Application of statistical physics methods to stochastic financial models

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As it is known the dynamics of financial assets, derivatives, and other indicators is described by stochastic equations. The Viner process (Brownian motion) plays the main role in financial stochastic analysis. As a rule the stochastic processes being considered are Markov processes. Hence it is sufficiently to know the conditional probability density to be able to describe characteristics of stochastic processes. Conditional probability density satisfies the FokkerPlanck equation. In dynamics of physical processes the stochastic equations are also known as Langevins equations. For imaginary time the FokkerPlanck equation is equivalent to a Schrodinger equation of a quantum-mechanical system. The method of Feynman (continuum) integrals is effectively used to solve FokkerPlanck equations and Schrodinger equations (system evolution). In method of continuum integration boundary conditions are taken into account quite effectively for propagator of a quantum system as well as for conditional probability density. Particularly, Dirichlet and Neumann boundary conditions can be taken into account by introducing additional potentials which are expressed through delta function $\delta(x)$ and its derivative $\delta'(x)$ respectively. In this article the mentioned approaches are used to model a structure of interest rate and bonds yield. Mentioned problems were solved for a number of stochastic models, Merton model, Vasicek modes and others. In Merton model the dynamics of interest rate is described by a Brownian motion with drift. Vasicek model of interest rate is an analogue to a known OrnsteinUhlenbeck model of statistic physics. However received solutions have significant drawbacks as they allow negative values of interest rate. As the result in Merton model the interest rate increases in time unlimitedly. In given approach the Dirichlet boundary condition for $r = 0$ is taken into account by introducing a potential $\delta(r)$ in continuum integral for conditional probability density, which limits the value of interest rate by domain $r > 0$. Continuum integral for conditional probability density is obtained by means of substitution in integral based on Wiener measure. As a result a time structure of interest rate was determined and a correct behavior of Merton model was obtained.