Self-averaging on annealed networks

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Self-averaging (SA) is a basic concept for systems with quenched randomness. In experiments, SA is realized by averaging over a sufficiently large sample. In theoretical description and numerical simulations, a physical property of a system is said to be SA if the relative variance of its value averaged over all disorder realisations tends to zero with an increase of systems size.

An annealed network is defined as an ensemble of all networks consisting of N nodes assigned to a given degree sequence $\{k\} = (k_1, ..., k_N)$ [1]. The linkage between nodes is taken to fluctuate for each fixed sequence, each particular linkage configuration being a realization of an annealed structural disorder. The prominent feature of an annealed network is that the partition function $\mathcal{Z}(\{k\})$ calculated for given degree sequence $\{k\}$ by averaging over random linkage does not depend on a particular choice of $\{k\}$: $\mathcal{Z}(\{k\}) = \mathcal{Z}$. This means that the free energy is a self-averaged quantity too.

In our analysis, we consider a generalized Ising model with random spin length on an annealed scale-free network and show that its free energy is a SA quantity too. We present an asymptotic analysis [2] of its thermodynamics and show that the model is characterized by a rich phase diagram with a variety of phase transitions in four different universality classes [3]. We discuss possible applications of the model considered.

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