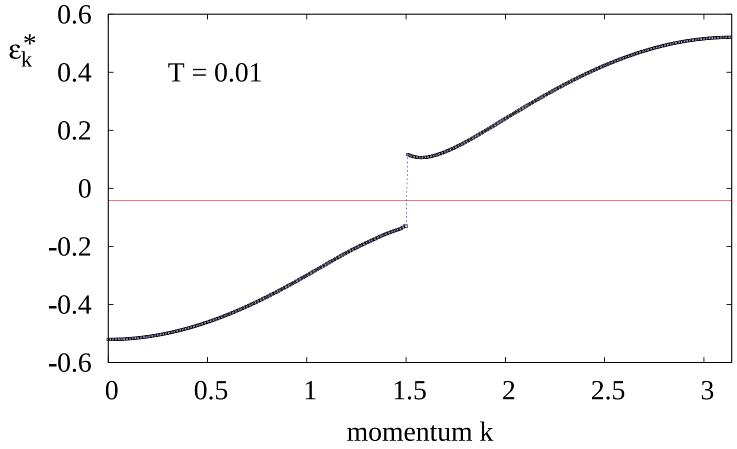
fermion dispersion



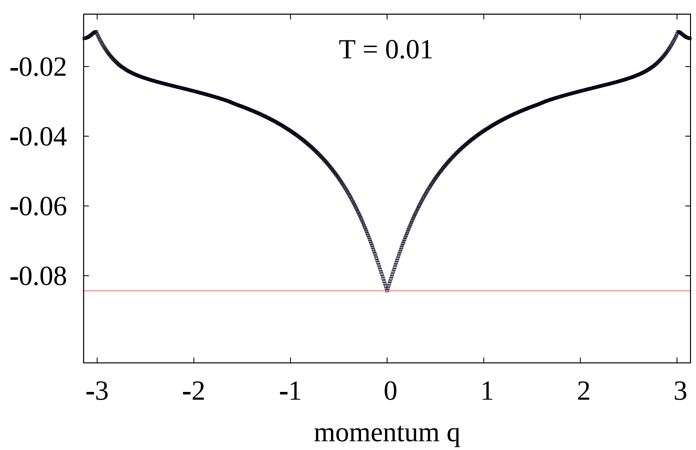
pair dispersion

Onset of diamagnetism above T_c

Onset of the diamagnetism coincides with appearance of the collective features in the fermion/boson spectrum.



