On solving the moment master equations of population dynamics for spatially inhomogeneous systems

I. Omelyan[a] and Y. Kozitsky[b]

[a] *Institute for Condensed Matter Physics, National Academy of Sciences of Ukraine, 1 Svientsitskii Street, 79011 Lviv, Ukraine, E–mail: omelyan@icmp.lviv.ua*

[b] *Instytut Matematyki, Uniwersytet Marii Curie-Skłodowskiej, 20-031 Lublin, Poland, E–mail: jkozi@hektor.umcs.lublin.pl*

The problems of solving the master equations for population dynamics of spatially inhomogeneous birth-and-death systems with nonlocal dispersal and competition are considered. It is shown that the standard numerical methods working well at spatially homogeneous conditions are unsuitable when studying the spatial inhomogeneity beyond the mean field approximation. The account of this inhomogeneity is mandatory for the wavefront and spread dynamics, in ecological invasions, in vitro cell invasion assays, embryogenesis and wound healing, malignant tumor proliferation, etc [1]. Despite the previous achievements, very little remains known about spatial moment dynamics of inhomogeneous systems.

In order to remedy such a situation, we propose a novel approach to find the desired solutions using analytically solvable decompositions of the time evolution operator. This has allowed us to calculate the inhomogeneous pair correlation functions of birth-and-death populations within the Kirkwood superposition moment closure for the first time. Also, we revealed a number of new subtle effects, possible in real systems. Namely, for populations with short-range dispersal and mid-range competition, strong clustering of entities at small distances followed by their deep disaggregation at larger separations are observed in the wavefront of density propagation. For systems in which the competition range is much shorter than that of dispersal, the pair correlation functions exhibit a long-tail behavior. Remarkably, the latter effect takes place only due to the spatial inhomogeneity and thus was completely unknown before. Moreover, both effects get stronger in the direction of propagation. All these types of behavior are interpreted as a trade-off between the dispersal and competition in the coexistence of reproductive pair correlations and the inhomogeneity of the density of the system.

[1] M.J. Plank, R. Law, *Bull. Math. Biol.* **77**, 586 (2015).