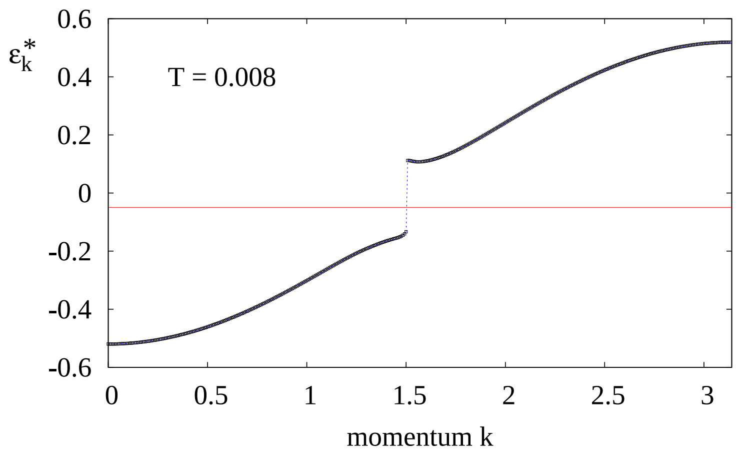
fermion dispersion

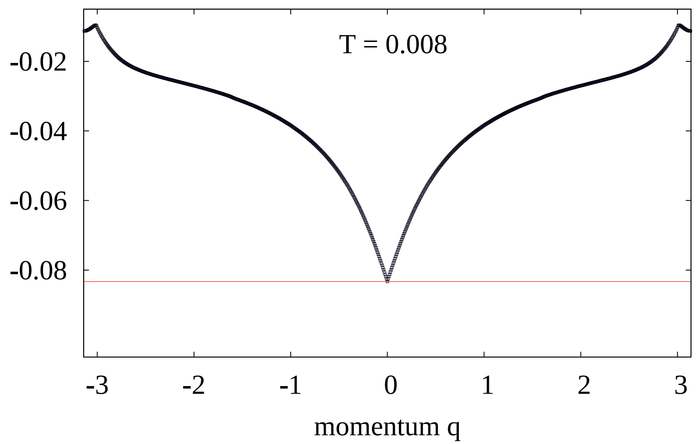


pair dispersion

Onset of diamagnetism above T_c

Onset of the diamagnetism coincides with appearance of the collective features in the fermion/boson spectrum.



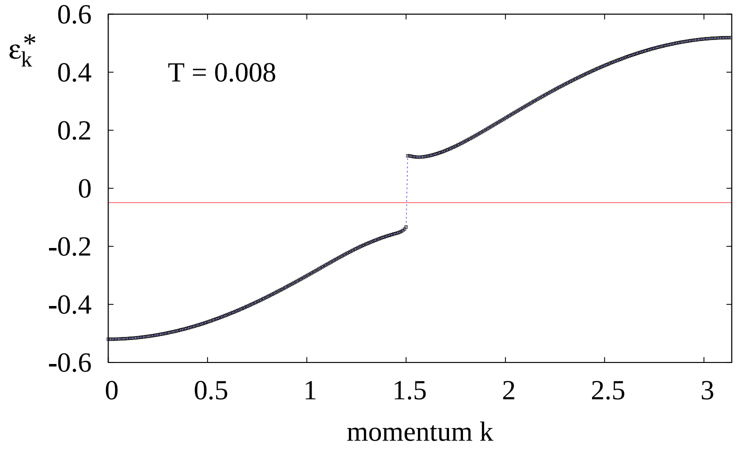


fermion dispersion

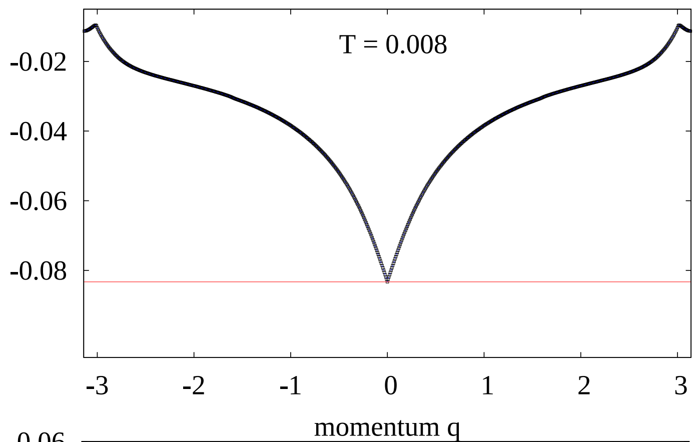
pair dispersion

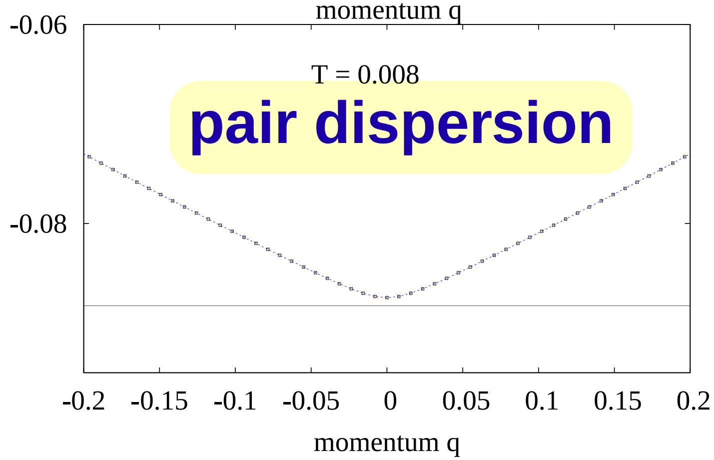
Onset of diamagnetism above T_c

Onset of the diamagnetism coincides with appearance of the collective features in the fermion/boson spectrum.



fermion dispersion





pair dispersion

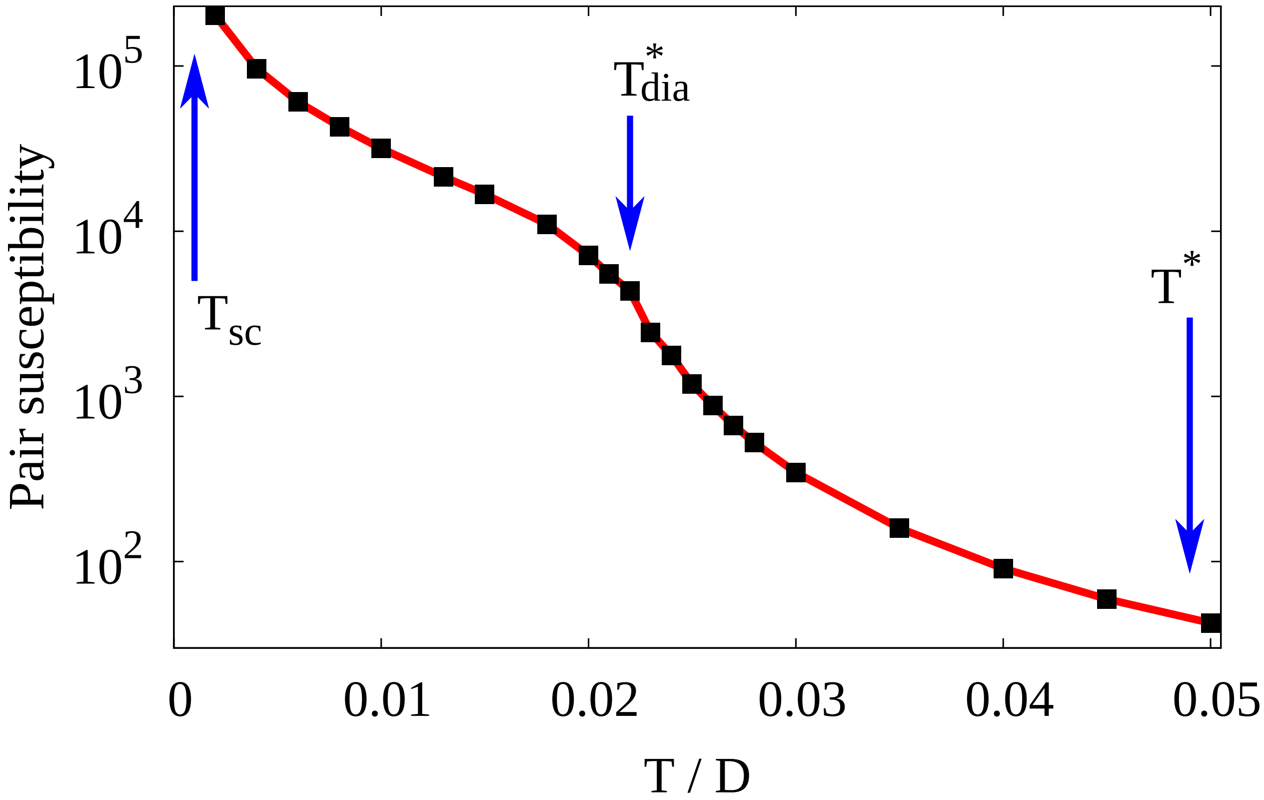
Diamagnetic response above T_c

Diamagnetic response above T_c

Residual diamagnetism originates from the collective behavior of pairs. It is closely related with increase of pair susceptibility, which is enhanced at $T^*_{\rm dia}$ and ultimately diverges at T_{sc}.

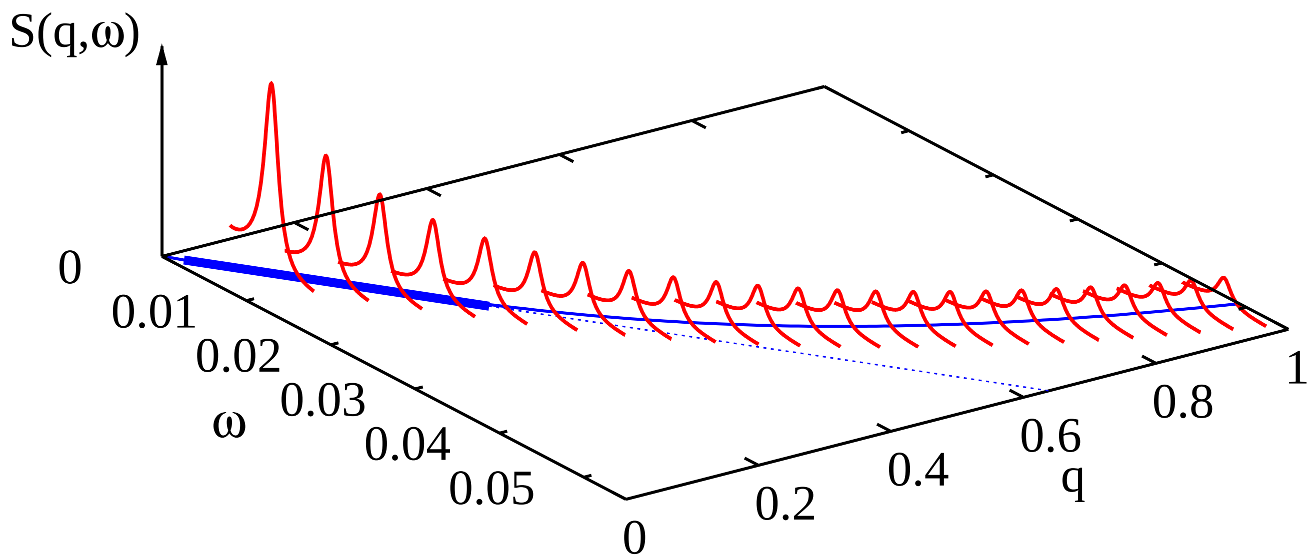
Diamagnetic response above T_c

Residual diamagnetism originates from the collective behavior of pairs. It is closely related with increase of pair susceptibility, which is enhanced at $T^*_{\rm dia}$ and ultimately diverges at T_{sc}.



Diamagnetic response above T_c

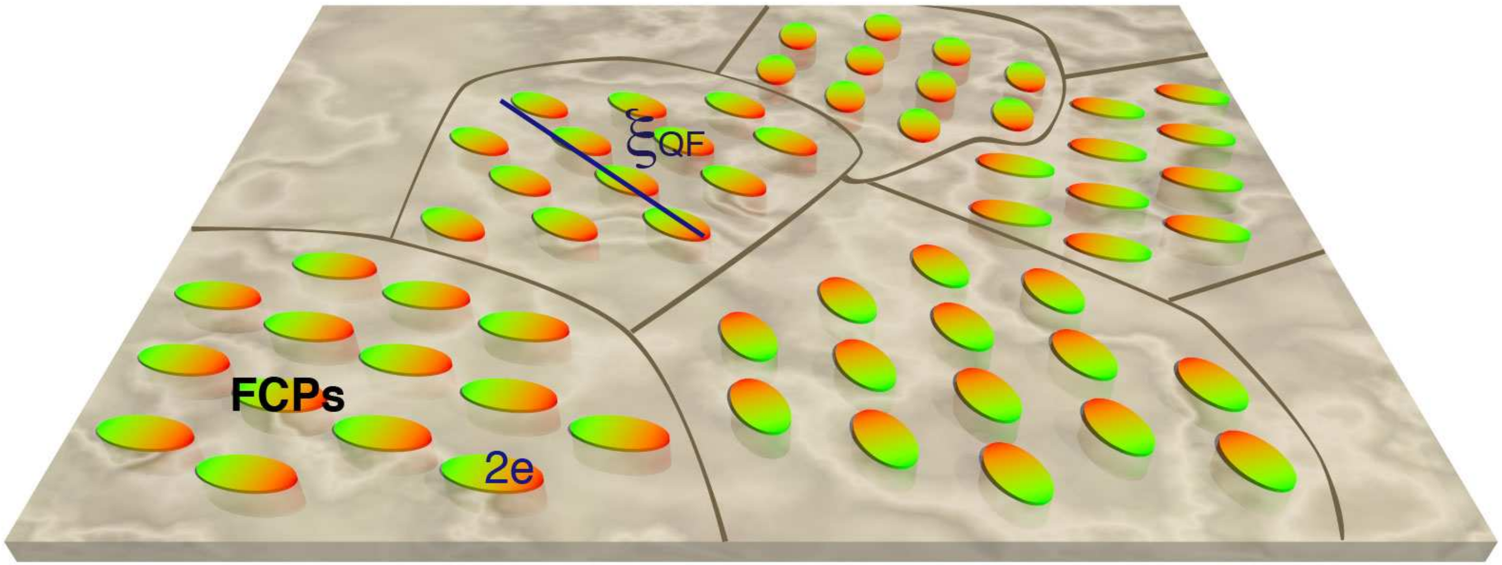
Residual diamagnetism originates from the collective behavior of pairs. It is closely related with increase of pair susceptibility, which is enhanced at $T^*_{\rm dia}$ and ultimately diverges at T_{sc}.



The structure function $\frac{1}{\omega}$ Im $\chi(q,\omega)$ showing a piece of the collective (Goldstone) branch for $q_{c1} < q < q_{c2}$.

Diamagnetic response above T_c

Residual diamagnetism originates from the collective behavior of pairs. It is closely related with increase of pair susceptibility, which is enhanced at $T^*_{\rm dia}$ and ultimately diverges at T_{sc} .



Cartoon illustrating the state of vortex liquid above T_c.

Summary

Summary

1) There is unambiguous evidence for pre-existing pairs above T_c due to

Summary

1) There is unambiguous evidence for pre-existing pairs above T_c due to

$\Longrightarrow$ **the residual diamagnetism**

/ torque magnetometry /

Summary

1) There is unambiguous evidence for pre-existing pairs above T_c due to

$\Longrightarrow$ **the residual diamagnetism**

/ torque magnetometry /

$\Longrightarrow$ **Bogoliubov quasiparticles**

/ ARPES, FT-STM, Josephson effect /

Summary

1) There is unambiguous evidence for pre-existing pairs above T_c due to

$\Longrightarrow$ **the residual diamagnetism**

/ torque magnetometry /

$\Longrightarrow$ **Bogoliubov quasiparticles**

/ ARPES, FT-STM, Josephson effect /

2) Superconducting fluctuations

Summary

1) There is unambiguous evidence for pre-existing pairs above T_c due to

$\Longrightarrow$ **the residual diamagnetism**

/ torque magnetometry /

$\Longrightarrow$ **Bogoliubov quasiparticles**

/ ARPES, FT-STM, Josephson effect /

2) Superconducting fluctuations

$\Longrightarrow$ **cause various collective features**

/ reminiscent of the sc state /

Summary

1) There is unambiguous evidence for pre-existing pairs above T_c due to

$\Longrightarrow$ **the residual diamagnetism**

/ torque magnetometry /

$\Longrightarrow$ **Bogoliubov quasiparticles**

/ ARPES, FT-STM, Josephson effect /

2) Superconducting fluctuations

$\Longrightarrow$ **cause various collective features**

/ reminiscent of the sc state /

$\Longrightarrow$ **although ODLRO is not yet established**

/ the short-range coherence /

Summary

1) There is unambiguous evidence for pre-existing pairs above T_c due to

$\Rightarrow$ **the residual diamagnetism**

/ torque magnetometry /

$\Rightarrow$ **Bogoliubov quasiparticles**

/ ARPES, FT-STM, Josephson effect /

2) Superconducting fluctuations

$\Rightarrow$ **cause various collective features**

/ reminiscent of the sc state /

$\Rightarrow$ **although ODLRO is not yet established**

/ the short-range coherence /

Thank you.